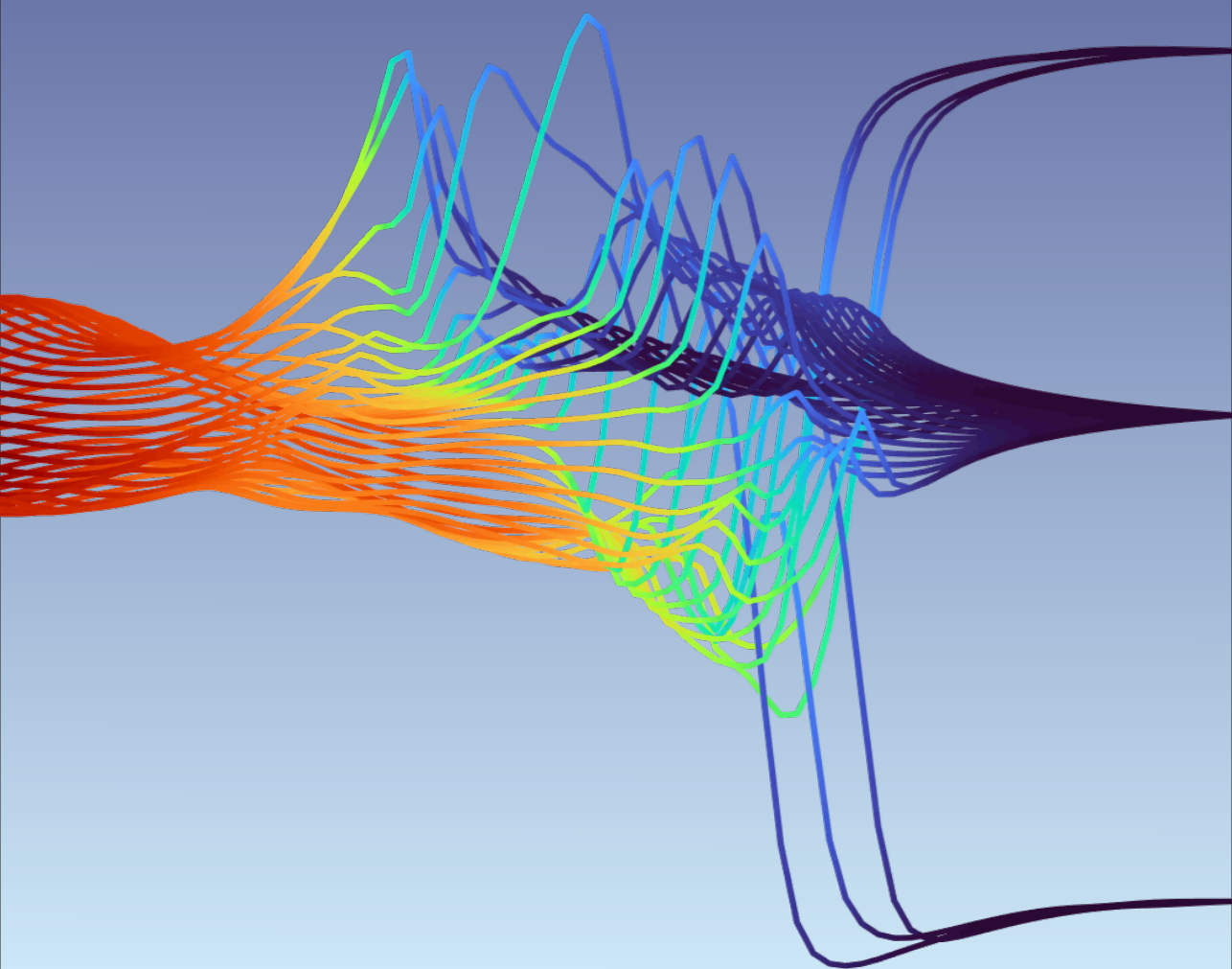


Advanced Diagnostics of Rutherford Cable Performance in Accelerator Magnets

– Current redistribution & Quench development –

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Summary

Particle colliders for High-Energy Physics research, such as the Large Hadron Collider (LHC) at CERN, require high-field magnets to operate. To generate high magnetic fields efficiently and compactly, superconducting coils carry a high current density at zero resistance. In the framework of the High Luminosity upgrade of the LHC (HL-LHC), a switch from cables with the workhorse superconductor Nb-Ti to cables with Nb₃Sn was made. Nb₃Sn allows to build magnets that can carry a high current density at higher magnetic fields, but it also brings several challenges. As opposed to the ductile Nb-Ti, Nb₃Sn is brittle, which makes it more susceptible to damage during manufacturing and operation. Nb₃Sn coils require a more complex production process including high-temperature heat treatment. And the higher magnetic field results in higher forces and mechanical stress on the conductor. Several Nb₃Sn demonstration and prototype magnets for the HL-LHC encountered performance limitations during testing. These limitations were accompanied by anomalous signals from diagnostic tools. This thesis work aims to help with the interpretation of such non-trivial signals in terms of possible root causes such as local strand damage. For this, the underlying physical phenomena on strand level within the Rutherford cable are studied with efficient numerical simulations, coupled with HL-LHC test data and dedicated experiments on a model coil.

Rutherford cables

For use in accelerator magnets, the superconducting wires, which consist of twisted Nb-Ti or Nb₃Sn filaments embedded in a copper matrix, are first assembled into a Rutherford-type of cable. A Rutherford cable is characterized by a bundle of twisted wires two-axes rolled into a high-aspect-ratio trapezoidal shape with two layers, which makes it suitable for the winding of $\cos\theta$ -type of magnets. The cable strands have a certain amount of mutual thermal and electro-magnetic coupling. This inter-strand coupling, which depends on the contact area and surface conditions of the strands, strongly influences the transient electric and thermal-electric behavior of the Rutherford cable.

Numerical tools

The two numerical simulation codes used in this thesis work are THEA and RuNe. THEA is an existing code for the thermal, hydraulic and electric analysis of superconducting cables and is used to study current redistribution effects over long length - and time scales. For localized transient effects, a higher spatial and temporal resolution is required, rendering THEA computationally less efficient. Therefore, in the framework of this thesis RuNe (Rutherford Network) was developed, a new 3D thermal-electric PEEC-FEM model that efficiently simulates quench dynamics, but is also still able to study current redistribution effects on a larger scale.

Experimental

This thesis aims to extract more information from the two main diagnostic tools used in accelerator magnets under test: voltage taps and quench antennas. By combining voltage taps with differential measurement schemes, the sub-microvolt precision that is needed to reliably detect the small voltages caused by conductor damage was achieved, even for 5.5 m long magnets developing sizeable inductive voltages during ramping or picking up significant environmental EM noise. Quench antennas consisting of arrays of pick-up coils that provide detailed additional insight into the current redistribution effects especially during the initial development phase of a quench, but also during its steady-state propagation. Systematic comparison between model predictions and test results showed the quench antenna signals to carry information regarding the detailed location of the initial hot spot; the presence and importance of possible damage-related non-uniformity of the current distribution; and thermal/electrical inter-strand resistances.

Moreover, a dedicated experiment was performed on a Short Model Coil (SMC), a two-layer racetrack magnet. Intentional damage was introduced in gradual stages, by locally interrupting an increasing number of strands over a series of tests. Strategically placed voltage taps were introduced to monitor the current redistribution process around the defect in detail. This controlled experiment provided a varied range of reference data confirming the model predictions and validating its accuracy.

Current redistribution

In a Rutherford cable with a local defect involving one or a few broken or degraded strands, current will bypass the defect through the other strands. Current redistributes mainly through strands adjacent to the defect rather than through crossing strands, because the former have a higher mutual inductance and lower electrical contact resistance with the affected strand(s). When the current in an adjacent strand is then driven to its critical value, that strand becomes saturated and the excess current redistributes through the next-neighboring strand. This effect proceeds in a cascading sequence of saturating strands.

Such current redistribution can be seen as a diffusion process governed by the values of the self - and mutual inductances of the strands and their electrical inter-strand contact resistances. The length over which current redistributes increases with time. This means that the current bypasses the defect through an increasing number of parallel resistances, since the non-affected strands themselves have zero resistance. The main physics of this process can be captured with a 1D analytical diffusion model, the predictions of which were reproduced with THEA, which allows to relate it to cable-specific parameters. In actual magnet tests, this type of local damage-induced current redistribution manifests itself as a resistive voltage recorded during a plateau in the current ramp, which decays gradually with time.

This type of decaying voltage was measured on several HL-LHC model - and prototype magnets with performance limitations and is thus indeed a clear indicator of local conductor damage in the cable. The decaying voltage can appear on current plateaus several kA below a magnet's quench limit and is therefore a useful predictor of magnet performance.

With the current redistribution process around the intentionally introduced defect in the SMC magnet monitored with the extra voltage taps, RuNe's model predictions could be confirmed in more detail. The local inter-strand contact parameters of the Rutherford cable in the coil were successfully estimated from comparison between model and experiment. Somewhat surprisingly, it was observed that the SMC magnet can reliably reach its operating current with as many as four strands adjacent to a defect fully saturated. This illustrates the importance of testing magnets all the way to the short-sample limit in quality assurance and acceptance tests.

Quench development and normal zone propagation

The evolution of a normal zone in a Rutherford cable is divided in two phases: the development phase and the propagation phase. During the development phase, the normal zone is still growing transversely in the cable until eventually all strands in the cable lateral cross-section have turned normal, at which point the propagation phase starts. These two phases can be distinguished in the experimental voltage data recorded during a quench by an accelerating voltage during the development phase, followed by a kink and then a linear voltage rise during the propagation phase. When electro-thermal coupling between strands is stronger the development phase is shorter.

Using harmonic quench antenna data in combination with RuNe simulations, quench origins can be reconstructed for both conductor-limited quenches as well as training quenches, but this is a computationally intensive analysis. A faster and easier method using the time integral of the magnetic dipole moment was therefore also developed for finding the quench start location in the cable cross-section of conductor-limited quenches directly from the quench antenna data.

Different normal zone propagation modes were studied using the model RuNe. Previously poorly understood anomalous measurements from an 11 T dipole magnet could be likely identified as due to a defect-dominated propagation mode, whereby the normal zone front propagates along overloaded strands due to the presence of a local defect. This propagation mode was later also observed in the SMC experiment, further validating the model and illustrating how the information contained in the diagnostic data can help identify the detailed root cause of magnet issues.

In summary, the unique combination of the model RuNe and the advanced diagnostic tools allowed to describe the earliest stages of a quench in Rutherford cable based accelerator magnets in much more detail than possible before.

Samenvatting (in Dutch)

Deeltjesversnellers voor hoge energie fysica, zoals de Large Hadron Collider (LHC) bij CERN, hebben hoogveld magneten nodig om te functioneren. Om een hoog magnetisch veld efficiënt en in een compacte ruimte te genereren, worden supergeleidende spoelen gebruikt die een hoge stroomdichtheid dragen met nul elektrische weerstand. In het kader van de High Luminosity upgrade van de LHC (HL-LHC), wordt een overstap gemaakt van Nb-Ti geleider naar Nb₃Sn. Met Nb₃Sn kunnen magneten gebouwd worden die een hoge stroomdichtheid kunnen dragen bij hogere velden, maar het brengt ook enkele uitdagingen met zich mee: in tegenstelling tot Nb-Ti, dat buigzaam is, is Nb₃Sn een bros materiaal, wat het gevoeliger maakt voor schade die kan ontstaan tijdens fabricage of operatie. Nb₃Sn spoelen hebben een complexer productieproces met een hoge temperatuur warmtebehandeling en het hogere magneetveld resulteert in hogere krachten en mechanische spanning op de geleider. Verschillende Nb₃Sn prototype magneten voor de HL-LHC liepen tegen prestatielimieten aan tijdens tests. Naast deze limitaties werden ook onverwachte signalen met diagnostische apparatuur opgevangen. Dit proefschrift heeft als doel de interpretatie van dit soort niet-triviale signalen te verbeteren met betrekking tot de onderliggende oorzaak zoals lokale schade aan draden in de kabel (ook wel strands genoemd). Met dit doel worden de onderliggende natuurkundige fenomenen bestudeerd op strandniveau in de Rutherford kabel met efficiënte numerieke modellen in combinatie met HL-LHC meetresultaten alsook een experiment op een modelspoel.

Rutherford kabels

Voor gebruik in versneller magneten worden de supergeleidende draden, die bestaan uit getwiste Nb-Ti of Nb₃Sn filamenten ingebed in een koperen matrix, eerst gebundeld in een Rutherford-type kabel. Een Rutherford kabel bestaat uit een bundel van getwiste draden die worden gevormd tot een trapezoidale vorm met een hoge aspect ratio bestaand uit twee lagen, welke het geschikt maakt voor het winden van $\cos\theta$ type magneten. De strands in de kabel hebben een bepaalde mate van elektromagnetische en thermische koppeling met elkaar. Deze koppeling, die afhangt van de oppervlakt condities van de strands, heeft een sterke invloed op het tijdsafhankelijke elektrische en thermo-elektrische gedrag van de kabel.

Numeriek

De twee numerieke simulatie codes die in dit proefschrift worden gebruikt zijn THEA en RuNe. THEA is een bestaande code voor thermische, hydraulische en elektrische analyse van supergeleidende kabels en wordt gebruikt voor het bestuderen van stroomherverdelingsprocessen over grote lengte - en tijdschalen. Voor lokale tijdsafhankelijke processen, is een hogere ruimtelijke en tijd resolutie noodzakelijk, welke THEA rekentechnisch minder efficiënt maakt. Daarom is in het kader van dit proefschrift RuNe (Rutherford Network) ontwikkeld, een nieuw 3D thermo-elektrisch PEEC-FEM model dat efficiënt quench dynamica kan simuleren, maar ook nog steeds stroomherverdelingsprocessen op grotere schaal.

Experimenteel

Dit proefschrift heeft als doel om meer informatie te halen uit de twee belangrijkste diagnostische tools voor versneller magneten: spanningscontacten en quench antenna's. Door spanningscontacten te combineren met differentiële meettechnieken kan de submicrovolt precisie die nodig is om geleiderschade te kunnen detecteren worden behaald, zelfs voor 5,5 m lange magneten die een aanzienlijk inductieve spanningscomponent hebben en ook gevoelig zijn voor elektromagnetische ruis vanuit de omgeving. Quench antenna's, bestaande uit een reeks pick-up spoelen, bleken in staat te zijn om extra inzicht te geven in de stroomherverdelingsprocessen tijdens de initiële quench ontwikkeling, maar ook tijdens de steady-state propagatie. Systematische vergelijkingen tussen modelvoorspellingen en meetresultaten toonden aan dat de quench antenna signalen informatie bevatten over de precieze locatie van de initiële hot spot; de aanwezigheid van een eventueel inhomogene stroomverdeling door toedoen van geleiderschade; en de thermische/elektrische inter-strand contact weerstanden.

Daarnaast is een experiment uitgevoerd op een Short Model Coil (SMC), een racetrack magneet bestaand uit twee lagen. Stapsgewijs is met opzet schade toegebracht aan de geleider, door lokaal een toenemend aantal strands te onderbreken gedurende een reeks experimenten. Met strategisch geplaatste spanningscontacten rondom het aangebrachte defect werd het stroomherverdelingsproces in detail gemonitord. Dit gecontroleerde experiment resulteerde in een gevarieerde set aan referentiedata dat modelvoorspellingen valideerde.

Stroomherverdeling

In een Rutherford kabel met een lokaal defect aan één of enkele strands zal stroom zich herverdelen door de andere strands. Stroom zal zich voornamelijk door de naastliggende - en niet de kruisende strands herverdelen, omdat eerstgenoemde een hogere mutuele inductie alsook een lagere contact weerstand met de getroffen strands hebben. Wanneer de stroom in een naastliggende strand dan zijn kritische waarde bereikt, zal de strand gesatureerd raken en de extra stroom doorgeven aan

de volgende naastliggende strand. Dit process zet zich voort in een cascade van saturerende strands.

Dit soort van stroomherverdeling kan worden gezien als een diffusieproces dat bepaald wordt door de waardes van de zelf - en mutuele inductie van de strands alsmede hun onderlinge elektrische contactweerstand. De lengte waarover stroom zich herverdeelt neemt toe met de tijd. Dit betekent dat de stroom het defect ontwijkt door een toenemend aantal parallelle weerstanden, aangezien de niet aangestaste strands zelf nul weerstand hebben. De belangrijkste fysica van dit process kan worden beschreven met een één-dimensionaal analytisch diffusie model, waarvan de resultaten met THEA zijn gereproduceerd en welke ons in staat stelt het gedrag te relateren aan kabelspecifieke parameters. In de werkelijke magneetttests leidt dit type van stroomherverdeling door toedoen van lokale schade tot een voltage gemeten op een plateau na het opregelen van de stroom, welke geleidelijk afneemt met de tijd.

Dit type van afnemend voltage is gemeten op verschillende HL-LHC model - en prototype magneten met gelimiteerde prestaties en is dus een duidelijke indicator van lokale schade aan de geleider in de kabel. Het afnemende voltage kan zich voordoen op stroomplateaus enkele kA onder de quenchstroom van de magneet en is daarom een bruikbare voorspeller van magneetprestaties.

Door het stroomherverdelingsproces rondom het met opzet aangebrachte defect in de SMC magneet te monitoren met de extra spanningscontacten, konden de modelvoorspellingen van RuNe in meer detail worden bevestigd. De lokale waardes van de inter-strand contact weerstanden konden worden geschat door middel van het vergelijken van model en experiment. Redelijk onverwacht werd vastgesteld dat de SMC magneet herhaardelijk in staat was om zijn operationele stroom te behalen met vier naastgelegen gesatureerde strands. Dit laat zien hoe belangrijk het is om een magneet helemaal tot zijn short-sample limiet te testen tijdens kwaliteits- en acceptatietests.

Quench ontwikkeling en normale zone propagatie

De tijdsevolutie van een normale zone in een Rutherford kabel is verdeeld in twee fases: de ontwikkelingsfase en de propagatiefase. Tijdens de ontwikkelingsfase is de normale zone zich nog transversaal aan het uitbreiden totdat op een gegeven moment alle strands in de dwarsdoorsnede van de kabel normaal zijn, waarna de propagatiefase van start gaat. Deze twee fases kunnen worden onderscheiden in experimentele spanningswaardes door een accelererende spanning tijdens de ontwikkeling gevolgd door een kink en dan een lineaire toename tijdens de propagatiefase. Wanneer de thermisch/elektrische koppeling tussen strands groot is, is de ontwikkelingsfase korter.

Met behulp van meetdata van een harmonische quench antenna in combinatie met RuNe simulaties, kunnen quench startcondities worden gereconstrueerd voor

zowel geleidergelimiteerde quenches alsook trainingquenches. Een snelle en makkelijke methode die gebruik maakt van het geïntegreerde dipoolmoment is ontwikkeld voor het bepalen van de quenchstartpositie van een geleidergelimiteerde quench.

Verschillende types van normale zone propagatie zijn bestudeerd met het model RuNe. Voorheen niet goed begrepen onverwachte metingen aan 11 T dipoolmagneten konden met grote waarschijnlijkheid verklaard worden door een defectgedomineerde propagatiemodus waarbij het front van de normale zone propageert langs strands die zijn gesatureerd door een lokaal defect. Deze propagatiemodus is later ook waargenomen tijdens het SMC experiment, welke het model verder heeft gevalideerd en heeft laten zien hoe de informatie die zich bevindt in diagnostische data kan worden gebruikt voor het indentificeren van de onderliggende oorzaak van prestatieproblemen in magneten.

Tot slot kunnen we concluderen dat de unieke combinatie van het model RuNe en de geavanceerde diagnostische technieken ons in staat stellen de vroegste stadia van een quench in op Rutherford kabels gebaseerde versneller-magneten in meer detail te beschrijven dan voorheen.

Contents

Acknowledgements	ix
Summary	xi
Samenvatting (in Dutch)	xv
1 Introduction	1
1.1 The Large Hadron Collider	1
1.2 Superconductivity in accelerator magnets	3
1.2.1 Nb-Ti and Nb ₃ Sn	4
1.2.2 Superconducting wires and stability	6
1.2.3 Rutherford cables	8
1.3 Accelerator magnets	9
1.3.1 Superconductor quench	11
1.3.2 Magnet training	12
1.4 Magnet test facility - SM18	13
1.5 Advanced performance diagnostics	13
1.5.1 Numerical analysis tools	14
1.6 Scope and structure of the thesis	15
2 Numerical techniques	17
2.1 Introduction	17
2.2 Rutherford cable geometry	18
2.3 THEA simulation code	21
2.3.1 Governing equations	21
2.3.2 Continuum model of the Thermal and Electrical network	22
2.3.3 Inductance matrix	24
2.3.4 Assumptions in THEA	25
2.4 RuNe - Rutherford Network simulation code	26
2.4.1 Electrodynamic domain	27
2.4.2 Superconducting-to-normal transition	28
2.4.3 Partial inductance calculation	30
2.4.4 Computational considerations	35
2.4.5 Singular Value Decomposition and Low Rank Approximation	36
2.4.6 Hierarchical matrices	37

2.4.7	Fast inductive voltage computation for periodic quasi-1D structures	38
2.4.8	Self-field calculation	41
2.4.9	Thermal domain	42
2.4.10	Coupling between domains	46
2.4.11	Solving strategy	47
2.4.12	Convergence, numerical stability and accuracy	49
2.5	Quench antenna calculation	51
2.5.1	Superconductor magnetization	52
2.5.2	Example calculation	53
2.6	Conclusion	54
3	Instrumentation and experimental techniques	57
3.1	Voltage tap measurements	57
3.1.1	Sources of voltages during magnet operation	59
3.1.2	Quench voltage	61
3.2	Quench antennas	61
3.2.1	Principle	61
3.2.2	Local quench antenna	62
3.2.3	Harmonic quench antenna	64
3.2.4	Transients during magnet ramp	66
3.3	Controlled damage experiment	68
3.3.1	Coil SMC 11T #1	70
3.4	Instrumentation	71
3.4.1	Voltage tap positioning	72
3.4.2	Local defect	76
3.5	Conclusion	76
4	Current redistribution in damaged Rutherford cables	77
4.1	Types of cable defects	77
4.2	Simulation of a local defect in a Rutherford cable	78
4.2.1	Simulation parameters	79
4.2.2	Current redistribution during a current ramp	81
4.2.3	Current distribution during a current plateau	83
4.2.4	Strand voltage during a current plateau	85
4.2.5	Effect of a ramp-down in current	87
4.2.6	Dependence on ramp rate and temperature	88
4.3	Parametric studies	91
4.3.1	Voltage over defect	91
4.3.2	Length-scale of the current redistribution	92
4.3.3	Effective time constants	93
4.4	Analytical diffusion model	95
4.4.1	Step-like ramps	96
4.4.2	Slow current ramps	99
4.5	Comparison between Network Model (RuNe) and Continuum Model (THEA)	102

4.6	Effect of inter-strand contact resistances and damage distribution .	103
4.6.1	Dependence of the current distribution and power dissipation on inter-strand contact resistances	104
4.6.2	Effect of damage distribution and core design	105
4.7	Controlled damage experiment	111
4.7.1	Ramp-rate and temperature variations	111
4.7.2	Ramp sequence	114
4.7.3	Comparison with RuNe	114
4.7.4	V-I measurements during ramp	122
4.7.5	Observation of current cascade	122
4.7.6	Collateral strand damage	126
4.7.7	Voltage build-up at 1.9 K	129
4.7.8	Voltage landscape	132
4.7.9	Strand saturation and stability	133
4.7.10	Effect on Magneto-Thermal Instabilities	134
4.7.11	Detecting strand saturation	134
4.8	Conclusion	135
5	Quench development and normal zone propagation	137
5.1	Introduction	137
5.1.1	Phases of a quench: development and propagation	137
5.2	Quench development	139
5.2.1	Quench diagnostic measurement results (in MQXF)	139
5.2.2	Comparison with RuNe simulation results	141
5.2.3	Influence of the quench's start position on magnetic dipole moment	149
5.2.4	Influence of superconductor magnetization	152
5.2.5	Training quench development	154
5.3	Normal zone propagation in Rutherford cables	160
5.3.1	Normal zone propagation in cables with a uniform current distribution	160
5.3.2	Normal zone propagation in cables with a non-uniform current distribution	164
5.4	Quench analysis of an SMC coil with introduced defect	177
5.4.1	Voltage build-up	178
5.4.2	Normal zone propagation velocity	179
5.4.3	Quench location	180
5.4.4	Comparison with RuNe	181
5.5	Conclusion	185
6	Conclusion	187
6.1	Numerical techniques	187
6.2	Experimental techniques	188
6.3	Current redistribution in damaged Rutherford cables	189
6.4	Quench development and normal zone propagation	191
6.4.1	Quench development	192

6.4.2	Normal zone propagation	193
6.5	Concluding remarks	194
Bibliography		195
Nomenclature		209

Chapter 1

Introduction

1.1 The Large Hadron Collider

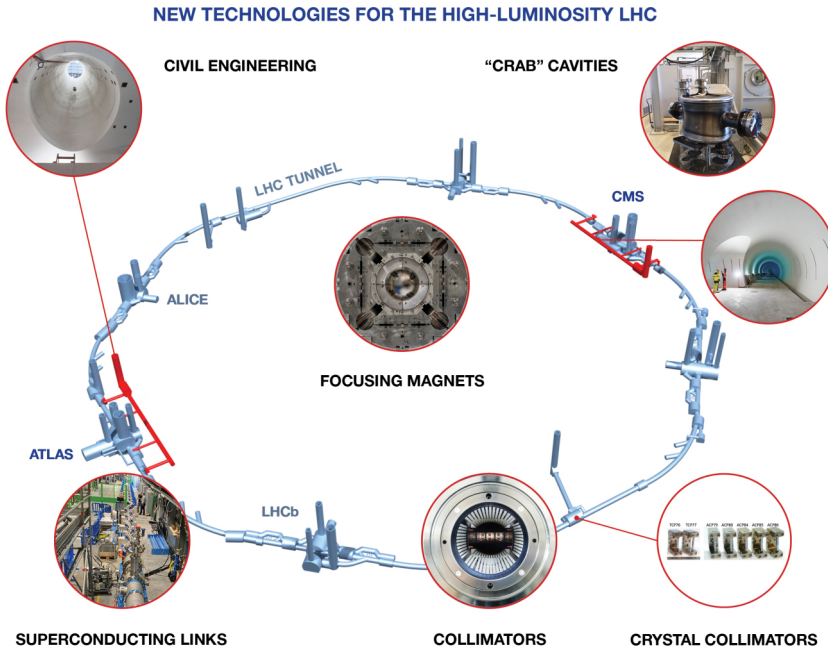
In High-Energy Physics, fundamental particles and their interactions through three of the four fundamental forces are studied. Over many decades of research this has led to the Standard Model of Particle Physics [1]. To study particle interactions, very high energy regimes need to be probed. This is achieved by bringing into collision relativistic charged particles like electrons, protons and heavy ions, after they have been accelerated almost to the speed of light using particle accelerators.

In 2008, the world's largest particle accelerator, the Large Hadron Collider (LHC), started operation at CERN. It was constructed in a circular tunnel with a length of 27 km, approximately 100 m underground, at the France-Switzerland border near Geneva [2].

The LHC is a particle accelerator of the synchrotron type. Protons or heavy ion bunches are injected in both directions into ultra-high vacuum beam pipes that form a ring, where they are gradually accelerated to relativistic speed with Radio-Frequency (RF) cavities [3]. The particle bunches are guided around the circular tunnel by 1232 15 m-long dipole magnets and are focussed by 392 quadrupole magnets, as well as a large number of higher-order orbit corrector magnets [4, 5]. The magnets are placed all around the circumference of the ring, with exception of the short straight sections where the collisions occur. The magnetic field is gradually ramped up, synchronized to the particle energy.

At four interaction points along LHC's tunnel, four main physics experiments, ATLAS, CMS, ALICE and LHCb are placed [6, 7, 8, 9], see Figure 1.1. At these locations, the two opposing beams cross paths and the particles interact with each other, producing high-energy collisions. The collision events produce showers of sub-atomic particles, which are tracked and identified based on their path's curvature as they pass through the detector's magnetic field. From the reconstructed particle trajectories, different types of physics hypotheses about sub-atomic par-

ticle interactions can be tested. The LHC produces so many collisions that it is impractical to store all gathered data. Therefore a trigger system is in place that quickly evaluates the collision data for potentially interesting interactions and discards the rest. The selection criteria are fine-tuned to the physics goals of the experiment.



CERN February 2024

Figure 1.1: Overview of the HL-LHC tunnel and its four experiments ATLAS, ALICE, CMS and LHCb. New technology developments for the HL-LHC are indicated. Image from [16].

Compared to previous particle accelerators, such as Tevatron and LEP, the LHC is able to probe an entirely new physics regime up to 14 TeV [10, 11]. Its most famous discovery, that of the Higgs-boson in 2012, was only possible because of this access to a higher energy regime [12, 13]. The particle beam energy E_b (TeV) is related to the magnetic field in the bore of the dipole magnets, B_d (T) and to the radius of the accelerator, r_a (km), following [14]:

$$E_b \simeq 0.3 B_d r_a. \quad (1.1)$$

The LHC was able to reach such high energies because of its larger radius, about 4 km, and because of the application of liquid superfluid helium cooled su-

perconducting magnets operating at 1.9 K, with a nominal dipole field of 8.3 T.

Besides the energy of the particles, the discovery potential of a particle accelerator is also related to the luminosity L_b ($\text{cm}^{-2} \text{s}^{-1}$) of the particle beam, which determines the probability of particle collisions. In High-Energy Physics, discoveries of particles or measurements of their properties are performed by analyzing sizeable amounts of collision data. It can take a long time before enough data is captured regarding a certain interaction to achieve the necessary statistical likelihood to claim a discovery. With a higher luminosity, interactions happen more often and the necessary amount of data can be gathered faster.

The research in this thesis work was performed in the framework of the High-Luminosity Large Hadron Collider (HL-LHC), which is an upgrade of the LHC that intends to increase the luminosity by a factor of 10 from its original design value. To achieve this, a number of new technologies needed to be incorporated in the LHC, in particular much stronger beam-focussing magnets in front of the collision points to achieve the smaller beam size, see Figure 1.1. In this thesis, the focus will be on new superconducting magnet technology that had to be developed, using Nb_3Sn superconductors for the first time in operational accelerator magnets [15]. More specifically, this work seeks to advance the diagnostics into possible performance limitations of these new magnets.

1.2 Superconductivity in accelerator magnets

To achieve the high current densities that are required in high-field accelerator magnets, it is not practical to use normal conductor windings of copper, due to excessive ohmic heating and electric power requirements. Therefore, the accelerator magnets of the LHC use superconducting cables, which carry a stationary current with practically zero resistance.

Superconductors can be classified into two types. Many pure metals, such as mercury, are so-called Type I superconductors. When cooled below a critical temperature T_c , they become superconducting. Besides having zero DC electrical resistance, the material then also excludes any applied magnetic flux from its interior by creating a high surface current that perfectly cancels the magnetic field in its bulk, a phenomenon which is called the Meissner-Ochsenfeld effect [17], see Figure 1.2. However, if the magnetic field is increased above a critical value B_c , superconductivity is lost. B_c is typically well below 0.1 T, making Type I superconductors unsuitable for high-field magnets.

All technical superconductors used in high-field magnets are therefore Type II materials, which are typically compounds of different elements, such as Nb-Ti, Nb_3Sn as well as $\text{ReBa}_2\text{Cu}_3\text{O}_7$ and $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$. Besides a first critical field B_{c1} , below which they also show the Meissner-Ochsenfeld effect, Type II supercon-

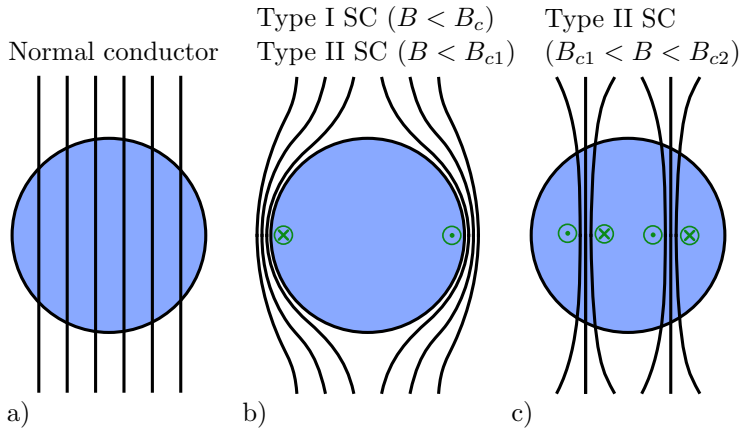


Figure 1.2: a) Magnetic field lines penetrate a normal conductor unimpeded. b) Type I and II superconductors shield their interior from external magnetic fields below $B < B_{c(1)}$. c) In addition, Type II superconductors allow partial magnetic flux penetration in the form of quantized flux lines for $B_{c1} < B < B_{c2}$.

ductors also have a second or upper critical field B_{c2} , which is much higher than B_{c1} . For magnetic field levels in-between B_{c1} and B_{c2} , the magnetic field partially enters the material in the form of quantized flux lines forming the so-called Abrikosov state [21], see Figure 1.2c. Each flux line comprises a single quantum of flux, $\Phi_0 = \frac{h}{2e}$, and penetrates the material in the form of a normal core surrounded by a circulating supercurrent. It is energetically more favorable for these flux lines to coincide with pre-existing disorder in the crystal structure, such as an impurity, lattice dislocation or grain boundary. This phenomenon is called flux pinning and superconducting materials for high-field magnets are designed to maximize the so-called pinning force, because it increases the critical current density of the superconductor, the maximum current it can carry before it reverts to a resistive state. In the presence of a transport current the vortices need to be able to sustain a density gradient, since $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, which will resort in a force on the flux lines due to the mutual repulsion between them. The presence of pinning centers counter-acts this force. Once the current density gets too high the flux lines get de-pinned and start moving, which dissipates energy, causing a so-called flux flow resistance.

1.2.1 Nb-Ti and Nb₃Sn

Currently, the LHC is equipped with magnets using Nb-Ti superconducting cables. For many years, Nb-Ti has stood as the ‘workhorse’ of the superconducting accelerator magnet community. Besides accelerator - and other types of high-field research magnets, Nb-Ti is also used extensively in NMR and MRI, which presently accounts for 80 % of superconducting magnets worldwide. Nb-Ti is a ductile alloy, which makes it easy to handle, and can be used in practice up to magnetic fields

of about 10 T, although in accelerator magnets this limit is reduced to about 8 T due to the high current density required. The Nb-Ti cables in LHC magnets are cooled with superfluid helium to a temperature of 1.9 K.

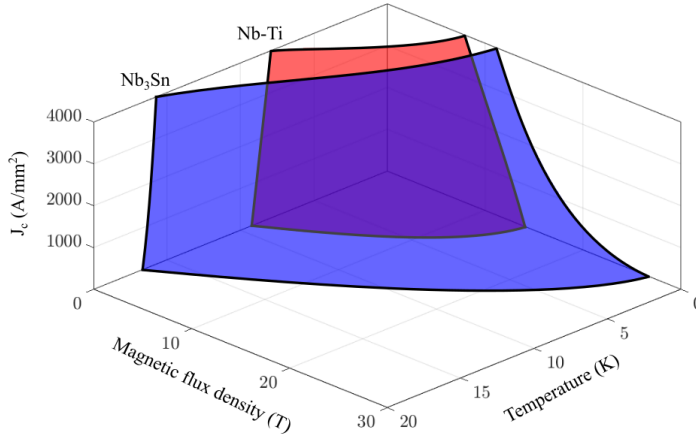


Figure 1.3: Critical current density as a function of temperature and magnetic flux density for Nb-Ti and Nb₃Sn.

For the HL-LHC, higher magnetic fields are needed in a few dipole magnets and especially in the final focussing quadrupole magnets. So for these magnets a shift in technology was made to Nb₃Sn, which has higher performance limits than Nb-Ti. The limits of a superconductor are described by its critical surface, which is a function of magnetic field, temperature, current density and strain. Figure 1.3 shows the critical surfaces of both Nb-Ti and Nb₃Sn in zero-strain state, showcasing the performance advantage of using Nb₃Sn. Nb₃Sn is a brittle inter-metallic compound that can be used in accelerator type magnets to about 16 T. Compared to Nb-Ti, the brittle nature of Nb₃Sn poses additional engineering challenges, since high-field accelerator magnets in operation can experience significant mechanical stress of about 150 MPa and the critical surface of Nb₃Sn also depends on its strain state. In addition, differences in the thermal expansion coefficient of the various materials in the magnet can introduce extra stress during cooling cycles. Furthermore, the brittle Nb₃Sn strands cannot be wound directly: cabling and magnet winding are completed with strands that contain precursors, after which the full magnet undergoes a heat treatment during which the Nb₃Sn is formed, a process that is further discussed in Section 1.2.2.

In the 1980's, another class of superconductors, the so-called cuprates or copper-oxides, were discovered. These materials, compounds of copper-oxide and rare-earth elements $\text{ReBa}_2\text{Cu}_3\text{O}_7$ or bismuth $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$, retain their supercon-

ducting state up to much higher temperatures, in the range of 90 K to 110 K, causing them to be named High-Temperature Superconductors (HTS) [22, 23, 24]. In addition to their high T_c -value, they also retain their superconducting properties at significantly higher magnetic field. Despite these obvious advantages, HTS materials have not yet found their way into operational accelerator magnets, mainly due to technological challenges related to their structure and form factor.

1.2.2 Superconducting wires and stability

For thermal stability, the superconducting material in a composite superconducting wire is embedded in a matrix with low electrical and thermal resistance, such as copper or aluminium. A superconductor in its normal state is a poor conductor, combined with a high current density, a modest local disruption of the superconducting state may cause an excessive amount of heating, which might even melt the conductor shortly thereafter. By shunting the superconductor with a good conductor, it can briefly sustain a local disruption of the superconducting state, as long as the normal zone is not too large. Superconductor stability is quantified by the concept of the Minimum Propagation Zone (MPZ) [18]. This is the minimum length of conductor that needs to revert to the normal state for this normal zone to propagate in a self-sustaining process instead of shrinking again and disappearing.

Within a typical superconducting wire, Nb_3Sn is embedded in a metal matrix, typically copper, in the form of relatively thin filaments, as seen in Figures 1.4a and b, which shows a so-called 192 PIT wire and a 108/127 OST RRP wire [20]. The numbering scheme indicates the number of locations occupied by the filaments in a hexagonal packing configuration. In the case of RRP wire, 19 filaments in the center are replaced by copper. The filaments need to have a small effective diameter to minimize the occurrence of so-called flux-jumps, which are a type of magneto-thermal instability. During a flux-jump, a local thermal fluctuation triggers flux flow, in which flux lines get de-pinned. This moving flux dissipates energy, leading to more flux flow in a positive feedback loop, so that a large amount of flux gets redistributed at once. The flux-jump stability of a wire is determined by its heat capacity, the copper resistivity and the stored energy in the persistent filament currents. In Nb-Ti and in bronze-type Nb_3Sn , filaments have a sufficiently small diameter of about 5 μm , but in high-current density Nb_3Sn wires they have a diameter close to about 50 μm due to the different manufacturing process. The Nb_3Sn magnets for the HL-LHC therefore still experience significant flux-jumps at low to intermediate field levels. These do not directly affect the magnet performance, but they do require modifications of the quench detection scheme, as discussed in Section 1.3.1. The filaments in the wires are twisted with a pitch of the order of 10 mm to minimize coupling currents between them in dynamic situations such as a magnet ramp.

Different types of manufacturing processes for Nb_3Sn wires exist, but for the high- J_c strands required for accelerator magnets the Powder-In-Tube (PIT) and

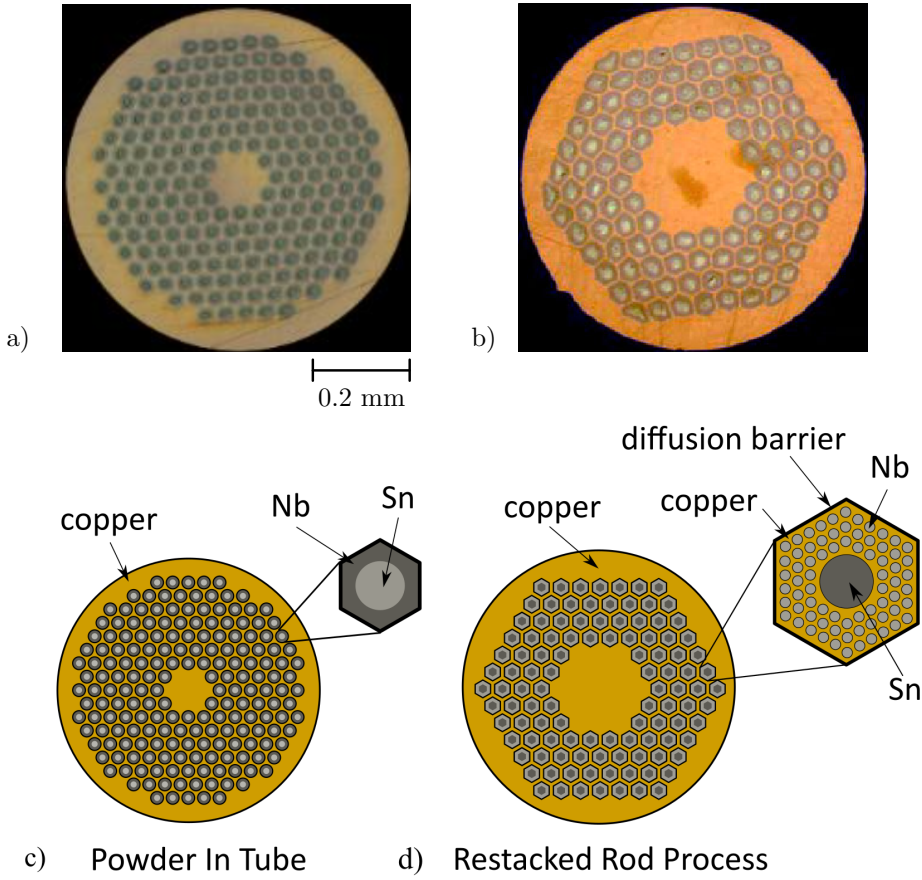


Figure 1.4: Cross-sections of PIT 192 (a) and 108/127 OST RRP (b) Nb_3Sn wires, image from [19]. The billet lay-out used for Powder-In-Tube (c) and Restacked Rod Process (d) type of wires, image adapted from [30].

the Restacked Rod Process (RRP), shown in Figure 1.4, are most common [25, 26, 27, 28, 29, 30]. These technologies result in a J_c value in the range of 2500 A mm^{-2} to 3000 A mm^{-2} at 12 T and 4.2 K. A PIT wire is formed from hollow Nb tubes filled with a Sn-rich powder, which are then stacked in a matrix tube and reduced by wire drawing. RRP wire is fabricated with an extra extrusion step, where first the sub-elements, consisting of a rod of Sn surrounded by rods of Nb in a copper structure, are extruded. The resulting rods are then restacked in a new copper matrix followed by a second extrusion step. The resulting thick wire is drawn down in several steps to arrive at the final wire diameter of the order of a mm or less. The magnets for the HL-LHC discussed in this thesis all use RRP type of Nb_3Sn wire.

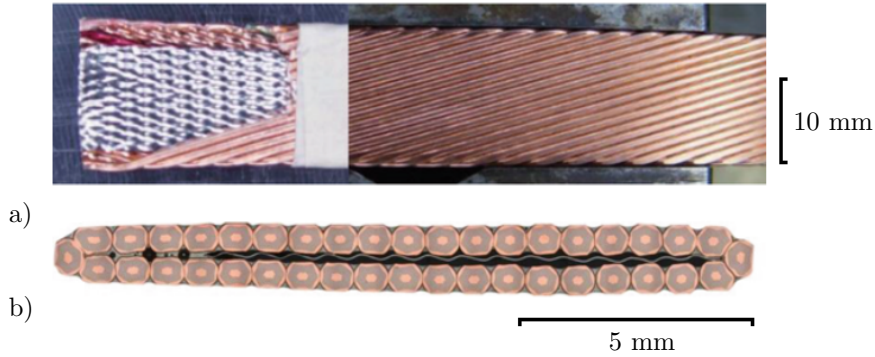


Figure 1.5: a) Top view of a Rutherford cable, on the left with some strands cut to reveal the stainless steel central core. b) Cross-section of a Rutherford cable. The stainless steel core is visible between the top and bottom layer. A slight ‘keystone’ angle is present to facilitate accommodation in the curved winding pack of a magnet, causing the edge on the left to be thinner than the one on the right. Image from [33].

1.2.3 Rutherford cables

In the accelerator magnets of the LHC, so-called Rutherford cables are used [31]. A Rutherford cable consists of a bundle of wires, called strands, that are twisted and two-axes rolled into a near-rectangular shape, see Figure 1.5. A slight keystone angle is introduced, making one short edge slightly thinner, yielding a trapezoidal shape that fits more naturally into the arching cable stack of a $\cos\theta$ winding layout, as presented in Section 1.3. The Rutherford cable configuration also allows for a high packing factor and good mechanical stability. By twisting the strands, a more even current distribution is achieved. This is vital, since between the superconducting strands, large coupling currents can be induced due to a lack of electrical resistance in the strands themselves and to the large magnetic field-sweep rates experienced by the cables. Another factor that strongly influences these coupling currents is the inter-strand contact resistance. The Nb-Ti wires in cables of the LHC were Sn-Ag coated to increase the contact resistance between strands, which reduces inter-strand coupling currents to acceptable levels [32]. In Nb₃Sn cables for the HL-LHC, such a coating cannot be used due to the required heat treatment at some 650 °C. Instead, a stainless steel ribbon with a thickness of 25 μm and a width of 12 mm is inserted as a highly resistive core between the two layers of the cable. The Rutherford cable is covered with a fiber-glass sleeve to electrically insulate it from neighboring turns in the winding pack. The cable in the 11 T dipole magnets also has a C-shaped glass-mica layer as extra insulation to improve dielectric strength.

1.3 Accelerator magnets

The dipole magnets in a particle accelerator need to produce a high-quality dipole magnetic field in their bore. A perfect dipole field in a circular aperture is achieved by a surface current density around its circumference that has a $\cos\theta$ distribution, as shown in Figure 1.6a. The surface current density K on the circumference is equal to:

$$K(\theta) = K_0 \cos(n\theta), \quad (1.2)$$

with $n = 1$ for a dipole field. Higher-order field components, such as quadrupoles or sextupoles, shown in Figures 1.6b and c, are then achieved by increasing the periodicity with $n = 2$ and $n = 3$, respectively. Another way of achieving a mathematically perfect magnetic field distribution, but this time with a finite conductor cross-section in a close-to circular aperture, is by overlapping circles or ellipses of opposing uniform volumetric current density J , as shown in Figure 1.7.

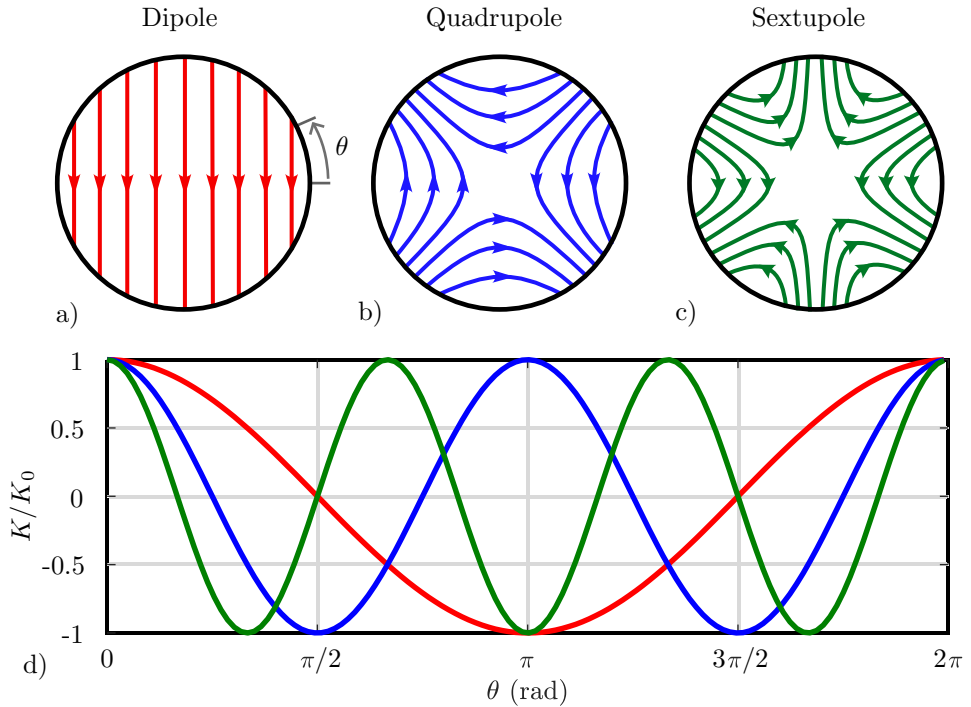


Figure 1.6: Different magnetic field distributions that can be achieved with a $\cos(n\theta)$ current distribution (d) around the circumference of a circle, for $n = 1, 2$, and 3 , respectively shown in (a), (b), and (c).

In a practical magnet, such mathematically perfect current distributions can never be achieved, due to the finite cross-sectional space that the cable needs to occupy in order to carry the sizeable currents that are necessary to achieve suffi-

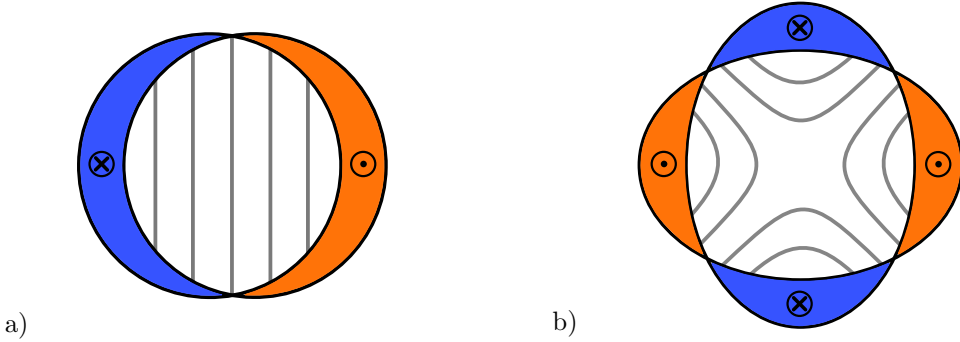


Figure 1.7: Dipole and quadrupole magnetic field distributions can also be achieved with overlapping circles (a) and ellipses (b) of opposing uniform current densities.

cient magnetic field strength. The cable's rated current also has to be sufficiently large so as to not require an excessive number of turns and thus keep the magnet's self-inductance sufficiently low. Figure 1.8 shows the cross-section of the winding pack of an 11 T dipole coil that was originally slated to be used in the HL-LHC, showing the individual turns of the Rutherford cable. The cable is placed in such a way as to approximate the perfect cosine-theta distribution, with a maximal current density near the mid-plane and zero current density at the poles.

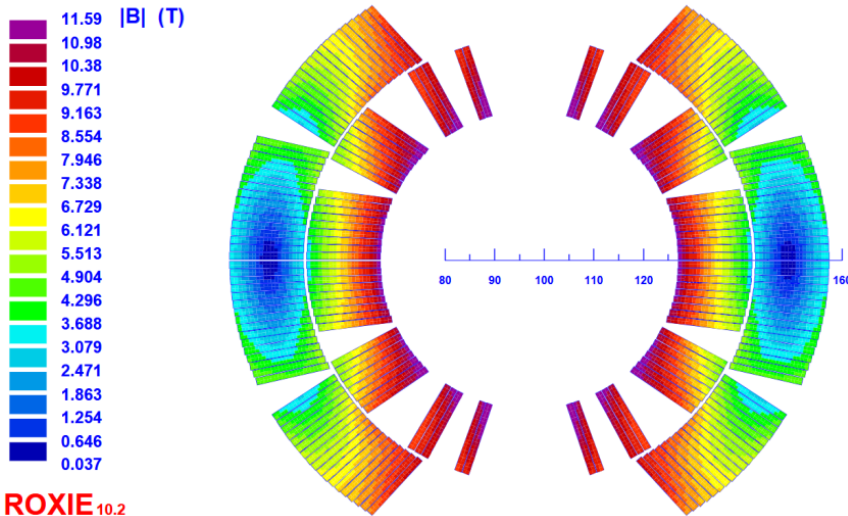


Figure 1.8: Cross-sectional view of the conductor lay-out in the coil of one aperture of an 11 T dipole magnet. The color scale represents the magnetic flux density experienced by the conductor at a nominal current of 11.85 kA. Image produced with ROXIE software.

In Figure 1.9 the cross-section of a so-called MQXF quadrupole magnet is

shown, including mechanical support structure and cryostat. MQXF magnets employ the so-called ‘bladder-and-key’ system, where steel bladders pressurized with water are used to open up space in order to insert keys in strategic locations to balance the stress in the magnet. This way, an almost even pre-compression on the coil turns can be achieved. An iron yoke around the coils contains the stray magnetic field while at the same time increasing the magnetic field strength in the bore by about 1 T for a given number of ampere-turns.

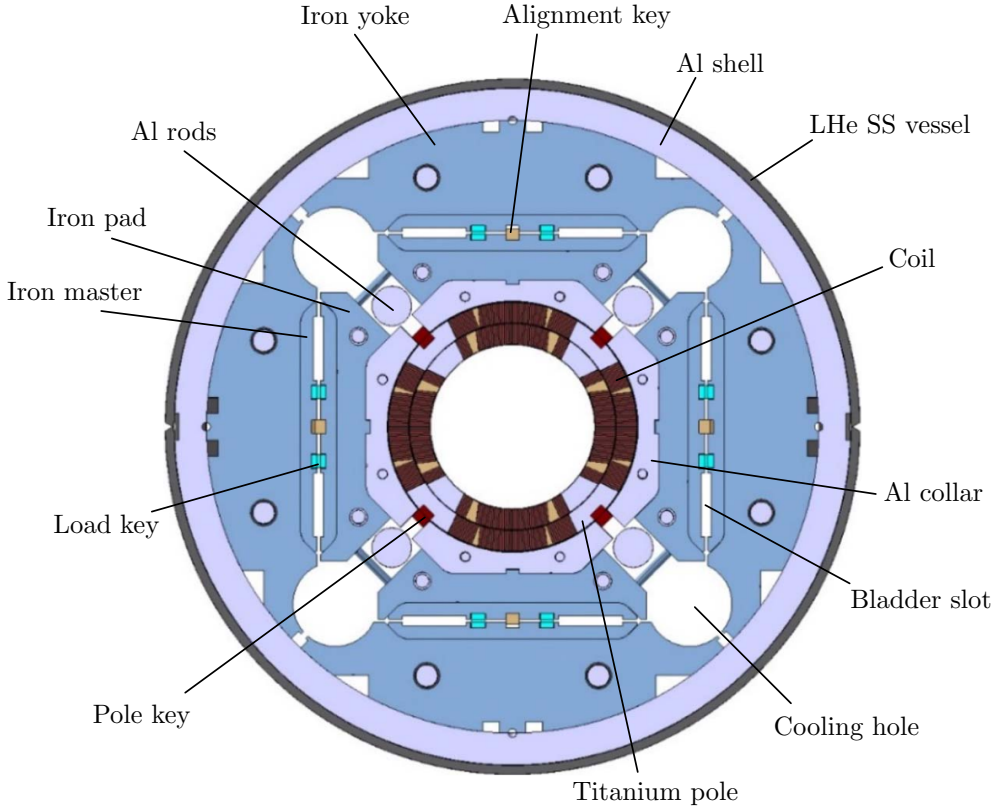


Figure 1.9: Cross-section of an MQXF quadrupole magnet, the mechanical support structure with the bladder-and-key technology is shown. Image from [20].

1.3.1 Superconductor quench

When the operational parameters of a superconducting cable locally exceed its critical surface, usually due to temperature caused by a heat load either from an external source or from a flux-jump instability, a normal zone forms that may keep expanding. Such a normal zone will then eventually propagate throughout the entire magnet. This process is called a quench: a sudden, unexpected and unrecoverable transition from the superconducting state to the normal state. Fully

excited superconducting accelerator magnets store a sizeable amount of energy. If no action is taken when detecting a quench, most of the stored energy will be dissipated resistively at the quench location, leading to a hot spot with a sufficiently high temperature to damage the cable. Therefore, all superconducting magnets need a protection scheme that limits this hot-spot temperature to a safe level.

Once a quench is detected, usually by monitoring the voltage over the coils, the current in the magnet must be brought down as quickly as possible. Classical methods for protecting a magnet are so-called quench heaters and dump resistors. Quench heaters are resistors in thermal contact with the winding pack that heat up the coils of the magnet to make sure the entire magnet reverts to the normal state within a short time. This prevents all the stored energy from being dumped into one single hot-spot, but instead spreads it out over the entire cold mass. A dump resistor is a bulky resistor that is placed outside the cryostat in parallel with the magnet. In the case of a quench, the power supply is disconnected from the magnet and the current is forced through the dump resistor. The current then quickly decays as the magnet's stored energy is dissipated in this resistor at room temperature.

Recently, an alternative method for quench protection, called Coupling Loss Induced Quench (CLIQ), was introduced [35, 36, 37, 38]. A fast current oscillation is applied superimposed on the transport current by means of a discharging capacitor. This alternating current and thus magnetic field induces sizeable coupling currents between the filaments in the strands, effectively heating up the coils from the inside [39]. MQXF magnets for the HL-LHC employ a redundant scheme of both CLIQ and quench heaters [40].

1.3.2 Magnet training

When superconducting magnets are tested for the first time, they often quench during the first ramp well before they reach their nominal current. Then, during the next excitation ramp, they usually quench at a slightly higher current. This cycle continues until the quench current reaches a plateau in a process called magnet training. It is a result of the settling of the conductor. Slip-stick motion between interfaces in the winding pack or cracking of the magnet-impregnant, usually epoxy, as the conductor moves to a slightly different position due to the large Lorentz force, may deposit a sufficiently large heat load to quench the magnet. Depending on the type of magnet, the number of training quenches can be quite large. This makes the training process expensive, both in time and in cost of helium. A magnet is said to have 'good memory' if after a thermal cycle it immediately reaches its previous maximum current again. In a particle accelerator with several thousands of magnets, such training behavior is highly undesirable and needs to be minimized.

1.4 Magnet test facility - SM18

Most Nb₃Sn magnets for the HL-LHC are tested prior to installation at the cryogenic test facility SM18 at CERN. Long magnets such as the MQXF quadrupoles and the 11 T dipole magnets are tested in a horizontal test configuration, see Figure 1.10, while shorter model magnets are tested in vertical test stations. The facility allows to test the magnet with super-fluid helium at 1.9 K, as well as liquid helium at a temperature of 4.5 K, with an operating pressure of 1.3 bar.



Figure 1.10: *Horizontal test stations of the SM18 cryogenic magnet test facility.*

1.5 Advanced performance diagnostics

During the test campaign of the new 11 T dipole - and MQXF quadrupole magnets, performance limitations were encountered. Some of the magnets were not able to reach their nominal design current, or showed degradation after thermal cycling. An example of the training behaviour of an 11 T dipole magnet is shown in Figure 1.11. During the first cool-down cycle the magnet performed acceptable, reaching its nominal current after only a few training quenches. But after a thermal cycle, the magnet quenched well short of its nominal current of 11.85 kA. In addition, anomalous diagnostic signals, such as non-zero voltages on current plateaus, were measured over some of the coils, indicating an issue with the superconductor. Unexpected behaviour was also seen in the quench diagnostic signals. To help understand the root causes of such observations, in this thesis work, the electro-thermal behaviour of superconducting cables with defects is studied extensively. This is performed by combining data from some full-scale magnets for the HL-LHC with numerical simulations. In addition, a dedicated experiment is performed on a small model magnet with intentionally introduced defects in its cable.

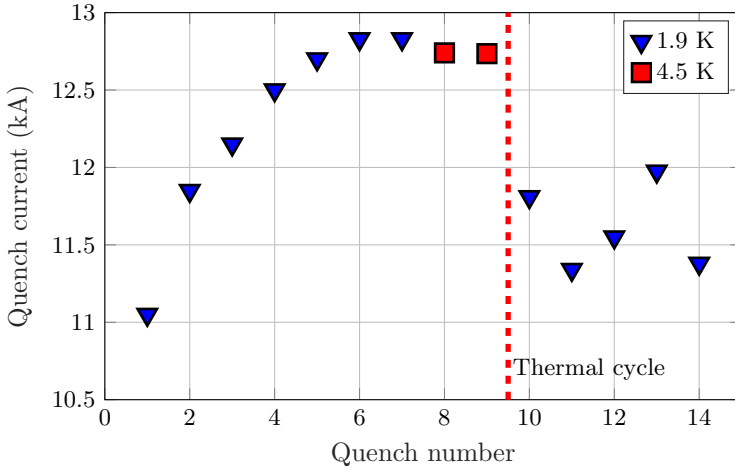


Figure 1.11: Quench current as a function of quench number for the 11 T dipole magnet designated 11 T - hybrid [41]. After the thermal cycle a significantly reduced quench current is observed.

1.5.1 Numerical analysis tools

In the past, several numerical and analytic tools were developed to study the current distribution and quench transients in accelerator magnets. These tools are mostly used to study and predict magnet behaviour under nominal operating conditions or specific fault conditions. For magnet protection during a quench transient, usually the full magnet and its interaction with the protection system has to be simulated to make sure the hot spot temperature does not exceed design limits [42, 43, 44, 45, 46, 47, 48, 50, 49]. Simplifications then need to be included to achieve practical simulation times. Often the cable is considered in 1D only or homogenization of the cable cross-section is performed. For this thesis work, a modeling philosophy that allows a more fine-grained approach was needed to study the cable behaviour on strand level in detail.

Several existing models can be used to simulate quenches in Rutherford cables with strand level detail, such as THEA, CUDI, and THELMA, which all have been used to study cable stability or normal zone propagation [51, 52, 53]. To quantify superconductor stability, the concept of Minimum Quench Energy (MQE) is used. This is the minimum energy that will result in normal zone propagation and a full quench of the magnet when locally deposited in a strand. For Nb-Ti and Nb₃Sn cables, only the first few strand quenches need to be simulated until a so-called Quench Decision Moment is reached where either a recovery or a propagating normal zone is observed [54, 55, 56, 57, 58].

In quench simulation, there are many parameters that are initially unknown, such as the thermal and electric inter-strand contact resistances, the initial current

distribution, and the location, size and shape of the initial heat load that leads to the quench. To study such a large parameter space, a computationally highly efficient numerical model that allows for fast simulations is needed. In the context of this thesis work, a new 3D thermal-electric PEEC-FEM model, called RuNe (Rutherford Network), was developed. RuNe allows for fast, parallelized quench simulations, completing more than a thousand simulation cases of a representative cable length within a day on a desktop PC.

To study the effects of local inhomogeneities or defects in strands, the cable necessarily needs to be modeled with strand level detail, similar to stability simulations, but for a longer duration past the quench decision moment. The simulations are used to clarify the origin of specific features in the experimental diagnostic signals and how they can be used to help to identify specific conductor issues.

1.6 Scope and structure of the thesis

In Figure 1.12, a schematic overview of the structure of this thesis is shown, illustrating the relations between the chapters.

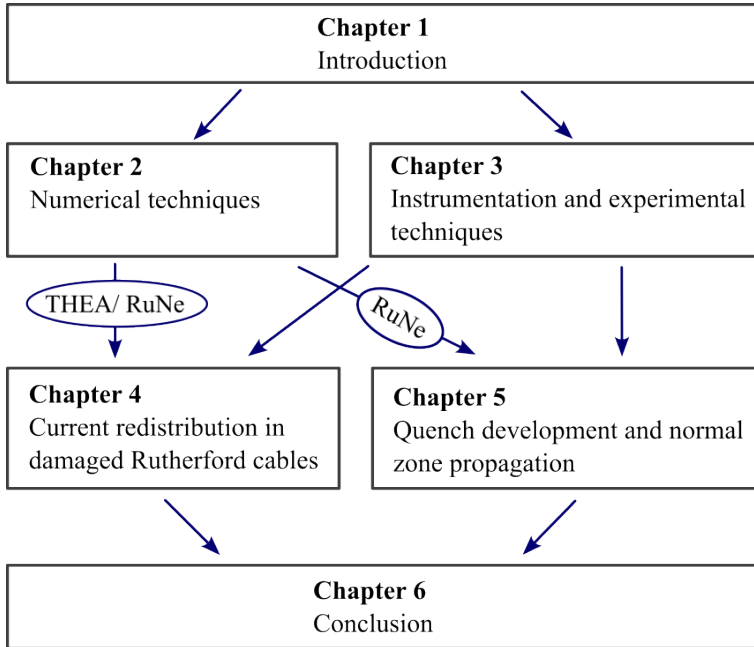


Figure 1.12: *Outline of the thesis.*

In Chapter 2, the numerical techniques that were used are described. For simulating current redistribution around defects in Rutherford cables, initially the

model THEA was applied. In addition, a new 3D thermal-electric PEEC-FEM model for the efficient simulation of quench transients and current redistribution during a ramp, called RuNe, is introduced. The focus is on ways to improve the performance of the model, which needs to process several highly non-linear material properties as well as dense integral operations for magnetic field and inductive voltage calculations.

In Chapter 3, the experimental techniques and diagnostic tools that are used to collect the data from HL-LHC magnets are presented. A dedicated experiment on a small SMC magnet with intentionally introduced defects in some of the cable strands is presented.

In Chapter 4, current redistribution effects in cables with a local defect are described. Several parametric studies were undertaken to study the effect of selected cable defect parameters on this process. The results from the SMC experiment are presented and compared to model predictions as well.

In Chapter 5, a detailed study is presented of quench development and propagation in Rutherford cables. Parametric studies into the effect of cable parameters and quench start location on diagnostic signals are performed. The effect of non-uniform current distributions, as described in Chapter 4, on the dynamics of quench propagation is reported. Then, a quench analysis of the experiment on the SMC magnet is presented.

Finally, in Chapter 6 conclusions are drawn on the features impacting the performance of Nb_3Sn magnets and on how the various diagnostics methods can be used in the future to analyze magnet performance with more detail.

Chapter 2

Numerical techniques

2.1 Introduction

In this chapter, the numerical tools that are used to predict the thermal and electrodynamic behaviour of Rutherford-type of cables are described. Broadly speaking, the phenomena studied can be divided in two timescales:

- **Long time scale** - Inter-strand current redistribution that occurs during the magnet current ramp takes place over timescales in the order of 10 s to 1000 s.
- **Short time scale** - Normal zone development in a Rutherford cable, from initiation to quench protection trigger, occurs within a time frame of 1 ms to 20 ms.

The two timescales are best studied with different types of models. In this thesis, the following two simulation codes are used:

- **THEA** - A simulation code for the Thermal, Electric and Hydraulic Analysis of superconducting wires and cables [52]. THEA is used to study current redistribution over long lengths of cable over the course of a magnet ramp.
- **RuNe** - A new thermal-electric network model using a coupled PEEC-FEM formulation. RuNe was developed in the frame of this PhD work to predict quench behaviour in a time scale up to 20 ms, but also for long-range current redistribution effects when the thermal domain is not included.

In principle, both codes can be used interchangeably for both types of cases if needed, but they both perform best in terms of accuracy and computational efficiency within their respective domains.

The study of current redistribution in superconducting cables has a long history. Simulation codes such as THEA, CUDI, Thelma, and Jack-Pot AC and their underlying models have been used to study current redistribution effects extensively [60, 51, 52, 53, 59]. Many of the techniques that were developed within these

models are applied here to the case of a Rutherford cable with conductor damage. THEA is used because it allows for a computationally efficient description of long cables using a distributed parameters approach.

Quench modelling is a notoriously complex topic, since it is a multi-physics problem with strong coupling between the physics domains. During a quench, several interdependent, highly non-linear material properties can change by multiple orders of magnitude. Nevertheless, several tools have been developed to numerically simulate the process in great detail. For this work, the first twenty milliseconds after quench initiation, i.e. the phase of normal zone propagation before the quench protection system triggers, are relevant. Once the quench protection has triggered, the minute details in the signals pointing to conductor damage are overwhelmed by other phenomena. In the present study it is of interest how the early stages of the quench process register on the available diagnostic tools, and what clues they can yield as to the quench origin and potential conductor damage.

This chapter starts with a parameterized description of the geometry of the Rutherford cable in Section 2.2. Then, in Section 2.3, the main features of the THEA code are discussed. Section 2.4 provides a detailed description of the newly developed code RuNe. Finally, in Section 2.5, some numerical considerations regarding quench antenna modelling and filament magnetization are discussed.

2.2 Rutherford cable geometry

The geometry of the Rutherford cable is defined in this section. In Figure 2.1, a cross-sectional and top-down view of a Rutherford cable are shown along with the relevant parameters. A Rutherford cable consists of a number of strands that are twisted and two-axes rolled into a high aspect ratio rectangular or trapezoidal shape. This makes the cable suitable for placement in a $\cos\theta$ magnet lay-out while at the same time accomplishing a reduction of coupling loss and improving magnetic field quality through the transposition of strands. The geometry of a Rutherford cable is thus characterized by two layers of strands that are electrically and thermally coupled through adjacent - and crossing inter-strand contact resistances [60, 64]. The (simplified) strand trajectories can be described by a set of instructions with the input parameters as listed in Table 2.1. In the description below, the twist pitch direction of the cable is clock-wise when looking in the positive z -direction but it can easily be changed by inverting the x -coordinate.

For modeling purpose, each strand is subdivided over the length of one twist pitch L_p in $f_d \cdot n_s$ segments, where n_s is the number of strands and f_d is the discretization factor. This discretization factor f_d is chosen based on the geometrical lay-out of the Rutherford cable. Note that an f_d value of at least two is necessary to describe all crossing contacts. Along with the longitudinal cable coordinate z_s , all strand trajectories can be described with the set of instructions described below.

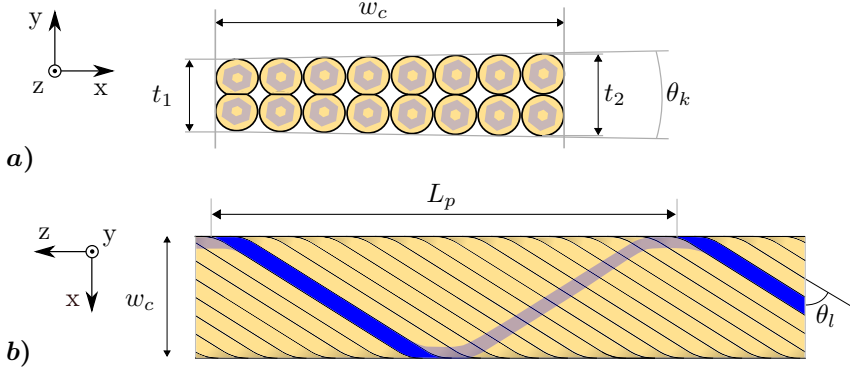


Figure 2.1: a) Schematic representation of the cross-section of a Rutherford cable. b) Top-down view of a Rutherford cable with trajectory of a single strand highlighted. The variables are listed in Table 2.1.

Table 2.1: Input parameters for the description of strand trajectories in a Rutherford cable.

Parameter	Symbol
Number of strands	n_s
Cable twist pitch (m)	L_p
Thin edge 1 (m)	t_1
Thin edge 2 (m)	t_2
Cable width (m)	w_c
Step size z-coordinate (m)	dz
Discretization factor	f_d
Keystone angle (rad)	θ_k
Strand lay angle (rad)	θ_l

First, an offset coordinate z_n is defined that is unique to each strand. This coordinate gives the strand a phase shift dependent on strand index number k , since all strands are defined by the same 3D line but phase shifted with respect to each other:

$$z_n(z_s, k) = z_s + (k - 1) \cdot dz \cdot f_d, \quad (2.1)$$

with:

$$dz = \frac{L_p}{f_d \cdot n_s}. \quad (2.2)$$

The following conditional statement makes sure that z_n always stays within one twist pitch, since the strand trajectories have a periodicity L_p :

$$z_n = z_n - L_p \lfloor z_n / L_p \rfloor \quad \text{when} \quad z_n > L_p, \quad (2.3)$$

where $\lfloor \cdot \rfloor$ denotes the floor function. The strand trajectories are then described with the following four conditional statements corresponding to the two broad faces (Equation 2.4, 2.6) and the two side faces (Equations 2.5 and 2.7):

$$\left. \begin{aligned} x_s(z_s, k) &= \frac{w_c}{n_s} + \frac{z_n n_s}{L_p(n_s/2 - 1)} \left(w_c - \frac{2w_c}{n_s} \right) \\ y_s(z_s, k) &= -\frac{t_1}{4} - \left(\frac{t_2}{4} - \frac{t_1}{4} \right) \left[\frac{2z_n n_s}{L_p(n_s - 2)} \right] \end{aligned} \right\} \quad \text{when } z_n < L_p \left(\frac{n_s - 2}{2n_s} \right), \quad (2.4)$$

$$\left. \begin{aligned} x_s(z_s, k) &= w_c \left(1 - \frac{1}{n_s} \right) \\ y_s(z_s, k) &= -\frac{t_2}{4} + \frac{t_2}{2} \left[n_s \left(\frac{z_n}{L_p} - \frac{1}{2} \right) + 1 \right] \end{aligned} \right\} \quad \text{when } z_n \geq L_p \left(\frac{n_s - 2}{2n_s} \right) \wedge z_n < \frac{L_p}{2}, \quad (2.5)$$

$$\left. \begin{aligned} x_s(z_s, k) &= w_c \left(1 - \frac{1}{n_s} \right) - \left(w_c - \frac{2w_c}{n_s} \right) \left[\frac{(2z_n - L_p)}{L_p \left(1 - \frac{2}{n_s} \right)} \right] \\ y_s(z_s, k) &= \frac{t_2}{4} - \left(\frac{t_2}{4} - \frac{t_1}{4} \right) \left[\frac{n_s(2z_n - L_p)}{L_p(n_s - 2)} \right] \end{aligned} \right\} \quad \text{when } z_n \geq \frac{L_p}{2} \wedge z_n < L_p \left(\frac{n_s - 1}{n_s} \right), \quad (2.6)$$

$$\left. \begin{aligned} x_s(z_s, k) &= \frac{w_c}{n_s} \\ y_s(z_s, k) &= \frac{t_1}{4} - \frac{t_1}{2} \left[n_s \left(\frac{z_n}{L_p} - 1 \right) + 1 \right] \end{aligned} \right\} \quad \text{when } z_n \geq L_p \frac{n_s - 1}{n_s}, \quad (2.7)$$

These instructions yield, for each strand, a matrix consisting of three arrays $\{x_s(k, z_s), y_s(k, z_s), z_s\}$ comprising the coordinates of the mesh elements along the strand trajectories. The resulting cable geometry in RuNe or THEA is similar to that of CUDI [51], but with the option to reduce the discretization size by increasing f_d and it can be used for cables with both an even and uneven number of strands. An example of a Rutherford cable consisting of 16 strands is shown in Figure 2.2. The presented geometry simplifies the strand trajectories as in reality the strands round the corner to the other layer smoothly, where in the model this cross-over between layers is described by two 90° turns with a straight section along the y-direction in-between on the side of the cable.

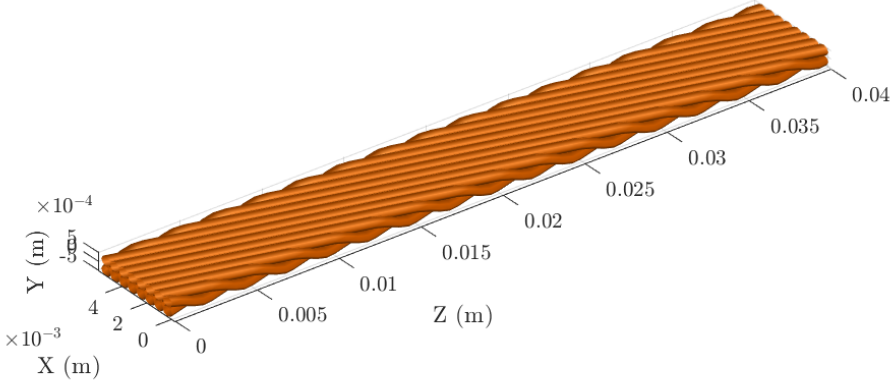


Figure 2.2: 3D view of a Rutherford cable with 16 strands as described by Equations 2.4 to 2.7.

2.3 THEA simulation code

THEA is a simulation code written in Fortran for the Thermal, Hydraulic and Electric Analysis of superconducting cables and components, making use of a continuum model approach [52]. In THEA, the strands are treated as 1D objects, meaning that any transverse features in the strand cross-section are assumed to be negligible. The 3D topology of a Rutherford cable is then described for each strand in terms of its local properties, such as critical current, strain state, background magnetic field, and coupling to other strands. The strand elements in the simulation are all transversely coupled both thermally and electrically in a continuous manner.

A short introduction to the underlying formulation of THEA is provided. Then, its application for simulating a Rutherford cable is described, following two references from literature [61, 62].

2.3.1 Governing equations

To calculate the current distribution in the components of the considered system, THEA assumes that the problem is 1-dimensional. Using the continuum model approach as described in [61], the current distribution can then be calculated from the 1-dimensional diffusion equation. In matrix form this yields the following equation:

$$\mathbf{L} \frac{\partial \mathbf{i}}{\partial t} + \mathbf{R} \mathbf{i} - \frac{\partial}{\partial z} \left(\mathbf{G}^{-1} \frac{\partial \mathbf{i}}{\partial z} \right) = \Delta \mathbf{v}_{\text{ext}}, \quad (2.8)$$

where $\mathbf{i}$ is the vector containing the currents of all elements, $\mathbf{G}$ is the transverse conductance matrix, $\mathbf{L}$ is the inductance matrix, $\mathbf{R}$ is the longitudinal resistance matrix, and $\Delta \mathbf{v}_{\text{ext}}$ is the externally applied voltage. The matrices $\mathbf{G}$, $\mathbf{R}$, and $\mathbf{L}$ are defined per unit length for each longitudinal spatial coordinate z and can thus vary in space. Here, the inductance matrix $\mathbf{L}$ is taken to be local, no upstream or downstream effects are taken into account.

The thermal domain is treated with the heat balance equation [52]:

$$A\rho c_p \partial_t \boldsymbol{\theta} - \partial_z (Ak \partial_z \boldsymbol{\theta}) = \dot{q}_i + \dot{q}_{\text{Joule},i} + \sum_{\substack{j=1 \\ i \neq j}}^N \frac{\boldsymbol{\theta}_j - \boldsymbol{\theta}_i}{H_{ij}}. \quad (2.9)$$

Here A is the cross-section of the component, ρ is the density, and c_p is the specific heat, k is the thermal conductivity, and $\boldsymbol{\theta}$ is the temperature. The source terms on the right represent the external heating $\dot{q}_i$, joule heating $\dot{q}_{\text{Joule},i}$ and inter-strand heat transfer, with H_{ij} the transverse thermal impedance. THEA then solves the resulting system of equations with a Finite Element Method in space and a Finite Difference Method in time [52].

2.3.2 Continuum model of the Thermal and Electrical network

For the simulation of long-range current redistribution effects, a distributed parameters approach as proposed in [61] is used. The discrete inter-strand contacts R_a and R_c (units in Ω) used in the network model, which will be discussed in Section 2.4, are averaged over the full length of the cable. In practice, R_a represents a continuous contact but R_c is discrete. The value of the adjacent inter-strand conductance per unit length (units in Sm^{-1}) in matrix $\mathbf{G}$ then becomes [61]:

$$g_a = \frac{2(n_s - 1)}{R_a L_p} \quad (\text{Sm}^{-1}). \quad (2.10)$$

This equation needs to be tweaked slightly when used for comparison with our model RuNe since the network is defined slightly differently than in the source reference:

$$g_a = \frac{2n_s}{R_a L_p} \quad (\text{Sm}^{-1}). \quad (2.11)$$

There are two more contacts R_a , because in network models such as RuNe and CUDI the cable short edges are also incorporated. The crossing inter-strand conductance in matrix $\mathbf{G}$ for both cases is defined as [61]:

$$g_c = \frac{2}{R_c L_p} \quad (\text{Sm}^{-1}). \quad (2.12)$$

Because of the long length scale of the current redistribution processes, this is considered a sufficiently accurate approximation that greatly reduces computational effort, since the intricate details of the network no longer have to be modelled

[61].

To simulate short-range current redistribution effects, a slightly different approach is used, as proposed in [62]. The crossing contacts in a Rutherford cable can be seen as discrete point contacts. In order to accurately describe these crossing contacts in THEA, the conductance matrix has to be varied along the longitudinal direction. For each pair of crossing strands there are two point where they cross-over for every twist pitch. To describe this contact point, a smooth continuous function has to be used instead of a block function. This way there are no sharp transitions that can cause computational instability in the solver [62]. A Gaussian function is therefore chosen to describe the cross-over point:

$$F(z) = \sqrt{\frac{a}{\pi}} e^{-az^2} \quad (\text{m}^{-1}). \quad (2.13)$$

It is easy to implement and it approximates the contact surface area reasonably well. The surface area under this function is equal to 1 and the parameter a (with units of m^{-2}) can be chosen such that the Gaussian peak has the desired width. For each crossing strand pair, two of these Gaussian peaks are present within one twist pitch, separated with a distance of $L_p/2$ from each other. So the crossing contact conductance is described as follows:

$$F(z) = \sqrt{\frac{a}{\pi}} \left[e^{-a(z-d_{i,j})^2} + e^{-a(z-d_{i,j}-L_p/2)^2} \right] \quad (\text{m}^{-1}). \quad (2.14)$$

Here the parameter $d_{i,j}$ depends on which combination of strands i and j are being evaluated and is given as:

$$d_{i,j} = \frac{L_p}{2n_s} (1 - i - j) \quad (\text{m}). \quad (2.15)$$

The factor F is then multiplied by the inverse of the discrete inter-strand contact resistance to obtain a continuum expression for the crossing contact conductance:

$$g_c(z) = \frac{1}{R_c} F(z) \quad (\text{Sm}^{-1}). \quad (2.16)$$

The contact conductance function along the length is then obtained for each strand pair. An example of $F(z)$ for two strand pairs is shown in Figure 2.3.

The thermal conductance κ_c of the crossing contact can be described similarly. Note that in THEA the thermal contacts are described by a thermal resistance value, so the inverse of the resulting function has to be used, taking care not to divide by zero in-between peaks by adding a negligible but finite thermal conductance value everywhere. The adjacent strand pairs are simply described by a constant conductance per unit length. By combining the conductance values of the adjacent and inter-strand contacts, the conductance matrix of the cable is constructed for each point along its length. In Figure 2.4 the conductance matrix for several positions along the cable is shown.

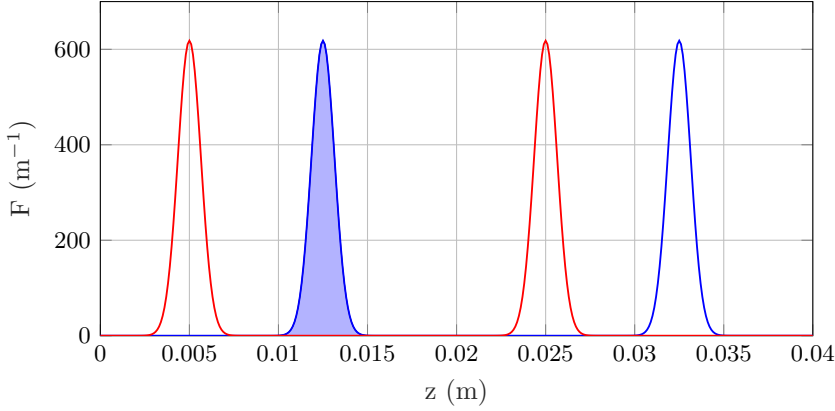


Figure 2.3: Inter-strand contact function F as a function of position z along the cable for contact between strands 1 and 6 (blue) and strands 1 and 12 (red), for a Rutherford cable consisting of 16 strands. The Gaussian function has the parameter a set to $1.2 \times 10^6 \text{ m}^{-2}$. The surface area of the blue shaded region is equal to one. There are two points of contact between each pair of strands per twist pitch.

2.3.3 Inductance matrix

In THEA the mutual inductance between strand sections is defined on a per unit length basis (H m^{-1}) and stored in an inductance matrix. This implies that the result will be less accurate if strong longitudinal gradients in the time derivative of the current are present, since the coupling is not global. Therefore the inductive voltages in the simulation are only approximate. To calculate the inductance matrix, the well-known analytical formulas of Grover are used [63]. The analytical solution for the self-inductance of a line section of current is given as:

$$L_s = \frac{\mu_0}{4\pi} \left[l \cdot \ln \left(\frac{l + \sqrt{l^2 + r^2}}{r} \right) - \sqrt{l^2 + r^2} + \frac{l}{4} + r \right]. \quad (2.17)$$

Here l is the length and r the radius of the line. The mutual inductance between two parallel line sections is given by:

$$L_m = \frac{\mu_0}{4\pi} \left[l \cdot \ln \left(\frac{l + \sqrt{l^2 + d^2}}{d} \right) - \sqrt{l^2 + d^2} + d \right]. \quad (2.18)$$

Here d is the perpendicular distance between the centers of the two sections. The self-inductances is assumed constant along the length in a Rutherford cable, but the mutual inductances between strands change periodically due to the twist pitch and to the trapezoidal cross-section. To acquire an approximate solution that is computationally easy to evaluate, a single inductance matrix for the entire cable length can be defined by averaging the inductance along the length of the cable [61]. This can be achieved by taking the average mutual inductance over several twist pitches to converge to a single inductance matrix for the entire cable length.

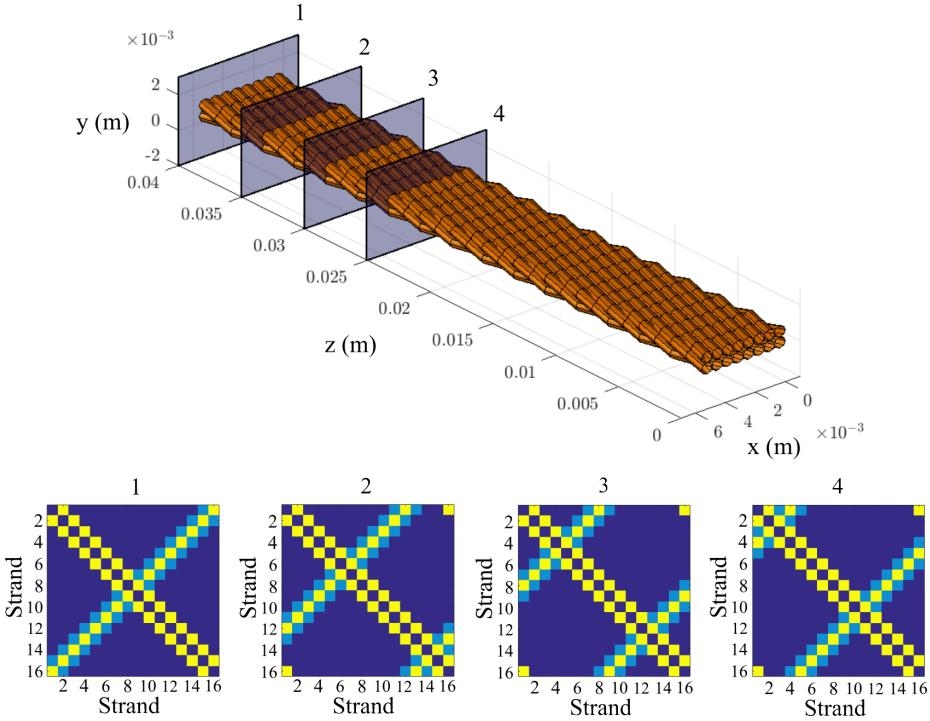


Figure 2.4: Color plots of the conductance matrix at several positions along the Rutherford cable. The two yellow off-center diagonals and the top-right and bottom left squares represent the adjacent inter-strand contacts. The other squares that change color based on position along the cable, represent the crossing inter-strand contacts. Both the values for the adjacent and crossing conductance are scaled to one in this example.

For simulating current redistribution effects over length scales much longer than the cable twist pitch this approach is valid. If desired, the inductance matrix can also be varied along the cable length with any chosen averaging length as shown in Figures 2.5a and b in a similar fashion to the conductance matrix in Figure 2.4. This approach is used when studying short length scale effects (less than the cable twist pitch) such as quench transients. From comparison with the 3D network model RuNe it was found that, for quench simulation of a typical cable, the optimal averaging length is some 20 mm.

2.3.4 Assumptions in THEA

The 1D assumption in THEA causes some uncertainties when modeling 3D structures. It is applicable to almost any cable as long as the characteristic length scale of the phenomenon under consideration is much longer than the cable width. The distributed parameters approach allows for flexibility in handling the simulation of long lengths of cable along which current can redistribute. For such current

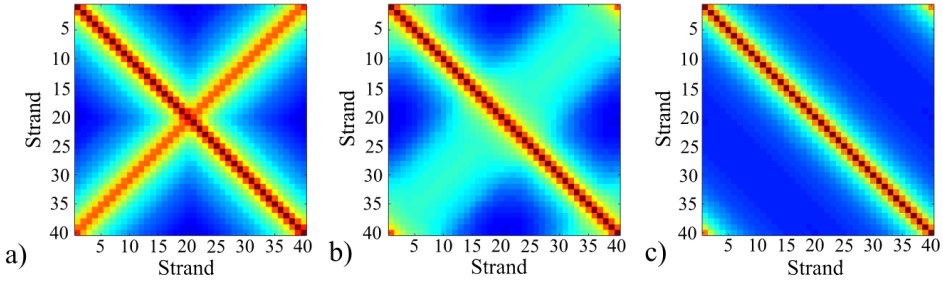


Figure 2.5: Inductance matrix for a Rutherford cable of 40 strands averaged over different lengths: a) 1 section, b) 50 sections, c) 200 sections. The discretization factor d_f is 5. In the case of a) and b) the inductance matrix has to be varied along the length while for c) this no longer needed.

redistribution effects with a length scale much greater than the cable width, the 1D assumption is justified.

However, for simulating the first milliseconds of a quench, during which transverse and longitudinal propagation effects occur on a similar length scale, this assumption may not hold. In addition, implementing network topology into the model incurs an overhead in terms of mesh element density, due to the need to describe the crossing inter-strand contacts smoothly [62].

The length of the strand sections has to be modified to account for the lay angle θ_1 . The lay angle also implies that Equation 2.18 is not entirely accurate because the parallel current sections are not next to each other but slightly shifted lengthwise or, in the case of opposite layers, pointing in different direction. More elaborate analytical solutions for these configurations are given in [63], but after averaging the inductances over long length scales, the difference of the final inductance matrix with Equation 2.18 is negligible.

2.4 RuNe - Rutherford Network simulation code

A 3D thermal-electric PEEC-FEM model for the study of quench transients and current distribution in Rutherford cables was developed. The model consists of an electric and a thermal domain with implicit coupling between them and borrows many elements from earlier simulation efforts. Notable models of similar design are CUDI, ElMaTh, THELMA, and THEA, all of which are able to model electro-thermal transients in superconducting cables [51, 70, 69, 52]. Certain aspects of these models are also implemented in the new model. Specifically, the solver sequence and domain coupling that is used for RuNe is adopted from ElMaTh [70, 71].

In RuNe the following features are combined:

- a full 3D network description of the Rutherford cable, including all current paths in the inductive voltage calculation;
- accurate modelling of the thermal domain with use of a Finite Element Method employing up to quadratic basis functions;
- acceleration of the computation of integral operators with hierarchical matrices.

RuNe is capable of simulating electrodynamic or coupled thermal-electric transients on both long and short length scales.

2.4.1 Electrodynamic domain

The electrodynamic domain is treated with the Modified Nodal Analysis (MNA) for electric circuits [65]. The implementation here is based on the Partial Element Equivalent Circuit formulation (PEEC), an integral method [66, 67]. Quasi-static conditions are assumed, meaning no charge accumulation through capacitive effects is taken into account. This method is applied to the Rutherford cable by discretizing it into a set of elements that are connected at nodes, where the strand elements follow the trajectories described in Section 2.2. The strand elements are then coupled in the transverse direction through two types of electrical resistance elements R_a and R_c .

The 3D network lay-out is shown in Figure 2.6, where two consecutive unit cells are depicted. A unit cell consists of $5n_s - 3$ elements with $2n_s$ strand elements, $2n_s$ ' R_a ' elements and $n_s - 3$ ' R_c ' elements. Though there are strong similarities to the network lay-out used in CUDI, for instance the discretization size and the number of elements, there are also a few key differences. As opposed to CUDI, the elements that represent R_a are now perpendicular to the longitudinal axis of the cable. In addition, besides the strand elements, the elements for R_a and R_c are now also included in the inductive voltage calculation, which is necessary to accurately describe current redistribution between strands at short length scales, especially for low values of R_a .

The network obeys Kirchhoff's current - and voltage laws. The voltage drop over each element is equal to the voltage difference between the two nodes that it is connected to and the sum of currents towards each node is zero. Each element has a self-inductance and is coupled to all other elements through a mutual inductance. The basic circuit equations in matrix form are then given as:

$$-\begin{bmatrix} 0 & 0 \\ 0 & \mathbf{L}_m \end{bmatrix} \begin{bmatrix} \partial_t \mathbf{v} \\ \partial_t \mathbf{i} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{M}_{\text{kcl}} \\ \mathbf{M}_{\text{kvl}} & \mathbf{R}_m \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix} + \begin{bmatrix} \mathbf{i}_{\text{ext}} \\ \mathbf{v}_{\text{ext}} \end{bmatrix}. \quad (2.19)$$

Here $\mathbf{L}_m$ is the inductance matrix, $\mathbf{M}_{\text{kcl}}$ is the incidence matrix representing Kirchhoff's current law and $\mathbf{M}_{\text{kvl}}$ is the incidence matrix representing Kirchhoff's

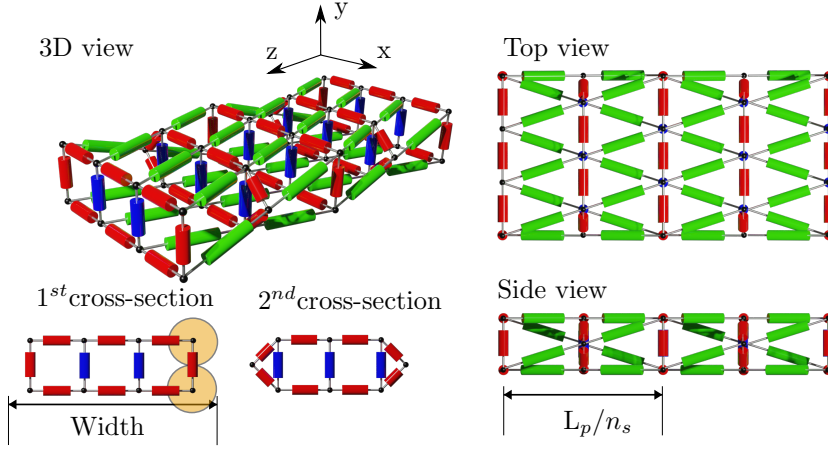


Figure 2.6: Network of a Rutherford cable with 8 strands, two unit cells are depicted here. The green, red, and blue elements represent the strand elements, adjacent contact resistances R_a and crossing contact resistances R_c , respectively.

voltage law. The matrix $\mathbf{M}_{\text{kcl}}$ has rows that represent the nodes and columns representing the elements. If an element is leaving a node its value in the matrix is -1, if it is entering the node it is 1, the rest of the matrix entries are zero. The two Kirchhoff matrices are transposed to each other: $\mathbf{M}_{\text{kvl}} = \mathbf{M}_{\text{kcl}}^T$. The vectors $\mathbf{v}$ and $\mathbf{i}$ contain the voltages at the nodes and the currents in the elements, respectively. ∂_t indicates the partial derivative with respect to time. $\mathbf{R}_m$ is the resistance matrix which contains the resistances R_a and R_c on the diagonal, the superconducting elements are set to zero. $\mathbf{i}_{\text{ext}}$ and $\mathbf{v}_{\text{ext}}$ are external voltage and current sources, such as the power supply. The resulting system of equations is a Differential Algebraic Equation in mass matrix form. In this form, the system is still linear since it does not contain the non-linear superconducting to normal transition. It can be used to study current redistribution effects in the cable under ‘weak excitation’, when the currents in the strand elements are far below their critical values.

2.4.2 Superconducting-to-normal transition

The superconducting-to-normal transition is highly non-linear in Nb_3Sn and is described by an empirical power law:

$$E(I_{\text{sc}}) = E_0 \left(\frac{I_{\text{sc}}}{I_c} \right)^n, \quad (2.20)$$

where E is the electric field in the superconductor, E_0 is the electric field criterion, which, by convention, is often set to $10 \mu\text{V m}^{-1}$, I_{sc} is the current in the superconductor and I_c is its temperature - and magnetic field dependent critical current. The n -value determines the steepness of the transition and for Nb_3Sn lies usually in the range of 30 to 40, but can locally be significantly lower in the

case of conductor damage [80, 81]. This empirical relation in essence describes the macroscopic manifestation of the dissipative flux flow regime of the Abrikosov state, discussed in Chapter 1.

Current sharing model

In a practical superconducting wire, the superconductor is embedded in a matrix with a relatively high electric and thermal conductivity for reasons of thermal stability [18]. This matrix usually consists of aluminium or copper. As the superconductor locally reverts to the normal state, it will start to share current with the matrix. Different models to describe this process exist in literature [52, 72]. Here, this behaviour is described with the parallel-path model. The metal matrix is represented by a resistance connected in parallel to the superconductor, leading to the following implicit relation:

$$\frac{I_{\text{tot}} - I_{\text{sc}}}{A_{\text{cu}}} \rho_{\text{cu}} = E_0 \left(\frac{I_{\text{sc}}}{I_c} \right)^n, \quad (2.21)$$

where ρ_{cu} is the resistivity of the metal, here assumed to be copper, A_{cu} its cross-sectional area, I_{tot} the total current and I_{sc} the current in the superconductor. The inductive contribution is neglected due to the close proximity of the current paths. To obtain the fractions of current flowing in the copper matrix and in the superconductor, the relation is solved with an iterative method for I_{sc} , which lies in the range $[0, I_{\text{tot}}]$. From this current, the voltage that is generated over the element, V_e , is then readily calculated:

$$V_e = \frac{(I_{\text{tot}} - I_{\text{sc}}) L_e \rho_{\text{cu}}}{A_{\text{cu}}}, \quad (2.22)$$

where L_e is the length of the element.

The voltages V_e are then calculated for all strand elements in the network and stored in the vector $\mathbf{v}_e$. This resistive voltage component due to current sharing with the matrix is then added to the system equation:

$$-\begin{bmatrix} 0 & 0 \\ 0 & \mathbf{L}_m \end{bmatrix} \begin{bmatrix} \partial_t \mathbf{v} \\ \partial_t \mathbf{i} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{M}_{\text{kcl}} \\ \mathbf{M}_{\text{kvl}} & \mathbf{R}_m \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{v}_e(\mathbf{i}) \end{bmatrix} + \begin{bmatrix} \mathbf{i}_{\text{ext}} \\ \mathbf{v}_{\text{ext}} \end{bmatrix}, \quad (2.23)$$

which is now able to simulate the behaviour of the cable also under ‘strong excitation’, as the strand elements now generate the appropriate electric field when they approach or exceed their critical current. The model in this form (Equation 2.23) can for instance be used to study current redistribution effects during current ramp in magnets, such as the current cascade effect, which will be described in Chapter 4.

2.4.3 Partial inductance calculation

The flux linkage between the elements is one of the most critical and challenging aspects of the modelling of transients in electrodynamic networks. To properly treat this aspect, the inductance matrix has to be populated with accurate approximations of the self- and mutual inductances of the cable elements. For these cable elements we are calculating *partial* inductances. Here, ‘partial’ means that the elements have a finite length in the current direction instead of forming closed current loops. Therefore, care must be taken to make sure that the overall network remains physically meaningful, i.e. that all elements have connected ends.

Exact closed-form expressions for the partial inductance exists only for specific cases but are less relevant because of uncertainties in the cable geometry and the non-uniform current distribution in the strand cross-section. However, due to the short length of the cable sub-elements in the model, using well-known standard expressions may result in unacceptable errors. Therefore, in this section, several analytical expressions and techniques for calculating partial inductances are introduced and applied to our case of a Rutherford cable.

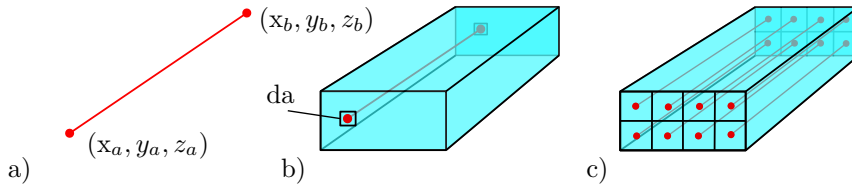


Figure 2.7: a) Line current between point a and b. b) Grover's filament expression can be analytically integrated over a surface. c) In most cases a crude Riemann sum is enough to obtain the desired accuracy.

Mutual inductance of cable elements

The exact mutual inductance for two solid conductors with an arbitrary current distribution is given by a 6-dimensional double volume integral of the current density and the free-space Green's function:

$$M_{ij} = \frac{\mu_0}{4\pi} \frac{1}{I^2} \int_{V_i} \int_{V_j} \mathbf{J}_i \cdot \mathbf{J}_j \frac{d\tau_i d\tau_j}{r_{ij}}, \quad (2.24)$$

where $\mathbf{J}_n$ is the current density in either conductor, I is the total current flowing in the conductors, $d\tau_n$ is an infinitesimal volume element and r_{ij} is the distance between such volume elements. In the case that the conductor sections behave as line currents, as in Figure 2.7a, the expression simplifies to Neumann's formula:

$$M_{ij} = \frac{\mu_0}{4\pi} \int \int \frac{d\mathbf{l}_i \cdot d\mathbf{l}_j}{r_{ij}}, \quad (2.25)$$

where $d\mathbf{l}$ is an infinitesimal line element vector. For the special case of two *straight* line currents with any relative orientation and length, the well-known analytical expression of Grover (in [63], page 56, Equation 52), denoted here by M_G , can be used. This analytical solution can then be used to simplify Equation 2.24 for the case of a *straight* conductor with any cross-section and a uniform current density, as proposed in [75, 76]. The 6-dimensional integral is reduced to 4 dimensions, see Figure 2.7b, the double surface integral of Grover's analytical expression over the conductors' cross-sections:

$$M_{ij} = \int_S \int_S M_{G,ij} d\mathbf{a}_i d\mathbf{a}_j. \quad (2.26)$$

In practice, an acceptable approximation can be achieved by considering the integral simply as a discrete Riemann sum of line elements as in Figure 2.7c [77]:

$$M_{ij} \approx \frac{1}{PQ} \sum_{i=1}^P \sum_{j=1}^Q M_{G,ij}. \quad (2.27)$$

As further discussed below, this expression is accurate ($< 1\%$) for even a modest amount of subdivisions P and Q .

With the expressions above, the mutual inductance for all elements in the Rutherford cable can now be calculated. First, the mutual inductances between strand elements are treated. For comparison, an accurate analytical expression from literature for the mutual inductance between two equal round wires in parallel with uniform current density is used [73]:

$$M_{\text{wires}} = \frac{\mu_0}{2\pi} \left[l \cdot \ln(W + l) - l \cdot \ln d - W + d + \frac{r^2}{4d} \right], \quad (2.28)$$

where d is the perpendicular distance between segments; l their length; r their radius and $W = \sqrt{l^2 + d^2 + r^2}$. A correction term ΔM is added to increase the accuracy to $< 0.1\%$ [73]:

$$\Delta M = \frac{\mu_0}{2\pi} \sqrt{d^2 + r^2} \left(\sqrt{d^2 + r^2} - d - \frac{r^2}{4d} \right) \frac{W - l}{W(W + l)}. \quad (2.29)$$

Comparing these analytical approximations to Grover's line current expression for the case of adjacent strand segments, the green elements in Figure 2.6, with dimensions as in the network model, yields an error of maximum 2% if only one line current is used, which considering uncertainties in the cable geometry is already a sufficiently good approximation.

For connecting strand elements in the longitudinal direction, the mutual inductance is calculated with the averaged sum of Equation 2.27 using a hexagonal packing of the filaments. This method converges to a value that is 15% larger than that of a single line current. Therefore, in the model an integration of the hexagonally packed filaments as shown in Figure 2.8b is always performed.

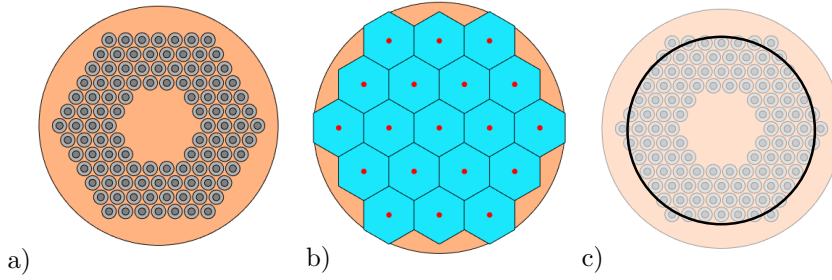


Figure 2.8: a) Cross-section of OST 108/127 RRP wire. b) The wire is approximated by a set of hexagonally packed filaments. c) Ring of current to approximate the high-frequency regime.

The current transfer between strands through the adjacent inter-strand contact is modelled with a sheet current as shown in Figure 2.9. For the near-field interaction of the adjacent inter-strand contacts, the red elements in Figures 2.6 and 2.9, approximating it as a single filament, results in an error of 40 %. To get a more accurate value for their mutual inductance, the averaged sum of Equation 2.27 is again used to approximate the sheet current. In Figure 2.10, the convergence of the solution with the number of line elements in the summation is shown. The amount of line elements needed for an accurate approximation drops sharply with the distance.

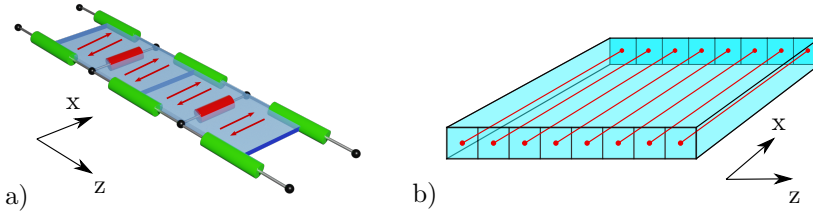


Figure 2.9: a) Two sheet current elements (light blue) shown with respect to the electric elements. The arrows indicate the current direction. b) The adjacent inter-strand current path is integrated using a summation of filament currents.

The mutual inductance between strand segments and inter-strand contacts is calculated with a summation over the hexagon of filaments and the sheet current.

There is only a weak dependence of the far-field interactions on the precise details of the local current distribution. Therefore, for far-field interactions beyond a range of three unit cells, the mutual inductance is simply calculated as:

$$M_{\text{ff}} = \frac{\mu_0}{4\pi} \frac{\mathbf{l}_i \cdot \mathbf{l}_j}{r_{ij}}, \quad (2.30)$$

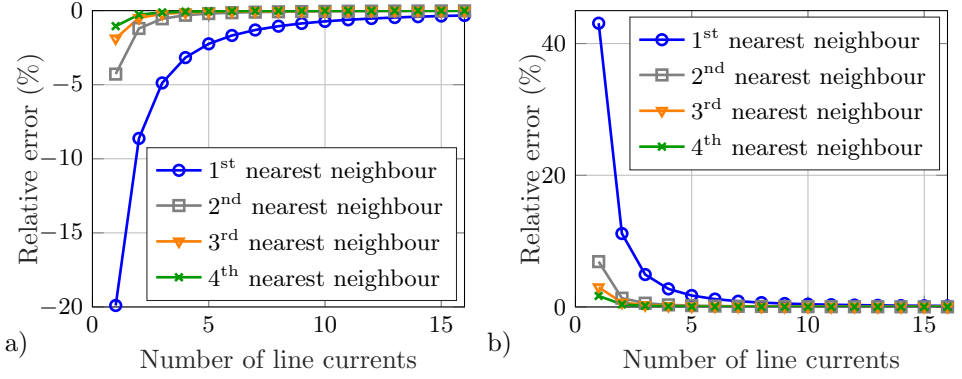


Figure 2.10: *a)* Relative error with respect to the converged solution in the mutual inductance between adjacent contact resistance elements versus the number of line currents in the averaged summation for the four nearest neighbours in the longitudinal direction. *b)* Relative error with respect to the converged solution in the mutual inductance between adjacent contact resistance elements versus the number of line currents in the averaged summation for the four nearest neighbours in the perpendicular direction.

where $\mathbf{l}_i$ is the current element vector.

Self-inductance of cable elements

The self inductance of a strand element is calculated with an analytical expression from literature [73]. This expression approximates the double folded volume integral of Equation 2.24 in the case of homogeneous current distribution, by means of a combination of the Geometric Mean Distance and Arithmetic Mean Distance methods:

$$L_{\text{wire}} = \frac{\mu_0}{2\pi} \left[l \cdot \ln \left(\frac{\sqrt{l^2 + r^2} + l}{r} \right) + \frac{l}{4} - \sqrt{l^2 + r^2} + 0.905415 \cdot r \right], \quad (2.31)$$

where l is once more the wire length and r is its radius. This relation is accurate also for short wires, within 0.1 % for $l/r > 2$ [73]. However, in a compacted cable, the shape of the wire cross-section may be closer to a square, for which the relation of the self inductance of a bar with side $a = \sqrt{\pi}r^2$ holds:

$$L_{\text{bar}} = \frac{\mu_0}{4\pi} 2l \left[\ln \left(\frac{2l}{r} \right) - 1 + \frac{r}{l} \right], \quad (2.32)$$

which yields a 2% reduction with respect to the case of a round wire [74].

Due to the distribution of superconducting filaments in the copper matrix and the low inter-filament resistance, the assumption of a radially uniform current

distribution is not valid in the case of a superconducting wire. The current is initially expected to run only in the outer filaments, moving inwards as they saturate. The self-inductance is therefore variable during the magnet current ramp and also during quench transients. To estimate the significance of this effect, an analytical high-frequency solution analogous to Equation 2.31 is used. Also in the high-frequency domain it is assumed that the current does not flow homogeneously in the cross-section but is concentrated in a ring at the edge of the filament bundle, see Figure 2.8c. The self-inductance of such a cylindrical current distribution is given as:

$$L_{\text{wire,hf}} = \frac{\mu_0}{2\pi} \left[l \cdot \ln \left(\frac{\sqrt{l^2 + 2r^2} + l}{r} \right) - \sqrt{l^2 + 2r^2} + \frac{4}{\pi} r \right]. \quad (2.33)$$

This relation predicts that if the current flows only on a ring at the outer filaments in the strands, the self-inductance will be very close to the fully homogeneous expression, with current equally distributed over the full cross-section including the copper matrix. However, in the case of a strand carrying close to its critical current, which is the relevant regime for a quenching cable, the current will be distributed almost evenly over all filaments. Such uniform current distributions will raise the self-inductance of the strand by some 5 %, since more current is now concentrated towards the center. This change in self-inductance is not included in the model, which always assumes a homogeneous current distribution over the full strand cross-section, but it has to be taken into account in the uncertainty analysis.

As discussed, the adjacent inter-strand contact is more like a sheet current flowing, on average, from strand center to strand center along the length of a strand segment, see Figure 2.9. The self-inductance for a sheet carrying a homogeneous surface current distribution of length l and width w is given by:

$$L_s = \frac{\mu_0}{6\pi w^2} \left[3w^2 l \ln \left(\frac{l + \sqrt{l^2 + w^2}}{w} \right) - (l^2 + w^2)^{\frac{3}{2}} \right. \\ \left. + 3l^2 w \ln \left(\frac{w + \sqrt{w^2 + l^2}}{l} \right) + l^3 + w^3 \right]. \quad (2.34)$$

In the cable this sheet current has the shape of a parallelogram and its self-inductance is calculated in the model using Equation 2.27. However, in that expression a singularity is present when the filaments overlap, i.e. for equal indices in the summation. Therefore, Equation 2.34 is used in the case of equal indices. For the self-inductance the averaged summation method converges much slower to the true solution than for the mutual inductance. To achieve an accuracy better than 1 %, 32 subdivisions are needed.

In the case of high cable compaction the sheet current may no longer be a good approximation and the sheet may need to be expanded in the y-direction (Figure 2.1) to account for the large contact area between strands.

2.4.4 Computational considerations

All elements of the network are inductively coupled. If the interactions were to be calculated directly for each pair, the computational load would increase with $\mathcal{O}(n^2)$, where n is the number of elements, resulting in an impractical simulation time for large systems. The size of the electro-thermal cable models used for quench analysis in this thesis is relatively modest, in the range of 1.5 to 4×10^4 mutually coupled elements. Nevertheless, a naive direct implementation would result in 80 to 90 % of the simulation effort being spent on the inductive voltage calculation. The electrodynamic PEEC model that is used for studying current redistribution during the ramp of a magnet may well consist of more than a million elements, for which it is not feasible at all to perform such a direct calculation.

In the past, this problem was often tackled by ignoring the contribution of elements further away than a certain distance [51]. This method certainly makes sense in the context of modelling cable segments, which have a high aspect ratio. Practically, this approach will result in an error of several percent, but it can also introduce truncation artifacts in the solution, and even cause numerical instability. Moreover, even when the truncation distance is chosen to be relatively small, the computational load of the dense block-matrix multiplication is not fully mitigated.

Fortunately, there are mathematically more elegant solutions available to tackle this problem. They make use of the fact that the Green's function (or other relevant kernel functions) is asymptotically smooth. This means that far-field interactions can be accurately approximated through a number of different techniques. One of these is the Multi-Level Fast Multipole Method (MLFMM). It utilizes multipole expansions in combination with a multi-level scheme to accurately approximate the contribution of the far-field interactions. This method has been applied to network models of Rutherford cables, CICC ITER cables, various *ReBCO*-HTS cables and even full-size magnets [59, 78, 79, 70, 71].

In this work, a similar technique is employed, the so-called hierarchical matrix or $\mathcal{H}$ -matrix is employed [83, 85]. Instead of analytical series expansions, it uses algebraic matrix compression techniques. With this technique, the computational load will scale with $\mathcal{O}(nf \log(n))$ for sufficiently large problems, where f is a factor proportional to the specified accuracy, which can be chosen freely in the options of the $\mathcal{H}$ -matrix algorithm. In addition to the performance-increase, memory requirements are reduced significantly as well.

Below, first the tools for matrix compression are introduced. This is followed by an example of a $\mathcal{H}$ -matrix routine available within the Julia programming language. Finally, an even more efficient implementation is introduced that is tailored to periodic structures such as cables.

2.4.5 Singular Value Decomposition and Low Rank Approximation

Our goal is to approximate a matrix $\mathbf{A}$ of rank n containing some underlying structure with a matrix of rank $r < n$. One of the methods that can be used to obtain such an approximation is Singular Value Decomposition (SVD). With SVD, any rectangular m by n matrix $\mathbf{A}_{mn}$ can be decomposed in the following way:

$$\mathbf{A}_{mn} = \mathbf{U}_{mm} \mathbf{\Sigma}_{mn} \mathbf{V}_{nn}^T. \quad (2.35)$$

Here, $\mathbf{\Sigma}$ is the scaling matrix which contains on its diagonal, in descending order, the singular values $\sigma_1 > \sigma_2 > \dots > \sigma_n$. $\mathbf{U}$ and $\mathbf{V}$ are called the right and left singular vectors, respectively. They are unitary matrices whose columns form orthonormal bases. The vectors can be thought of as containing information about the underlying structure of the matrix $\mathbf{A}_{mn}$ with the first vector containing the most important component, followed by the rest in descending order of importance. The original matrix can then be reconstructed from these vectors. The trend of the decay of singular values in $\mathbf{\Sigma}_{mn}$ is an indicator for the compressibility of matrix $\mathbf{A}_{mn}$. A fast decay indicates that the original matrix is of low rank and suitable for compression.

By truncating the sequence of singular values at the appropriate place an approximation $\tilde{\mathbf{A}}$ to $\mathbf{A}_{mn}$ can be constructed:

$$\tilde{\mathbf{A}} = \tilde{\mathbf{U}} \tilde{\mathbf{\Sigma}} \tilde{\mathbf{V}}^T. \quad (2.36)$$

This concept is visually sketched in Figure 2.11, where it is immediately apparent that the new form requires much less memory, since $\tilde{\mathbf{U}}$ and $\tilde{\mathbf{V}}$ are now ‘skinny’ rectangular matrices. The Eckhart-Young theorem states that given a rank r of the approximation matrix $\tilde{\mathbf{A}}$, the Truncated Singular Value Decomposition provides the optimum approximation to $\mathbf{A}_{mn}$ under the Frobenius norm.

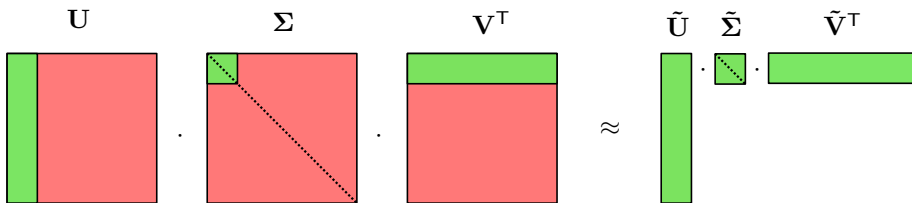


Figure 2.11: Visual representation of a low rank matrix approximation with the Singular Value Decomposition. The sequence of singular values is truncated allowing for an approximate representation of the original matrix as a product of skinny matrices.

$\tilde{\mathbf{A}}$ can now be used as an approximation to $\mathbf{A}_{mn}$, requiring less memory and allowing for faster matrix-vector products. The matrix-vector product with a

vector $\mathbf{x}$ is then performed directly with the reduced versions of the left and right singular vectors:

$$\tilde{\mathbf{A}} \cdot \mathbf{x} = \tilde{\mathbf{U}} \cdot \tilde{\mathbf{\Sigma}} \cdot \tilde{\mathbf{V}}^\top \cdot \mathbf{x}. \quad (2.37)$$

In the following sections, this low rank approximation is used for the construction of a hierarchical matrix, which allows for an efficient representation of the integral operators in the model.

2.4.6 Hierarchical matrices

The hierarchical matrix representation approximates the original matrix by splitting it into a set of sub-matrices of lower rank. The sub-matrices, also called blocks representing far-field interactions, are then compressed using the low rank approximation described above. The splitting is performed by forming a cluster tree that contains all the blocks of the matrix in a hierarchical ordering. There are many different algorithms available to perform such splitting, which all use different criteria, but all are based on the spatial distribution of the source - and target elements.

The choice of which sub-matrices are then compressed is based on an admissibility criterion η , based on the average distance R_d between- and the diameter D_b of the element groups associated with the block. A block is compressed if:

$$D_b \geq \eta R_d. \quad (2.38)$$

If the admissibility criterion is too low, groups that are too close will also be approximated. In this case the specified tolerance is still guaranteed, but the matrix-vector product of the block will be slower.

The technique described above is based on the work of Hackbush [82] and Bebendorf [83] and was implemented in the Julia programming language by Faria in the package `Hmatrices.jl` [84].

An example of this implementation in Julia with a visual representation of the underlying block structure is shown in Figure 2.12a, where the fully dense inductance matrix of five unit cells of a 40-strand Rutherford cable is shown. Next to it, in Figure 2.12b, the hierarchical matrix representation of the same matrix is shown. Finally, the hierarchical matrix for two twist pitches is shown in Figure 2.12c. The density of sub-matrices reduces as one moves further away from the diagonal, where the elements are farther away from each other. A $\mathcal{H}$ -matrix of this size has a set-up time of several minutes on a single thread of an Intel i7-6700 CPU.

Note that the kernel function in this case does not only depend on the distance between the two elements, but also on their relative orientation. By naively applying the $\mathcal{H}$ -matrix on such a kernel function it is still possible to obtain good results. However this only works with method like the *a posteriori* truncation of the SVD is used [84]. Methods that rely on sampling the kernel, while making assumptions

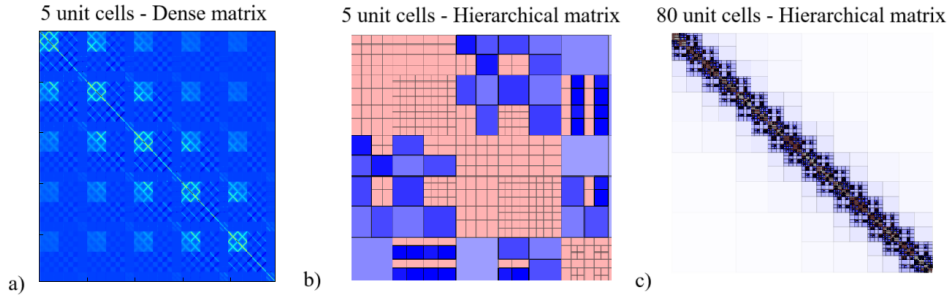


Figure 2.12: **a)** Inductance matrix of the first five unit cells of a Rutherford cable consisting of 40 strands. **b)** Block structure of the hierarchical matrix for five unit cells. The red blocks are dense, whereas the blue blocks are compressed, with a lighter color indicating a higher compression. **c)** Block structure of the hierarchical matrix for 80 unit cells or two twist pitches of the cable.

about the analytic properties, such as the Adaptive Cross Approximation, can fail in this case.

In general, Fast Multipole Methods have less set-up time than $\mathcal{H}$ -matrices. However, since the inductive coupling in the model is purely geometric, the $\mathcal{H}$ -matrix does not change during or between simulations of one specific type of cable. Therefore, some time may be invested in generating an efficient $\mathcal{H}$ -matrix, after which it is stored on the hard disk and can be used many times for different kinds of simulations. No direct comparison is made here, but it is stated in literature that the matrix-vector product performance of an $\mathcal{H}$ -matrix can out-perform the equivalent calculation of an FMM implementation [86].

2.4.7 Fast inductive voltage computation for periodic quasi-1D structures

The concept of the $\mathcal{H}$ -matrix described above is now applied to the Rutherford cable in such a way as to exploit its periodicity. In fact, this method may be applied to any quasi-1D structure with a periodic geometry such as a Rutherford cable. As in the case of $\mathcal{H}$ -matrices, a subdivision of the main matrix is performed, but this time it is specifically determined by the dimensions of the block of a single unit cell of the cable, which is equal to the number of elements it contains (N_e). Each block in the structure has a length of $2^n N_e$ for $n = 1, 2, 3, \dots$

First, dense blocks of length N_e , representing the near-field interactions, are placed on the diagonal. In the case of a typical Rutherford cable it was found that besides the main diagonal, the first two lower and upper diagonals must also be dense. The singular values of these diagonals do not decay fast enough for efficient compression. With these blocks as the base, the block structure is grown further from here. The size of the blocks increases with distance from the diago-

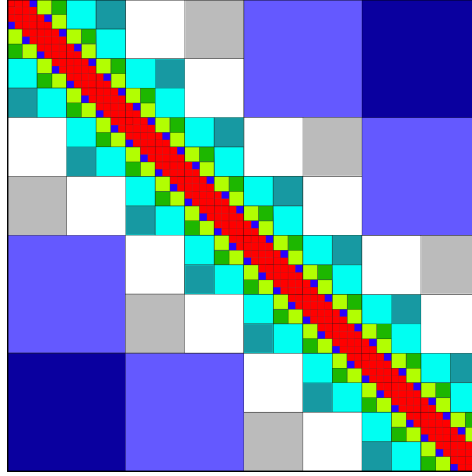


Figure 2.13: *Block structure of the specialized $\mathcal{H}$ -matrix for a Rutherford cable model with 64 unit cells. Except for the dense red blocks, all blocks with the same color and size are identical or transposed.*

nal. Finally, a block structure as shown in Figure 2.13 is obtained. All non-dense blocks with same color are identical or transposed, which is a property that will be exploited. As discussed in Section 2.4.6, all admissible blocks are compressed with a low rank approximation obtained through a Truncated Singular Value Decomposition (TSVD). The TSVD is computationally quite expensive, but in this case it only has to be performed on each unique block once. Further acceleration is achieved because the TSVD algorithm truncates after finding the first n singular values. The largest effort to the set-up will be caused by the largest far-field blocks. To determine at which point the sequence of singular values has to be truncated the sequence is normalized with respect to the first and largest singular value σ_1 . The sequence is then simply truncated at the rank for which the corresponding normalized singular value is less than the user defined relative tolerance. An example is shown in Figure 2.14, where all values in the red zone are truncated. As the blocks represent interactions that are increasingly further away, their singular values decay faster.

Because of the repeating block structure, the matrix-vector product can now be performed in a more optimal way. For each block with length h , the vector is reshaped into a dense matrix with h rows. Reshaping a matrix does not allocate memory and therefore poses a low computational burden. This way the matrix-vector product of the full matrix is transformed into a sequence of dense matrix-matrix products with the dense and compressed blocks for each unique diagonal block. Figure 2.15 schematically illustrates this process. Matrix-matrix products have a higher FLOPS count than matrix-vector products, allowing for significantly faster execution.

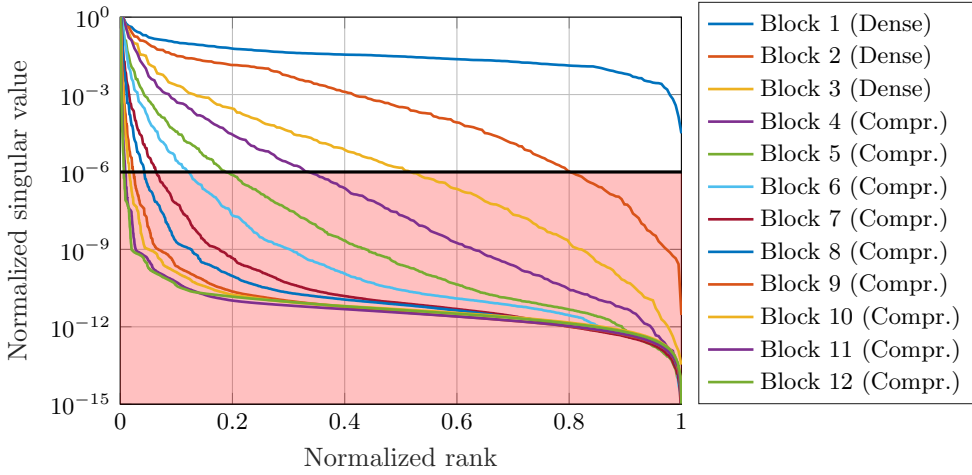


Figure 2.14: Decay of singular values for the first ten blocks of the inductance matrix shown in Figure 2.13. All singular values in the shaded region below 10^{-6} are truncated to obtain the low rank approximation with $\varepsilon = 10^{-6}$.

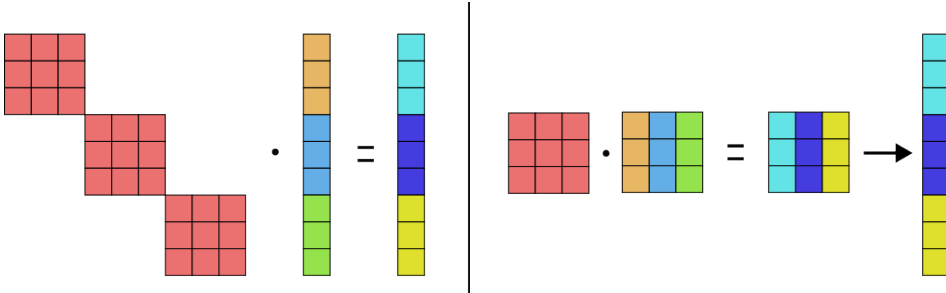


Figure 2.15: Matrix-vector products of the repeating block diagonals are replaced with dense matrix-matrix products.

In Figures 2.16a, the time-cost of the matrix-vector product is shown for Rutherford cables of different lengths with the three methods discussed above. Clearly, the specialized approach (denoted LowRankBlocks) is faster than the standard $\mathcal{H}$ -matrix method in this case, by a factor of five.

The block partition is grown from the base block, representing the unit cell, outward. This also means that there is a limitation on the size of the inductance matrix. It can only consist of a specific number of blocks that fit the block partition. A few more options can be included by adaptively changing the block structure of the far-field blocks. To allow all possible combinations, the matrix has to be padded with rectangular far-field blocks at the sides.

In Figure 2.16b, the execution time is plotted as a function of the specified

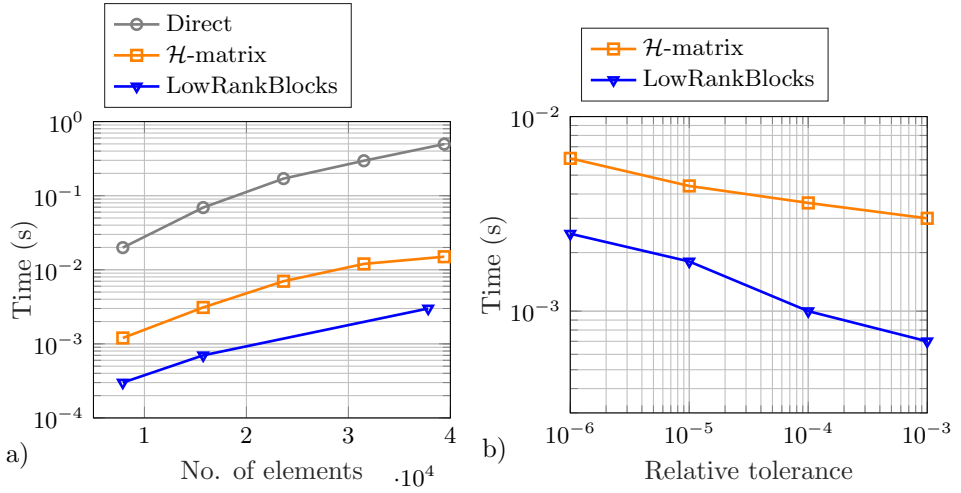


Figure 2.16: a) Execution time versus number of elements of different methods for calculating the inductive voltage with a relative tolerance of 10^{-3} . b) Execution time versus relative tolerance of the two fast methods. The number of elements is 1.58×10^4 .

relative tolerance. For both methods the full indicated range of tolerance criteria is met, but for the specialized method the actual tolerance achieved was almost an order of magnitude better than for the $\mathcal{H}$ -matrix. All methods were executed on an Intel(R) Core(TM) i7-12700 running at 2.10 GHz.

2.4.8 Self-field calculation

During a quench, current redistribution causes the self-field of the cable to change with time. To account for this effect, the self-field contribution to the magnetic field is updated each time step. To avoid singularity effects in the strand magnetic self-field, this is achieved by setting the targets of the self-field calculations in-between the strand elements. The magnetic field at the strand element locations is then interpolated. Figure 2.17 shows the position of the magnetic self-field calculation points with respect to the strand elements. Only the magnetic field at the strand elements needs to be calculated, as these are the only elements in the model with field-dependent properties. To speed up the self-field calculation, the magnetic field generated by each source element at each target point is calculated once for a unit current. The calculation is performed using the Biot-Savart law. The calculated value is then put in a matrix, which is later multiplied by the vector of currents to obtain the magnetic field. Three matrices are needed for the three magnetic field components. The matrices are compressed in a similar fashion as the inductance matrix using low rank approximations.

The main background magnetic field of the entire magnet is calculated using ROXIE software [89, 90]. At the start of the simulation, the cable's self-field for

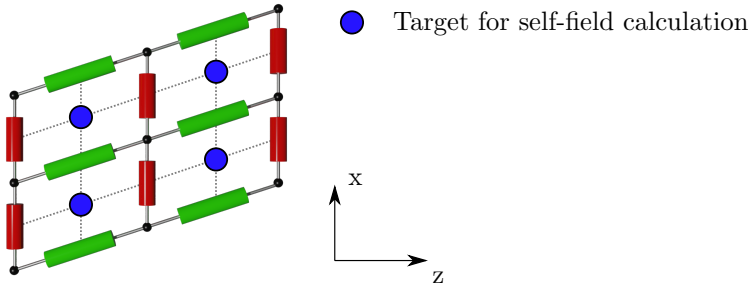


Figure 2.17: Target locations for the self-field calculation in the network of current elements.

a homogeneous current distribution is calculated and its contribution is removed from the values calculated with ROXIE and then replaced each time step with the newly calculated values.

2.4.9 Thermal domain

To calculate the thermal evolution of the system, many network models use a lumped-element network approach. However, such lumped models usually have poor mesh convergence properties so that a small subdivision of the mesh is needed to obtain an accurate result. For quench simulation this problem is exacerbated when strands carry close to their critical current and the quench propagation velocity is high. Therefore in RuNe, a 1D Finite Element Method (FEM) is used, similar to THEA. The FEM is only applied to the strands in the longitudinal direction, because this is a continuous material heat conduction path, resulting in n_s parallel 1D FEM calculations. Since the transverse conductance at the interface of the inter-strand contacts is assumed to be much larger than the bulk conductance of the strand itself, for the cross-section of the strand a uniform temperature distribution is assumed. So the strand-to-strand heat flux is modelled with a lumped approach.

Finite Element Method Derivation

The 1-dimensional time-dependent heat equation including internal and external source terms is [91]:

$$\rho c_p \partial_t \theta = \partial_z (\kappa \partial_z \theta) + p_{\text{int}} + p_{\text{ext}}, \quad (2.39)$$

where ρ is the mass density, c_p is the specific heat, θ is the temperature, and κ the thermal conductance. The heating terms are p_{int} for internal and p_{ext} for external heat sources or sinks, respectively.

The finite element method is applied by first rewriting Equation 2.39 into the weak form [92]. Equation 2.39 is multiplied by a weighting function $\mathcal{F}(z)$ and

integrated over the domain Ω :

$$\int_{\Omega} [\rho c_p \partial_t \theta \mathcal{F}(z) - \partial_z (\kappa \partial_z \theta) \mathcal{F}(z) - p_{\text{int}} \mathcal{F}(z) - p_{\text{ext}} \mathcal{F}(z)] d\Omega = 0. \quad (2.40)$$

To reduce the differential order of the second term, it is integrated by parts. Applying the fundamental theorem of calculus then yields:

$$\underbrace{\int_{\Omega} \rho c_p \partial_t \theta \mathcal{F}(z) d\Omega}_{\text{Thermal inertia}} = - \underbrace{\int_{\Omega} \kappa \partial_z \theta \partial_z \mathcal{F}(z) d\Omega}_{\text{Thermal conductance}} - \underbrace{\int_{\Gamma} \mathcal{F}(z) \kappa \partial_z \theta d\Gamma}_{\text{Heat flow BC}} - \underbrace{\int_{\Omega} (p_{\text{int}} + p_{\text{ext}}) \mathcal{F}(z) d\Omega}_{\text{Heat load}}. \quad (2.41)$$

The differential order of the temperature is decreased by one whereas the weighting function is increased by one order. Only first-order derivatives are now present, thus the weak form is obtained.

To discretize the solution, $\mathcal{F}(z)$ is represented by a sum of *local* basis functions, in FEM more commonly called shape functions:

$$\mathcal{F}(z) = \sum_{i=1}^N \theta_i \phi_i(z), \quad (2.42)$$

where θ_i is the node temperature and $\phi_i(z)$ is the nodal shape function. The nodal shape functions are nonzero only in a specific part of the domain. A common type of shape function employs Lagrange polynomials of any specified order to describe the solution over the entire domain. Here, we employ linear or quadratic Lagrange type basis functions as depicted in Figure 2.18. The linear elements only have two nodes at their boundaries, while the quadratic elements have an additional internal node. To map the thermal nodes onto the voltage nodes of the PEEC model, the quadratic elements have to be twice as long as the strand elements, spanning one unit cell of the electrical network.

From the nodal shape functions the *stiffness* matrix $\mathbf{K}$, which comprises the thermal conductance between the nodes, and the *mass* matrix $\mathbf{C}$, which describes the thermal capacitance of the nodes, are constructed. These matrices are constructed as follows:

$$\mathbf{C}_{i,j} = \int_{\Omega} \rho c_p \phi_i \phi_j d\Omega, \quad (2.43)$$

$$\mathbf{K}_{i,j} = \int_{\Omega} \kappa \partial_z \phi_i \partial_z \phi_j d\Omega. \quad (2.44)$$

The external and internal heat loads are then included as:

$$\mathbf{f}_i = \int_{\Omega} (p_{\text{int}} + p_{\text{ext}}) \phi_i d\Omega. \quad (2.45)$$

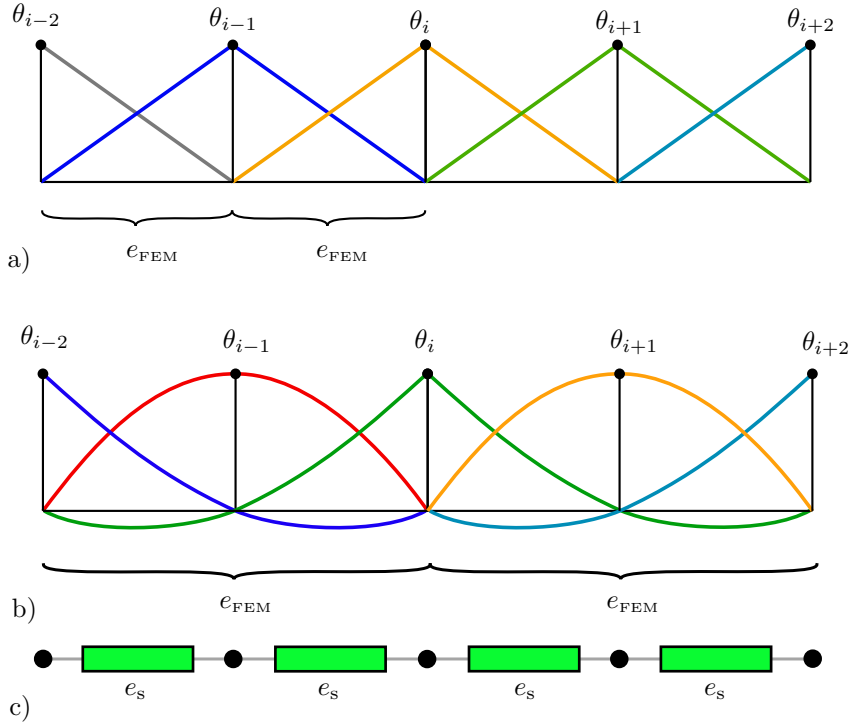


Figure 2.18: Linear (a) and quadratic (b) nodal shape functions in relation to the electric network elements (c). One linear FEM element corresponds to one electric strand element, whereas one quadratic FEM element corresponds to two electric strand elements, thus spanning one unit cell of the Rutherford cable network model.

With these matrices, the differential equation that describes the time evolution of the system is obtained:

$$\mathbf{C} \partial_t \boldsymbol{\theta} = \mathbf{K} \boldsymbol{\theta} + \mathbf{f}. \quad (2.46)$$

Matrices $\mathbf{K}$ and $\mathbf{C}$, as well as the load vector $\mathbf{f}$ all strongly depend on the thermal material properties and need to be updated at each function evaluation.

Thermal conductance

The longitudinal thermal conductance κ of the strand *itself* is a function of the copper properties only, since the thermal conductivity of Nb_3Sn is orders of magnitude lower and is therefore not considered here. κ depends on temperature, local magnetic flux density and the RRR value of the copper. The thermal contact resistance *between* strands k is more difficult to quantify, even more so than the electric contact resistance. The thermal contact resistance of a copper-to-copper interface contains both an electron and a phonon contribution. In the case of a

fully oxidized surface, the phonon contribution dominates, since electron transport is hampered by the oxide layer. However, only a small fraction of the surface area has to make a ‘clean’ metal-to-metal contact for the electron contribution to become the dominant one, since it constitutes a much more efficient heat transport process. In practice, a mix of both contributions is likely to be present [93].

In the relevant temperature range of 1.9 K to 30 K, the electron contribution scales linearly with temperature, whereas the phonon contribution scales with the third power. For practical contacts, this results in a temperature scaling factor b ranging from 1.5 to 2.2 [88, 96, 97]. The thermal contact conductance k can then be expressed as:

$$k = \alpha T^b. \quad (2.47)$$

Due to the lack of proper reference values for the parameters α and b , in this work they will be used as fitting parameters and a parametric sweep within physically meaningful limits will be performed.

Nb₃Sn cables in accelerator magnets are usually impregnated with an epoxy resin, which has a relatively poor thermal conductivity. The heat transfer to adjacent cables and to helium on the timescale of a quench can therefore be considered negligible for the purpose of studying normal zone development. Normal zones in Nb₃Sn magnets do also propagate transversely turn-to-turn, but this happens at a much slower pace than longitudinal propagation. Therefore, when studying relatively short cable segment containing only a few twist pitches, the domain may be considered adiabatic. In other words, no thermal coupling with neighbouring cables or helium is included.

At the simulation’s domain boundaries (at the cable ends) a constant temperature or temperature gradient is enforced, i.e. Dirichlet or Neumann boundary conditions.

Comparison of methods

In Figure 2.19 the temperature profile after 1 ms in a single quenched strand is shown for different models, methods, and mesh element sizes. The strand current is set equal to its critical current. This constitutes a numerically worst-case scenario, since the fast quench propagation combined with the non-linear material properties can lead to significant errors. A numerical ‘ground truth’ is shown, which is calculated with both RuNe and THEA with a mesh element size that is fully converged and is about ten times smaller than RuNe’s element size. All the other cases have an element size of 1.25 mm, which is similar to the typical element size for a Rutherford cable in RuNe.

The lumped element method has the largest error and is not suitable for this use case. The FEM with linear shape function performs a lot better, with RuNe’s

implementation outperforming THEA's. When employing quadratic shape functions, both THEA and RuNe achieve an error of less than 2%. For all methods, the numerical error always resulted in an overestimation of the normal zone propagation velocity.

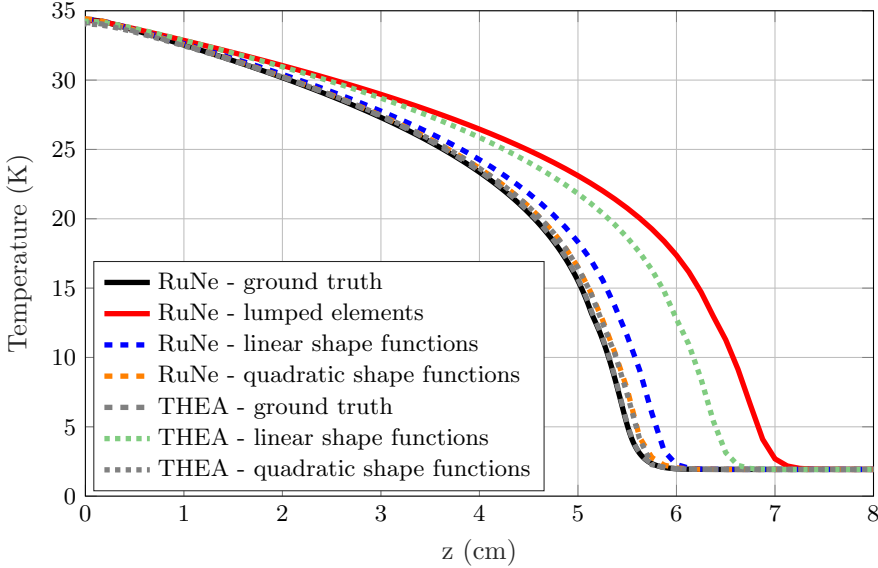


Figure 2.19: Temperature of a quenching strand carrying I_c a function of position after 1 ms of a strand at I_c for different configurations of the model. The quench is initiated at $x = 0$ m with a heat pulse of $100 \mu\text{s}$ in one node. A comparison to THEA is also shown. The strand carries 659 A (I_c) at 11 T and 1.9 K. The RRR of the copper matrix is 100.

2.4.10 Coupling between domains

The thermal and electric domains of the model are coupled through the heat generation in strands and contact resistances, and through the temperature dependence of the critical current and copper resistivity. In numerical modelling, separate physics domains are often solved separately. This is called ‘soft-coupling’, or ‘explicit coupling’ and it is often used when the time scales of the two domains differ considerably, which means it is more efficient to have different time-stepping schemes for the two domains. However, this does imply that a good criterion has to be chosen for when to initiate a new time-step for both domains.

Another way of coupling the domains is by solving both concurrently in the same system of equations. This is called ‘hard-coupling’ or ‘implicit coupling’. The advantage here is that the interplay between the physics domains is always accurately modelled, at least when the models of the two domains are accurate and the quantities are properly scaled. The down-side of hard-coupling is that it may lead to significantly longer simulation times. During a quench, the time

scales of the different processes vary wildly between different stages of the quench and may differ between simulations with different initial conditions, such as highly inhomogeneous current distributions. To guarantee numerical accuracy, here all simulations have hard-coupled domains unless stated otherwise.

The power generation in the combined thermal-electric model comes from heating of strand elements when passing from the superconducting state to the normal state and from the coupling loss due to inter-strand current transfer through the inter-strand resistances. The power generated in the elements, stored in the vector $\mathbf{p}_e$, is calculated as $I^2 \cdot R_{a/c}$ for the resistance elements and as $I \cdot V_e$ for the strand elements. The power dissipated in each thermal node from all connected elements is then readily calculated using the KCL matrix, which already contains the element-to-node connectivity:

$$\mathbf{M}_{e \rightarrow n} = \frac{1}{2} \mathbf{M}_{\text{kcl}}^{|\cdot|}. \quad (2.48)$$

Here $|\cdot|$ signifies that the element-wise absolute value of the matrix is taken. Each node takes up half of the power generated in its connected element. The heat deposited into the nodes is then calculated as:

$$\mathbf{p}_n = \mathbf{M}_{e \rightarrow n} \mathbf{p}_e, \quad (2.49)$$

The coupled system equation, assuming first-order basis functions for the thermal FEM, then takes the following form [70, 71]:

$$\begin{aligned} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{L}_{\mathcal{H}} & 0 \\ 0 & 0 & -\mathbf{C} \end{bmatrix} \begin{bmatrix} \partial_t \mathbf{v} \\ \partial_t \mathbf{i} \\ \partial_t \boldsymbol{\theta} \end{bmatrix} + \begin{bmatrix} 0 & \mathbf{M}_{\text{kcl}} & 0 \\ \mathbf{M}_{\text{kvI}} & \mathbf{R}_m & 0 \\ 0 & 0 & \mathbf{K} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \\ \boldsymbol{\theta} \end{bmatrix} \\ + \begin{bmatrix} \mathbf{i}_{\text{ext}} \\ \mathbf{v}_{\text{ext}} \\ \mathbf{p}_{\text{ext}} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{v}_e(\mathbf{i}, \boldsymbol{\theta}, \mathbf{B}) \\ \mathbf{p}_n(\mathbf{i}, \boldsymbol{\theta}, \mathbf{B}) \end{bmatrix} \cong \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (2.50) \end{aligned}$$

where $\mathbf{L}_{\mathcal{H}}$ is the hierarchical inductance matrix discussed in Section 2.4.6.

2.4.11 Solving strategy

The final form of the system equation is expressed as an implicit Differential Algebraic Equation (DAE), which is solved with the Sundials IDA solver [100, 101] implemented in the Julia programming language [102]. The initial current distribution in the cable is found by solving a steady-state version of the network model with user-supplied boundary conditions. In the case some of the strand elements are near the non-linear superconducting-to-normal transition a non-linear solver is used. The initial temperature of all the nodes is set either to 1.9 K or 4.5 K. When the system is non-linear a form of Newton iteration has to be performed to

obtain the solution of each time step iteratively. The Sundials IDA solver is also used by JackPot-AC and by ElMaTh [59, 70]. The solver sequence is similar to these models, following the basic structure of the DASLL and DASPK algorithms, on which Sundials IDA is based [98, 99]. It takes the following three functions as input:

- **Residual function** - This function calculates residuals of the full system Equation 2.50.
- **Preconditioner function** - This function solves the linearized Jacobian of the residual function. This includes the matrix-free calculation of the inductive voltage and is therefore solved iteratively. For electro-thermal problems the system matrix is asymmetric so the Generalized Minimal Residual method (GMRES) is used. To solve the non-linear differential algebraic equation the Jacobian is needed. For a DAE this is defined as [98]:

$$\mathbf{J} = \frac{\partial F_{\text{res}}}{\partial \mathbf{y}} + \gamma \frac{\partial F_{\text{res}}}{\partial \mathbf{y}'}, \quad (2.51)$$

where F_{res} is the residual function, the left-hand side of Equation 2.50, with $\mathbf{y}$ it's solution vector, and γ is the inverse of the present step size dt .

In [99] this preconditioning method was included in the solver in the form of a matrix-free subspace Krylov method, which in this case allows for including the hierarchical inductance matrix, like in [70, 71]. $\mathbf{J}$ is then represented by the following linear operator:

$$\mathbf{J} = \gamma \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{L}_{\text{nf}} & 0 \\ 0 & 0 & -\mathbf{C} \end{bmatrix} + \begin{bmatrix} 0 & \mathbf{M}_{\text{kcl}} & 0 \\ \mathbf{M}_{\text{kvl}} & \mathbf{R}_{\text{m}} + \mathbf{R}_{\text{nl}} & 0 \\ 0 & \mathbf{M}_{\text{e} \rightarrow \text{n}}(\mathbf{I} \odot \mathbf{v}_{\text{r}}) & \mathbf{K} \end{bmatrix}. \quad (2.52)$$

The term $\mathbf{M}_{\text{e} \rightarrow \text{n}}(\mathbf{I} \odot \mathbf{v}_{\text{r}})$ represents the coupling between the electric and thermal domain. $\mathbf{I} \odot \mathbf{v}_{\text{r}}$ represent a diagonal matrix with the vector $\mathbf{v}_{\text{r}}$, the resistive voltage over the elements, as its diagonal. $\mathbf{R}_{\text{nl}}$ contains the resistance of the non-linear strand elements.

- **Preconditioner set-up function** - This function calculates the factorization of the simplified version of the linearized Jacobian with an inductance matrix $\mathbf{L}_{\text{nf}}$ that only contains specified near-field interactions. This factorization is used to precondition the iterative solver. The choice of which near-field terms are included is a fuzzy consideration that has to be tuned by hand. With too many terms the factorization and linear solution of the matrix become slow. When including too few, the iterative solver may converge slowly or even not at all. In RuNe, only elements with a distance less than $\frac{1}{5}w_{\text{c}}$ that lie on the same longitudinal coordinate are included. The

simplified matrix then takes the form:

$$\mathbf{P}_l = \gamma \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{L}_{\text{nf}} & 0 \\ 0 & 0 & -\mathbf{C} \end{bmatrix} + \begin{bmatrix} 0 & \mathbf{M}_{\text{kcl}} & 0 \\ \mathbf{M}_{\text{kvl}} & \mathbf{R}_m + \mathbf{R}_{\text{nl}} & 0 \\ 0 & \mathbf{M}_{\text{e} \rightarrow \text{n}}(\mathbf{I} \odot \mathbf{v}_r) & \mathbf{K} \end{bmatrix}. \quad (2.53)$$

In essence, the matrix $\mathbf{P}_l$ captures the dominant sources of stiffness of the system. This sparse matrix is factorized with the LU-decomposition, which encodes the Gaussian elimination process into the upper triangular matrix $\mathbf{U}$ and the lower triangular matrix $\mathbf{L}$. The solution to different right-hand sides can then be found through a forward and backward substitution process performed on these matrices.

The matrices are solved with the LU-factorization method provided by the KLU solver package [95]. This solver is specifically tailored to the unstructured sparsity patterns arising from circuit simulations.

The matrix factorization is a computationally expensive part of the solving sequence. Most solvers separate this phase into two steps, the symbolic factorization and the numerical factorization. In the case of a system for which the sparsity pattern does not change between factorizations, the symbolic factorization step can be reused, which is a feature that is applied in RuNe. The superconducting elements that can go from zero to a finite resistivity can be included in the resistance matrix $\mathbf{R}_m + \mathbf{R}_{\text{nl}}$ for the initial symbolic factorization, while being numerically zero. The same holds for their associated losses in the matrix product $\mathbf{M}_{\text{e} \rightarrow \text{n}}(\mathbf{I} \odot \mathbf{v}_r)$.

2.4.12 Convergence, numerical stability and accuracy

At the heart of the Sundials IDA solver lies the linear solver of the inner iteration loop for the non-linear solver used for the time step. This linear solver is a subspace Krylov method, which is preconditioned by a direct linear solver that solves an approximation of the system matrix. Most of the runtime is spent here and the numerical properties of this linear system largely determine the efficiency of the overall solving strategy.

Scaling

The rate of convergence of a solver using the Krylov subspace method such as GMRES depends on the distribution of eigenvalues of the system matrix. Ideally, the eigenvalues need to be clustered and be far away from the origin. For these kinds of linear solvers in the inner iteration loop of a non-linear solver, our type of matrices often do not adhere to this requirement and are ill-conditioned. This is especially true for a hard-coupled multi-physics simulation. To keep the condition number of the system matrix low, the three different output quantities of the model, namely voltage, current, and temperature, are scaled based on their magnitude. For quench simulations, the current is left as is, while one unit of voltage

corresponds to 100 μV and a unit of temperature corresponds to 0.1 K. With these scaling factors the solver converges faster. This scaling is then also applied to the tolerances.

Factorization update

Another factor in the GMRES convergence rate is the preconditioner that is supplied to the solver. The Sundials IDA solver determines when an update of the matrix factorization is necessary, however, it has no direct communication with the iterative solver and therefore has no information on its convergence characteristics. It generally only updates this factorization when there is a meaningful change in the size of the time step. When a stable quench front has developed in the cable, an update of the timestep generally is no longer necessary and the iteration count of GMRES slowly drifts up due to a lack of factorization updates. For a typical quench simulation, this effect can lead to more than double the simulation time. There is no automatic load-balancing between the cost of matrix factorization and GMRES convergence. To mitigate this issue, a forced update of the preconditioner matrix factorization is performed at preset intervals. The optimum frequency of these forced updates depends on the cable properties and on the specified tolerances. For this particular model, the matrix factorization is not a dominant factor in the execution time, so this update frequency is not optimized. A more refined approach would be to update the factorization based on a heuristic algorithm that monitors the iteration count of the iterative solver. Generally, more updates are required when thermal and electric coupling between strands is high and when the tolerance is low.

Tolerances

The solver takes up to two arguments for the allowed tolerance of the *local* error that needs to be adhered to for each time step. For the quench simulations the *relative* tolerance is set to 10^{-5} , whereas the *absolute* tolerance is set to 10^{-4} . It is necessary to set an absolute tolerance because certain state variables are zero at the start of simulation, making the relative tolerance meaningless. The enforced minimization of the *local* error does not guarantee that the *global* error behaves similarly. In addition, errors may accumulate during the time integration. To assure that unacceptably large errors are not accumulated, the model was tested with both the relative and the absolute tolerance set two decades smaller, which resulted in a negligible difference in the computed solution.

Lumped mass matrix

In many finite element models, a lumped mass matrix is used to ease the computational burden. The off-diagonal components in the mass matrix, which increase the stiffness of the matrix, are removed and instead lumped together with the diagonal ones. The most simple method to achieve this is row-sum lumping, which simply takes the sum of each row as the diagonal entry. In a lot of applications,

this has only a limited effect on the accuracy of the solution. For quench simulation this is not always the case, since the thermal capacitance can vary over orders of magnitude for characteristic cases. To retain accuracy, it is therefore best to avoid it in this application. However, the matrix $\mathbf{P}_l$, which serves as the preconditioner for GMRES, can still be lumped without losing accuracy in the final solution and with only minimal impact on the convergence of GMRES. Since $\mathbf{P}_l$ is solved with a direct method, this is also where a lumped mass matrix yields the largest performance gain.

Accuracy

Given some of the simplifications in the model, such as the assumptions for the self- and mutual inductances and the homogeneous material properties over the strand cross-section, an accuracy of 5 % to 10 % is expected. In comparison, the largest uncertainty is expected to arise from the unknown input parameters, such as the inter-strand contact resistances. Moreover, these values can also have a spatial distribution in the cable, whereas in the model they are assumed to be uniform.

2.5 Quench antenna calculation

Once the current evolution within the Rutherford cable is known in 3D space, the resulting voltage response of the quench antennas can be calculated. A quench antenna consists of pick-up loops that experience an e.m.f. proportional to the rate-of-change of flux linkage through the loop, see Figure 2.20. A more detailed description of the quench antennas used with HL-LHC magnets is given in Section 3.2. This e.m.f. can be calculated by taking the time derivative of the flux through the pick-up loop. The flux is calculated by taking the surface integral of the magnetic field $\mathbf{B}(x, y, z, t)$ or by the contour integral of the magnetic vector potential $\mathbf{A}(x, y, z, t)$:

$$\Phi(t) = \int_S \mathbf{B}(x, y, z, t) \cdot d\mathbf{a} = \oint \mathbf{A}(x, y, z, t) \cdot d\mathbf{s}. \quad (2.54)$$

The induced e.m.f. is then calculated as:

$$\mathcal{E} = -n_{\text{ant}} \frac{d\Phi}{dt}, \quad (2.55)$$

where n_{ant} is the number of turns in the pick-up coil of the quench antenna, $d\mathbf{a}$ is an infinitesimal vector element of the surface bound by the quench antenna and $d\mathbf{s}$ is an infinitesimal vector element of the contour. The current distribution in the Rutherford cable is stored in a matrix containing the current in each element and the coordinates of the begin - and end points of this element: $\{I, x_a, y_a, z_a, x_b, y_b, z_b\}$. The magnetic field or vector potential as a result of these line current can then be calculated. With the 3D coordinates of a line element's begin - and end points as well as its current we can calculate the magnetic field

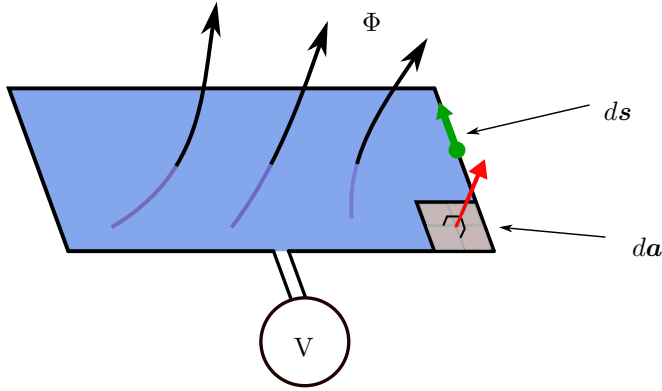


Figure 2.20: Magnetic flux through a pick up coil with vector elements ds and da for line - (green) and surface (red) integrals, respectively.

components that it generates at a specific target point with the Biot-Savart law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \oint \frac{I d\mathbf{l} \times \mathbf{r}}{|\mathbf{r}|^3}, \quad (2.56)$$

where μ_0 is the permeability of free space, I is the current, $d\mathbf{l}$ is the line element vector, and $\mathbf{r}$ is the displacement vector from the source to the target. The treatment of the Biot-Savart law for the the magnetic field of a straight line element between two points a and b is taken from [103]. A similar treatment for the magnetic vector potential is taken from [104]. If the pick-up coil consists of straight wire elements, a direct calculation of the mutual inductance between the cable elements and the wire elements of the pick-up coil using Grover's formula's can be performed [63].

2.5.1 Superconductor magnetization

Besides the redistribution of transport current, the superconducting magnetization currents that get erased during a quench also cause a magnetic moment. In the quench antenna calculation, the superconductor magnetization M assigned to each strand element is modelled as a point-like magnetic dipole placed at the location of that strand element. Each strand element has a magnetic moment m which is given by:

$$m = M l_s A_s \quad (Am^2), \quad (2.57)$$

where l_s is the length of a strand element and A_s is the cross-sectional surface area of the strand. The magnetic field around a magnetic dipole placed in the origin can be calculated with the following equation:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{m}) - \mathbf{m}}{|\mathbf{r}|^3} \quad (T), \quad (2.58)$$

where $\mathbf{r}$ is the location vector and $\mathbf{m}$ is the vector with the x-, y-, and z-components of the magnetic moment. The quench antenna is sufficiently far away from the strand elements for this far-field approximation to be valid.

For the simple case of a round superconducting filament fully penetrated by the magnetic field, the magnetization is:

$$M_{\text{round}} = \frac{2}{3\pi} J_c d_{\text{fil}} f_t \quad (\text{Am}^{-1}), \quad (2.59)$$

where d_{fil} is the filament diameter, J_c is the critical current density and f_t is a correction factor for the presence of transport current in the strand [18]:

$$f_t = 1 - \left(\frac{J_t}{J_c} \right)^2, \quad (2.60)$$

where J_t is the transport current density. Filaments in Nb_3Sn wires are more akin to hollow cylinders, since during the heat treatment the A15 phase of Nb_3Sn is formed in a ring around the outside of the filament. Following [105], the magnetization of such a hollow filament with inner radius r_{in} and outer radius r_{out} is calculated from the result of Equation 2.59 as follows:

$$M_{\text{hollow}} = (1 - \nu^3) M_{\text{round}}, \quad (2.61)$$

with $\nu = r_{\text{in}}/r_{\text{out}}$.

Alternatively, values of the magnetization at the strand positions in the cross-section of a cable in a magnet can also be calculated using ROXIE [89, 94], which takes into account the magnetic history, transport current and evolution of J_c throughout a ramp.

In the quench antenna calculation, the magnitude of the magnetic moment due to the magnetization is then calculated by updating J_c and J_t in Equation 2.59 each timestep. In practice, the contribution of the collapse of filament magnetization during quench only has a small influence on the total induced voltage in the pick-up coils.

2.5.2 Example calculation

In Figure 2.21, an example of the result of such a quench antenna calculation is shown. The pole turn cable, which in this specific case has a non-uniform initial current distribution, and the quench antenna are placed in their correct relative positions and the field normal to the surface spanned by the quench antenna is calculated. The time derivative of this magnetic field is visualized with a color plot. Simultaneously, the color plot on the surface of the Rutherford cable shows a snapshot of the current distribution during the quench.

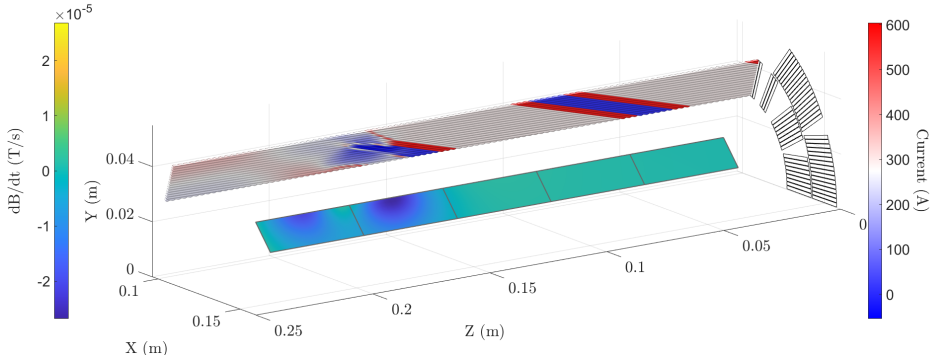


Figure 2.21: 3D visualization of the current evolution of a developing quench in a Rutherford cable with an inhomogeneous current distribution. The calculated dB/dt is shown as a color plot on the surface spanned by the pick-up coils of the quench antenna.

2.6 Conclusion

The numerical tools used in this work for predicting the thermal and electric behavior of superconducting Rutherford-type of cables were presented. Two models, corresponding to the two relevant time scales studied here were discussed. THEA is best suited for relatively long time scales, corresponding to the magnet ramp and cycling operation. RuNe is newly developed to study the shorter timescales involved in quench development and propagation. In both cases, a particular focus is placed on the current redistribution between the strands in the Rutherford cable.

A parametric description of the Rutherford cable geometry, which is used for both models, was provided. All geometric parameters, including the discretization size can be modified.

The THEA software, with its continuum model approach, is used to describe current redistribution over longer lengths and time scales. The cable properties are averaged over the length, which results in a numerically efficient cable description. THEA can also simulate network topology on short length scales, but incurs an extra overhead in terms of mesh density in doing so, and uncertainties arise in simulating fast transients due to the localized inductance matrix.

A new 3D PEEC-FEM model of a Rutherford cable for simulating quench development and propagation in Rutherford-type of cables was presented. The model combines methods previously used in CUDI, THEA, JackPot and ElMaTh. The electric domain is treated with the Partial Element Equivalent Circuit (PEEC) method, where the voltage at the nodes and the current in the elements are calculated using the Modified Nodal Analysis. To mitigate the computational cost of the integral operators, which result in dense matrices, a hierarchical matrix specifically tailored to periodic structures was implemented. The thermal domain

is treated with the Finite Element Method (FEM). For the FEM, up to second-order basis functions were implemented to specifically deal with the need for high accuracy in simulating quenches with a high quench propagation velocity. Both domains are hard-coupled and solved implicitly in one system matrix, similar to ElMaTh, using the Sundials IDA solver.

In Chapters 4 and 5, these models will be used to study the mechanisms underlying the anomalous diagnostic data that were recorded from HL-LHC magnets and to explain how likely conductor damage in their coils can be concluded from these measurements.

Chapter 3

Instrumentation and experimental techniques

In this chapter, the experimental techniques that were used to study magnet performance and to diagnose performance issues, are introduced. In terms of physical measurement tools, the two most important types are:

- **Voltage taps** - Voltage taps are placed at various locations on the cable of a magnet to measure the voltage drop over specific conductor segments.
- **Quench antennas** - Quench antennas consist of arrays of inductive pick-up coils that are typically placed in the magnet bore. They pick up disturbances in the magnetic field caused by a quench, but can also be used to study different types of dynamic phenomena.

By combining the measurement *tools* with different measurement *strategies*, such as measuring at different ramp rates and temperatures, the performance of a magnet can be studied with sufficient spatial and temporal resolution to clarify certain R&D issues. In this thesis work, the focus is on Nb₃Sn magnets with special attention to conductor damage.

First, a general description of the working principal of the diagnostic tools and their application in accelerator magnets at CERN is given. Then the instrumentation and the design of the main experiment presented in this thesis, in which damage is introduced in a magnet in a controlled manner, is described.

3.1 Voltage tap measurements

Depending on the type of magnet, specific lay-outs of the voltage taps can be applied. Demonstration magnets that are still in the development phase usually have a multitude of voltage taps in their winding pack, enabling validation of their

design and the study of specific performance. Magnets intended for series production generally only have diagnostic voltage taps at their splices, where they are placed on both sides or only on the Nb-Ti current lead side of the splice, and are used mostly, but not exclusively, for magnet protection and for monitoring the splice resistance, see Figure 3.1.

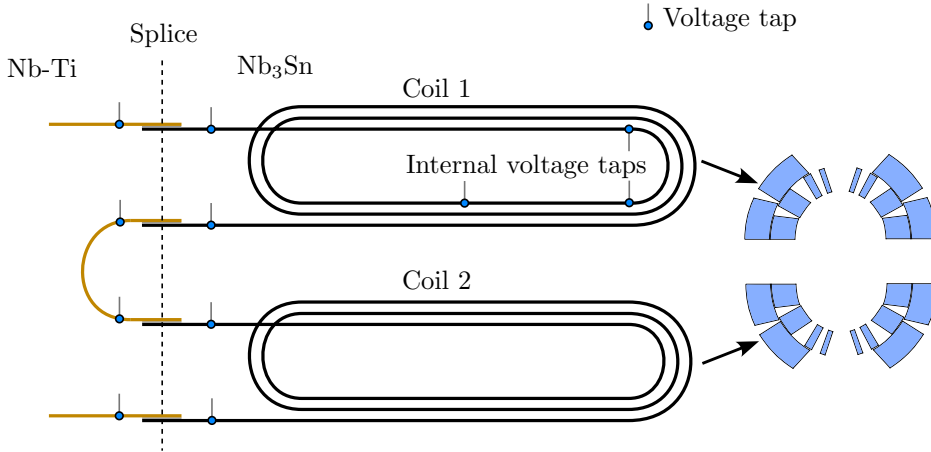


Figure 3.1: Schematic overview of voltage tap locations in a Nb₃Sn accelerator magnet. The Nb₃Sn coils of the magnet are connected to the power converter through Nb-Ti cable-based current leads, while also coil-to-coil inter-connections are realized with Nb-Ti cables.

Two main types of measurements are performed with these voltage taps:

- **Low-frequency, high-accuracy V-I measurements** - These measurements are used to assess the conductor performance within the magnet, usually performed with a sampling rate of 1 Hz to 5 Hz, with a precision of 0.1 μ V to 0.5 μ V. The magnet current is increased in steps and the voltage is measured on the constant current plateaus. Characteristic voltage levels for magnets with damaged conductors are in the range of 0.1 μ V to 20 μ V.
- **High-frequency, medium-accuracy quench voltage measurements** - The voltage that develops during a quench event serves as the trigger for the protection system. The sampling rate is in the range 10 kHz to 100 kHz, read out with a precision of about 1 mV. However, the development of the quench voltage can also be used for diagnostic purposes. The quench voltage builds up to a maximum of about 500 mV before the protection system triggers, whereas the detection threshold usually already triggers somewhere in the range of 50 mV to 200 mV, with a validation time of 3 ms to 10 ms to avoid false positives.

3.1.1 Sources of voltages during magnet operation

Voltages measured over a full coil or over long conductor segments of the magnet, may originate from a number of distinct sources. While performing V-I measurements, some "parasitic" sources may dominate over "relevant" voltages generated by conductor damage that we wish to study:

- **Noise** - Several factors can contribute to noise, but the dominant contribution is related to the stability of the power converter, which exhibits a non-negligible current ripple that produces a spectrum of inductive voltage noise components.
- **Inter-Filamentary Coupling Currents (IFCC)** - Coupling current arising between twisted filaments cause resistive voltages within the strands. The magnitude of these IFCCs is determined mainly by the filaments' twist pitch and by the matrix resistivity [60].
- **Inter-Strand Coupling Currents (ISCC)** - Coupling currents are also induced between the strands of the Rutherford cable. Their magnitude is largely a function of the inter-strand contact resistance, of the cable width, and of the cabling twist pitch [60].
- **Boundary Induced Coupling Currents (BICC)** - These are coupling currents that are induced by internal 'boundaries' within the magnet, such as a sharp change of the magnetic field sweep rate in the coil head or a sudden change in the cable's R_c value due to spatial variations in the winding conditions [60].
- **Magnetization** - Persistent currents in the superconducting filaments magnetize the superconductor. Thermal fluctuations sometimes trigger a sudden collapse of this magnetization, which is called a flux jump and produces a characteristic voltage spike. For Nb_3Sn magnets, flux jumps mostly occur at magnetic field levels below 6 T [60].

With voltage taps that span long conductor segments or full coils, it is usually not possible to measure a meaningful voltage during the ramp up or - down. Only on the plateaus, where most inductive contributions to the parasitic background subside, can the small (sub)micro-volt level signals that are tell-tale for conductor performance be measured sufficiently accurately. The voltages for the V-I measurements are captured with the National Instruments PXI-4071 card.

Differential measurements

When measuring long conductor segments, usually a differential measurement is performed, to remove the noise component as well as the contribution of coupling currents. The signals from nominally identical coils or conductor segments are subtracted from each other, thereby cancelling these contributions. Any differences between the two coils, such as local conductor damage, then becomes apparent. To make sure that two coils with similar damage are not compared to each other,

such a comparison can be made for all coil combinations. In Figure 3.2a, the current ramp for a V-I measurement of an HL-LHC 11 T dipole magnet (magnet MBHB003) is shown. Figure 3.2b shows the coil lay-out. In Figure 3.2c the voltage as measured directly across two of the coils is shown. Only the voltages measured during the current plateaus are shown, since during the ramp there is too much noise and inductive background. Even on the current plateau, the noise present in the measurements makes any identification of conductor damage impossible. Figure 3.2d shows the voltage over the same coils, but this time with the voltage signal from the opposing identical coil subtracted. This time, a clear indication of conductor damage is seen for coil D2U.

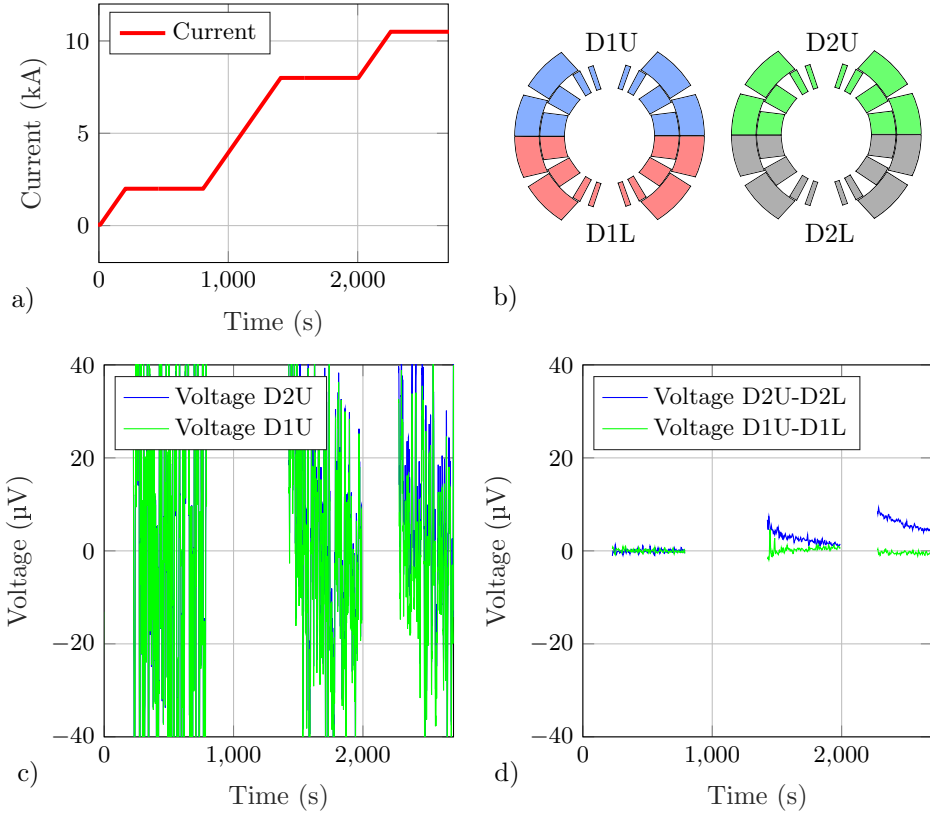


Figure 3.2: Illustration of differential voltage measurements on a HL-LHC dipole magnet. a) Magnet current ramp sequence. b) Coil lay-out. c) Non-compensated voltage measured over coils D1U and D2U. d) Compensated voltage obtained with differential measurements over identical coil pairs.

Local measurements

As mentioned above, demonstration magnets usually have many extra voltage taps on specific conductor segments, concentrated around performance-critical ar-

eas such as pole turns. The monitored segments can be short, consisting of just a few twist pitches, or relatively long, comprising multiple turns. The shorter segments, in the order of 0.2 m to 2 m, are plagued by a lot less noise and the voltage component due to conductor damage can be measured directly. In some cases it is even possible, with such short segments, to measure this voltage component during the magnet ramp.

Strand voltage

Voltage taps are attached to the cable through a copper strip that is pressed down on the wide side of the cable, electrically contacting several strands. It will be shown in Chapter 4 that close to a damaged spot in the cable, the voltage can differ significantly between strands. The voltage that the tap then measures is equal to the voltage of the strand that is closest to the instrumentation wire connected to the copper strip. The strip is simply clamped between cables, with the precise location of the contacts and the distribution of contact pressure not known. It is assumed that the measured voltage is of one strand and not the average of all strands. This is the most likely scenario given the dimensions of the copper strip and the expected relatively large contact resistance between strip and strands. For the controlled damage experiment, which will be introduced in Section 3.3, knowledge of the voltage tap position is ensured by soldering the voltage taps directly onto selected strands only.

3.1.2 Quench voltage

When a superconducting cable is developing a quench, a rising voltage, corresponding to the size of the normal zone, is measured over the affected segment. This voltage, see Figure 3.3, is used to trigger the quench protection system of the magnet when passing a set threshold. Initial development of such a voltage is one of the topics of this study, since it can hold clues to presence and even nature of local conductor damage. When multiple voltage taps are close together, the quench propagation velocity can be determined using the ‘time-of-flight’ method. The initial voltage build-up can also be used to study the dynamics of a quench development. The quench voltage signals are captured with an INCAA TR14 data acquisition card.

3.2 Quench antennas

Results obtained with two different types of quench antennas are studied: so-called ‘local’ and ‘harmonic’ quench antenna systems. Below, their general design and working principle are explained.

3.2.1 Principle

When a quench occurs in one of the cables in the coils, part of the cable becomes resistive. The dynamics of how this process unfolds are discussed in more detail

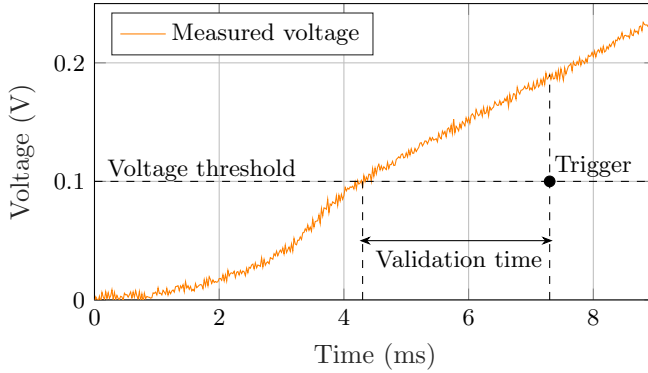


Figure 3.3: Normal zone voltage as measured on the MQXFBP3 quadrupole magnet. The quench protection system only activates when the threshold level has been exceeded for a set amount of time. Sometimes an extra delay is programmed that postpones the protection system activation after the trigger event for several milliseconds.

in Chapter 5, but for the quench antenna the important point is that it entails a significant amount of current redistribution. This redistribution of current results in a changing magnetic moment that is sensed by the pick-up coils. By combining a number of pick-up coils in specific configurations, different aspects of the quench can be studied. Most quench antennas are designed to determine the quench origin and its propagation velocity.

3.2.2 Local quench antenna

The first type is a local quench antenna with a cross-sectional lay-out as shown schematically in Figure 3.4 (left) [106, 107]. This quench antenna is partitioned lengthwise in 11 parts that each consists of four pick-up coils (A, B, C, and D), with one coil in each quadrant of the circular bore. In the axial direction of the magnet, see Figure 3.5, the coils have a length of 40 mm, with each one consisting of 200 turns. By measuring the coils in series as A-C and B-D or A+B+C+D, the contribution of the main magnetic field is cancelled and only magnetic field disturbances are picked up.

The relatively small axial length of the individual pick-up coils allows for a good longitudinal resolution in determining the quench position. By combining the data from the quench antennas with the voltage taps, the quadrant of the quench location can be determined. In the past, this type of quench antenna was also used to determine the angle of the dipole moment produced by the quench, which allowed to determine the direction in which the current redistributes [106].

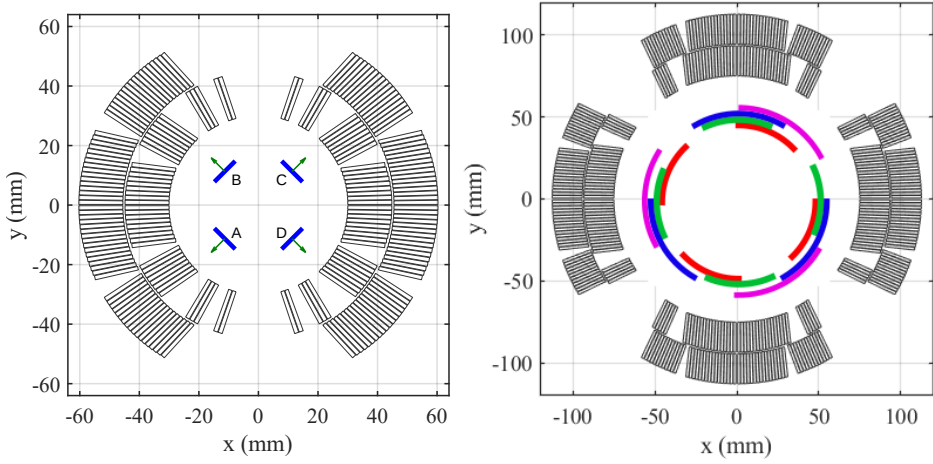


Figure 3.4: Left: Cross-sectional view of the position of the pick-up coils of the local quench antenna [107]. The arrows indicate the direction in which a positive magnetic field change induces a positive voltage. Right: Schematic of the cross-sectional lay-out of the harmonic quench antenna system [110]. The four windings patterned on the flexible PCB are designed to be sensitive only to specific harmonic content.

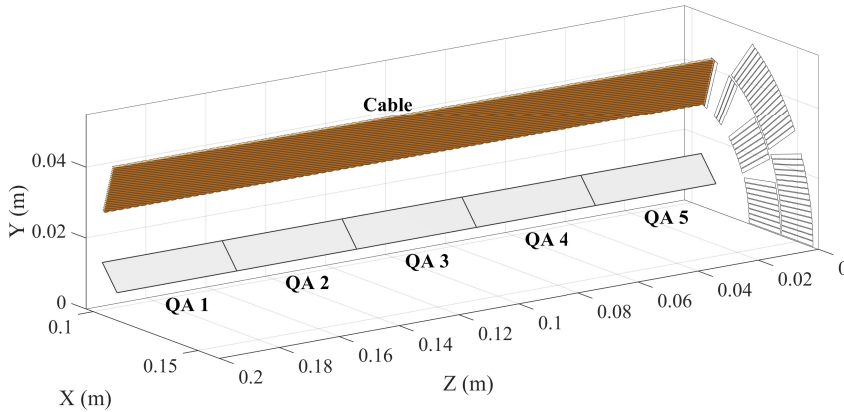


Figure 3.5: View of one quadrant of the lay-out of the first 5 pick-up coils of a local quench antenna.

3.2.3 Harmonic quench antenna

The second type of quench antenna has a multipole lay-out that is designed to be sensitive only to specific harmonic components of the magnetic field in the magnet bore [109], see Figure 3.4 (right). This design was first conceived and implemented in a prototype dipole magnet for the Superconducting Super Collider [110]. The design of this quench antenna allows for precise localization of the quench in the cross-section of the magnet [110, 111, 112]. Recently, at CERN, a new version using flexible printed circuit boards was developed and implemented for use in the HL-LHC quadrupole magnets [113].

The magnetic field inside the bore of an accelerator magnet can be described in 2D by means of a Fourier expansion of the radial magnetic field component in cylindrical coordinates. This will yield the following sets of coefficients:

$$B_n(r_0) = \frac{1}{\pi} \int_0^{2\pi} B_r(r_0, \phi) \sin(n\phi) d\phi, \quad n = 1, 2, 3, \dots, \quad (3.1)$$

$$A_n(r_0) = \frac{1}{\pi} \int_0^{2\pi} B_r(r_0, \phi) \cos(n\phi) d\phi, \quad n = 1, 2, 3, \dots. \quad (3.2)$$

The set B_n is called the *normal* coefficients and set A_n the *skew* coefficients. From these coefficients, the magnetic field harmonic C_n is defined as:

$$C_n = B_n + iA_n. \quad (3.3)$$

These magnetic field harmonics form an orthonormal basis that allows to describe any magnetic field in the magnet bore. The magnetic field harmonics are thus linearly independent, which is central to the design of the harmonic quench antenna.

The harmonic quench antenna at CERN is meant to diagnose and monitor the performance of quadrupole accelerator magnets. Therefore, its coils are designed to be sensitive to the first two higher-order components: the sextupole C_3 and octupole C_4 harmonics. In total four pick-up coils are needed, to measure both the normal and the skew components of these harmonics.

By measuring the voltages induced by the time-varying linearly independent multipoles, the quench position can be reconstructed, if one assumes that the quench initially produces a single point-like magnetic dipole moment. The direction in which this magnetic dipole moment points can also be determined.

The harmonic coefficients at a reference radius r_0 generated by a magnetic dipole moment $\mathbf{m} = m e^{i\theta_c}$ that is situated at $z_c = x + iy$, see Figure 3.6, are given by [90]:

$$C_n = m \frac{i\mu_0 n}{2\pi} \frac{e^{i\theta_c}}{z_c^2} \left(\frac{r_0}{z_c} \right)^{n-1}. \quad (3.4)$$

The harmonic quench antenna is designed to be sensitive to C_3 and C_4 , so if we divide these harmonics using Equation 3.4, this yields [113]:

$$\frac{C_3}{C_4} = \frac{3}{4} \frac{z_c}{r_0}. \quad (3.5)$$

Hence the location z_c can be derived from the harmonics as:

$$z_c = \frac{4}{3} \frac{C_3}{C_4} r_0. \quad (3.6)$$

Using this relation, also the direction θ_c of the magnetic dipole moment $\mathbf{m}$ can

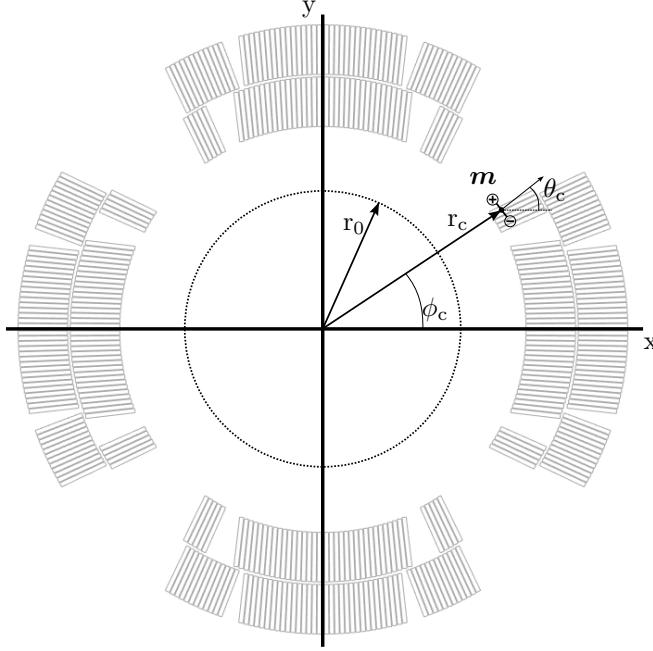


Figure 3.6: Position of the magnetic dipole moment $\mathbf{m}$, given by the cylindrical coordinates r_c and ϕ_c , with direction θ_c . r_0 is the reference radius, where the windings of the quench antenna are located.

be determined. Expressing Equation 3.4 in cylindrical instead of cartesian coordinates, the harmonic coefficients at reference radius r_0 are given by [90]:

$$C_n = m \frac{i\mu_0 n}{2\pi r_c^2} \left(\frac{r_0}{r_c} \right)^{n-1} e^{i[\theta_c - (n+1)\phi_c]}. \quad (3.7)$$

Given that $C_n = B_n + iA_n$ and using Euler's formula:

$$e^{i\phi} = \cos \phi + i \sin \phi, \quad (3.8)$$

Equation 3.7 can be rewritten as:

$$B_n + iA_n = m \frac{i\mu_0 n}{2\pi r_c^2} \left(\frac{r_0}{r_c} \right)^{n-1} \{ \cos[\theta_c - (n+1)\phi_c] + i \sin[\theta_c - (n+1)\phi_c] \}. \quad (3.9)$$

By separating the real and the complex components, the following two relations are obtained for the normal and skew harmonic coefficients:

$$B_n = -m \frac{\mu_0 n}{2\pi r_c^2} \left(\frac{r_0}{r_c} \right)^{n-1} \sin[\theta_c - (n+1)\phi_c], \quad (3.10a)$$

$$A_n = m \frac{\mu_0 n}{2\pi r_c^2} \left(\frac{r_0}{r_c} \right)^{n-1} \cos[\theta_c - (n+1)\phi_c]. \quad (3.10b)$$

Dividing the normal coefficient by the skew coefficient then yields:

$$\frac{B_n}{A_n} = \frac{-\sin[\theta_c - (n+1)\phi_c]}{\cos[\theta_c - (n+1)\phi_c]} = -\tan[\theta_c - (n+1)\phi_c], \quad (3.11)$$

which can be rewritten to retrieve an expression for the angle:

$$\theta_c = -\arctan(B_n/A_n) + (n+1)\phi_c. \quad (3.12)$$

From Equation 3.7 we can also obtain an expression for the magnitude of magnetic moment m :

$$|C_n| = m \frac{\mu_0 n}{2\pi r_c^2} \left(\frac{r_0}{r_c} \right)^{n-1}, \quad (3.13)$$

$$m = |C_n| \frac{2\pi r_c^2}{\mu_0 n} \left(\frac{r_0}{r_c} \right)^{1-n}. \quad (3.14)$$

A degree of uncertainty is present, since the quench antennas are also sensitive to odd multiples of the used harmonics. Specifically, the 9th and 12th harmonic have a significant contribution to the signal [109], which results in a localization error of ± 5 mm.

In Figure 3.7 a characteristic example of the harmonic quench antenna captured during a quench of a quadrupole magnet is shown. The four signals proportional to B_3 , A_3 , B_4 , and A_4 are shown in the top graph, while in the bottom graph a cross-section of the magnet is shown with the reconstructed position of the quench. The position is reconstructed from the signal between the two vertical dashed lines in the top plot. The calculated direction of the dipole moment is also shown.

3.2.4 Transients during magnet ramp

While the magnet is ramping up or down, different types of transients may occur, such as flux jumps and mechanical displacement events. Such events are also picked up by the voltage taps and quench antennas and can be localized and quantified throughout the ramp.

For example, in Figure 3.8, heat maps of flux jump events measured during a magnet ramp, are shown for two time instances. Figure 3.8a during the relatively

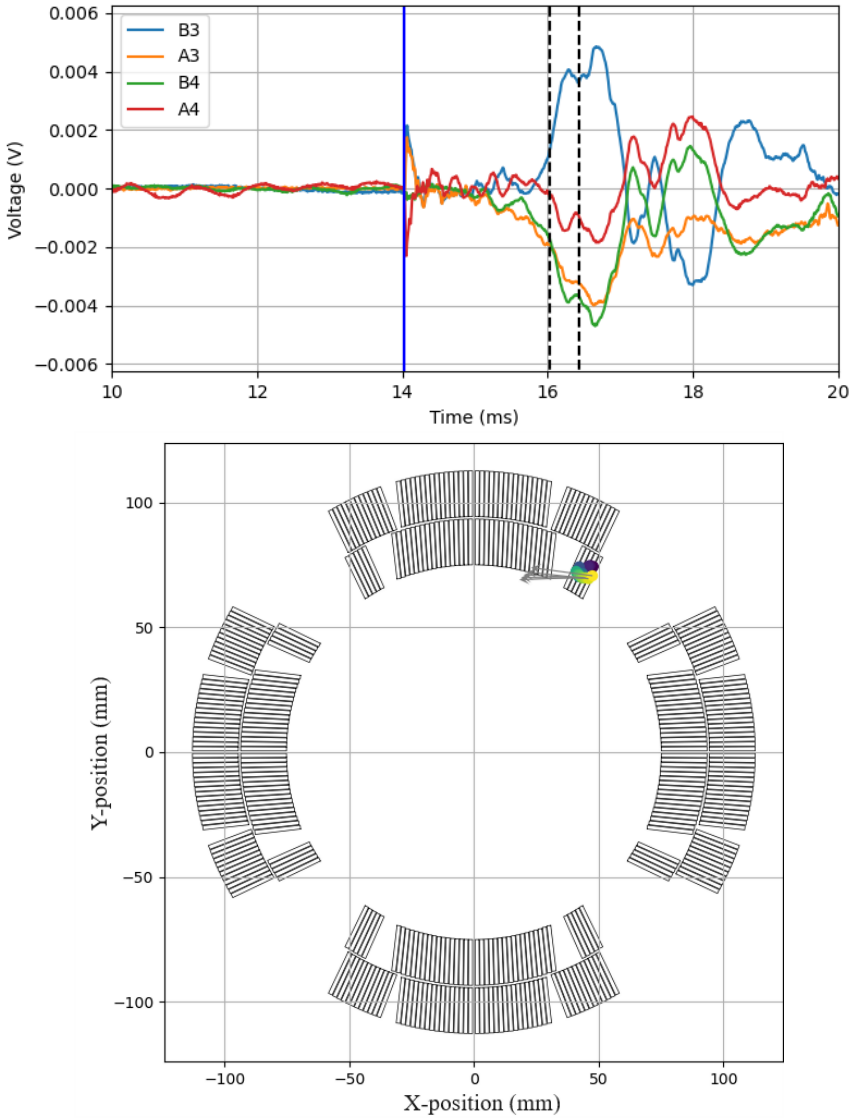


Figure 3.7: *Top: Example of measured components of the third and fourth harmonic magnetic field contributions picked up by the quench antenna during a quench. The voltages correspond to the harmonic content of the disturbance of the magnetic field in the bore. Bottom: Reconstructed location of the quench in the magnet's cross-section. The arrow indicates the direction of the dipole moment generated by the quench. The vertical dashed lines in the top picture indicate the data range used for this reconstruction, the vertical blue line indicates the quench start.*

low magnetic field part of the ramp between 1.5 kA and 2.5 kA and Figure 3.8b between 7.5 kA and 8.5 kA. During the early stages of the ramp a high density of

flux jump events is present near the poles. As the current increases, the flux jump activity is reduced and migrates towards the mid plane.

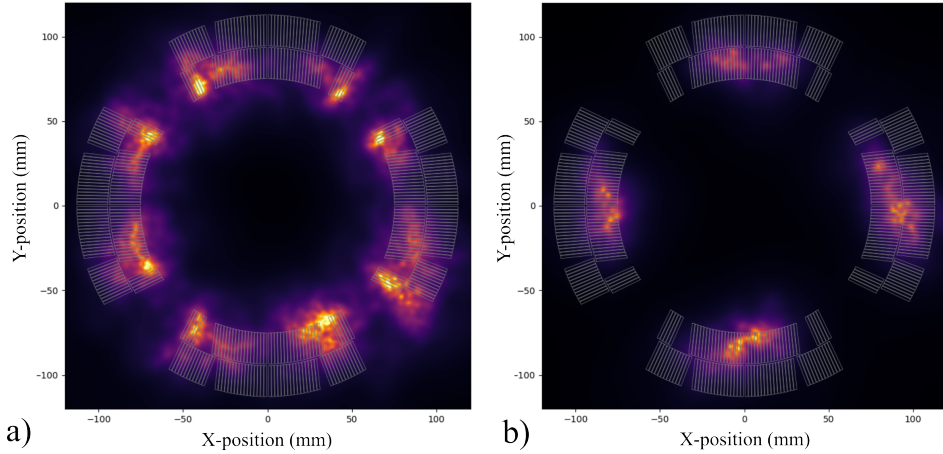


Figure 3.8: *a) Heat map of flux jump activity at start of a magnet ramp, around $I = 2$ kA. b) Heat map of flux jump activity in the middle of a magnet ramp, around $I = 8$ kA.*

3.3 Controlled damage experiment

A local inhomogeneity in the winding pack, such as a cable with some degraded strands, will influence the current distribution in - and around it. To study the effect of such variations on the low-frequency V-I curves and on the high-frequency quench dynamics, a controlled experiment is performed with the so-called Short Model Coil (SMC) [114]. The SMC magnet is a test bed for studying cable performance under magnet-relevant conditions. It consists of two stacked racetrack coils contained in a mechanical support structure, see Figure 3.9. To apply a properly balanced pre-stress, the structure employs the so-called ‘bladder-and-key’ technology [114]. The coil is wound from a Rutherford cable and the winding pack is impregnated with CTD epoxy resin.

The SMC type of magnet was chosen for this experiment because its cable is longer than the characteristic length scale of the current diffusion process around a defect, which normally for Rutherford cables during a magnet ramp exceeds 10 m, although, as we will show in Chapter 4, this value strongly depends on the inter-strand contact resistance. In cable test stations such as FRESKA, the sample length is about 2 m and the splices at the cable ends affect the diffusion process [117]. This influences the time constants of the current redistribution process too and thus also affects the measured voltage differential between strands. A further advantage of an SMC test rather than a FRESKA experiment is the magnetic field range that is accessible by each: the HL-LHC magnets have a peak magnetic

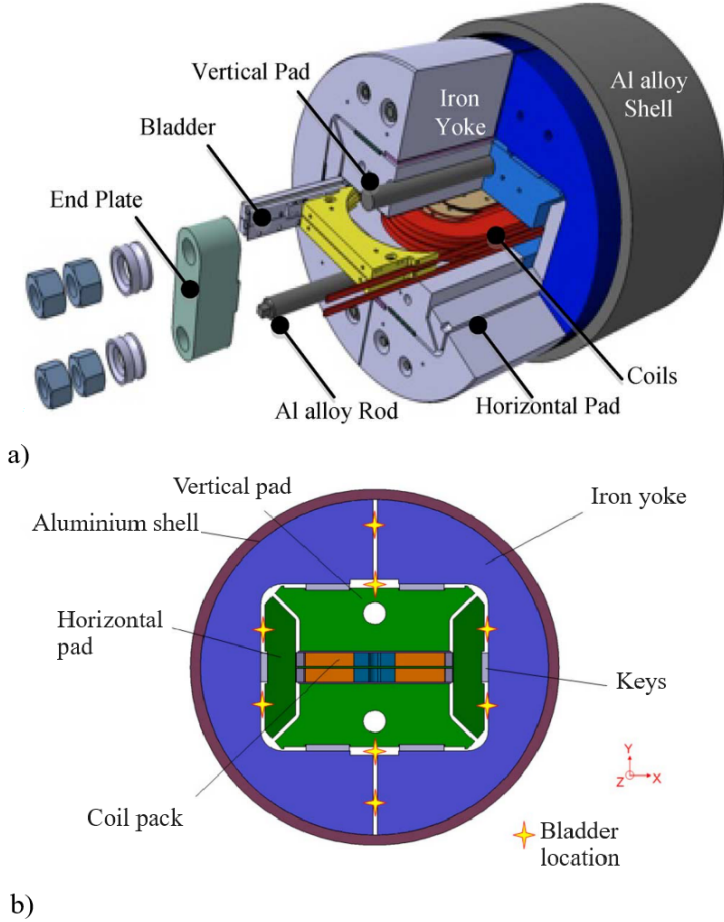


Figure 3.9: a) Mechanical design of the Short Model Coil, image from [114]. b) Cross-section of the SMC showing its bladder and key structure, image from [115].

field on the conductor exceeding 11 T, whereas the FRESCA test station cannot achieve a magnetic field higher than 9 T. The SMC can reach 13 T, and is thus a better analogue for an HL-LHC magnet.

The experimental campaign comprises five runs:

- **Run 1:** Reference test of the coil and conductor without additional instrumentation or broken strands.
- **Run 2:** Test of the additional instrumentation, without broken strands.
- **Run 3:** Test with a single broken strand.
- **Run 4:** Test with three broken strands.

- **Run 5:** Test with five broken strands.

The first run serves to verify the performance of the magnet. The second run is used to validate the extra instrumentation and to make sure its implementation did not damage the conductor. It also provides a reference for the signals from the new voltage taps. In runs 3 through 5 then incrementally the number of broken strands in the cable was increased. As discussed in Section 3.4.2 the strands are all broken at the same longitudinal cable location and are situated adjacent to each other.

3.3.1 Coil SMC 11T #1

The coil chosen for the experiment is SMC11T#1, a first-generation coil of the Short Model Coil Program [116]. The coil's Rutherford cable consists of 40 strands. The strand type is OST RRP 108/127 and its properties are listed in Table 3.1. The cable in this particular SMC coil does not have a stainless steel core, unlike the cables in HL-LHC coils. It is therefore expected to have a significantly lower R_c value compared to the latter. The total length of the cable is about 70 m.

Table 3.1: *Coil SMC 11T #1 cable properties.*

Parameter	Value
No. of strands	40
Strand type	OST RRP 108/127
Twist pitch	100 mm
Cable width	15 mm
Cable thickness	1.25 mm
Strand diameter	0.70 mm
Filament twist pitch	14 mm
Filament diameter	36 μ m
Cu/non-Cu ratio	1.25
RRR	80 - 105

Load line and critical surface

In Figure 3.10, the load-line of the SMC coil is shown along with the critical current of its strands at 1.9 K and 4.5 K. Due to differences in the magnet's size and geometry, the load-line of the SMC is steeper than those of the 11 T dipole - and MQXF quadrupole magnets, i.e. the cable needs to carry a higher current to attain a specific magnetic flux density. This can cause the SMC magnet to behave somewhat differently than the HL-LHC magnets.

Magnetic field gradient

Since the current imbalance between strands is expected to be maximum at the defect location, this is where the critical surface of the superconductor will in-

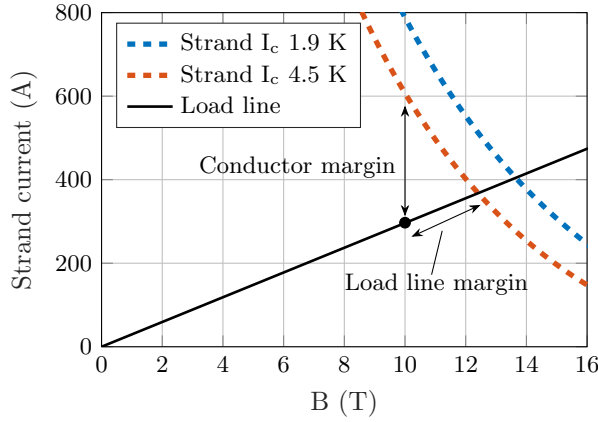


Figure 3.10: Load line of the SMC magnet and the critical current of its strands at 1.9 K and 4.5 K.

fluence the current redistribution process the most. For accurate comparison to model predictions, the critical current distribution in this region must be known with sufficient accuracy, so a detailed field map is necessary in this location. The magnetic field in the cross-section of the magnet has been calculated with ROXIE and is shown in Figure 3.12a. The pole turn, where the defect is introduced, is highlighted (the rectangular box in Figure 3.12a). Here, the magnetic field varies by some 1.5 T over the width of the cable, see Figure 3.12a.

The magnetic field increases as the magnet current increases, but due to iron saturation this rate slows down at higher currents, see Figure 3.12b. The transfer function of the magnet that links the conductor peak magnetic field to the magnet current will later be used in the modeling of the magnet behavior during the ramp.

3.4 Instrumentation

The SMC coils were originally instrumented with a limited number of voltage taps. These voltage taps were used to monitor the voltage over cable segments and for the interpretation of the signals the cable was considered as a monolithic entity, without consideration for which strand the voltage tap is touching. To study the current diffusion process around a local defect, a more fine-grained approach is necessary. The voltage between strands and over specific strand sections needs to be measured. Therefore, a system of new voltage taps sensing well-defined selected strands are added to the coil.

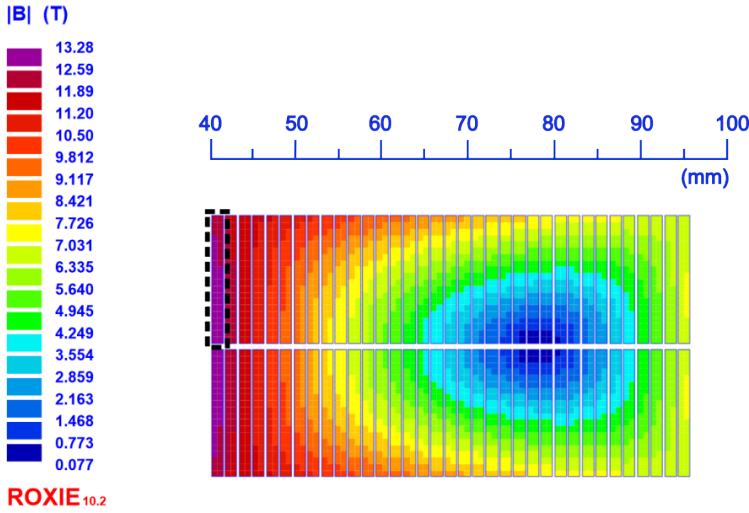


Figure 3.11: Magnetic flux density in one half of the cross-section of the winding pack of the SMC magnet, as computed by ROXIE for a current of 16 kA. The horizontal scale indicates the distance to the magnetic center.

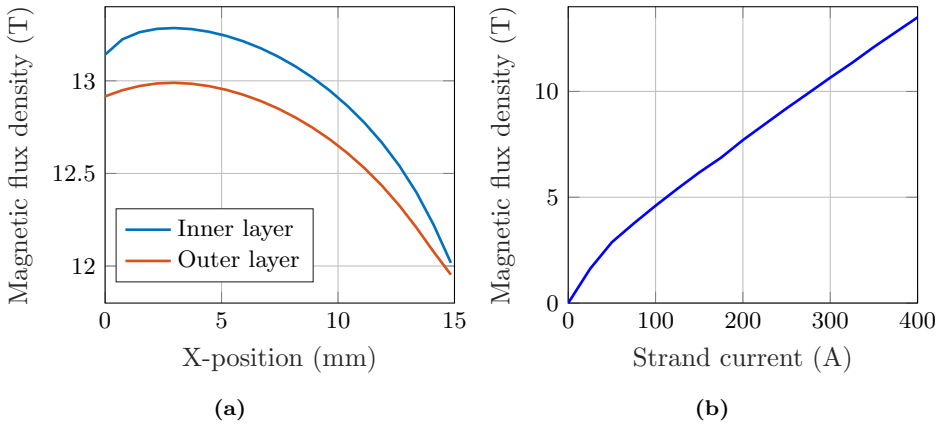


Figure 3.12: a) Magnetic flux density on the pole turn for the inner and outer layers of the Rutherford cable, calculated with ROXIE. b) The transfer function of the peak magnetic field on the conductor of SMC 11T #1 coil with respect to the strand current, calculated with ROXIE.

3.4.1 Voltage tap positioning

Since the new voltage taps were added to an existing magnet, the only accessible part of the Rutherford cable is the short edge facing the pole. This imposes some constraints on the placement of the new voltage taps. Any given strand will only be accessible once every twist pitch of the cable. Furthermore, due to the lack of

space, voltage taps cannot be placed on directly adjacent strands and there has to be at least one strand in between.

To add the new voltage taps, first the epoxy and the insulation covering the cable have to be locally carefully removed. Then, the new voltage taps were soldered directly to the exposed strands at the cable edge. The remaining void is then filled with Stycast 2850FT epoxy resin to make sure the area remains mechanically stable and also to restore electrical insulation.

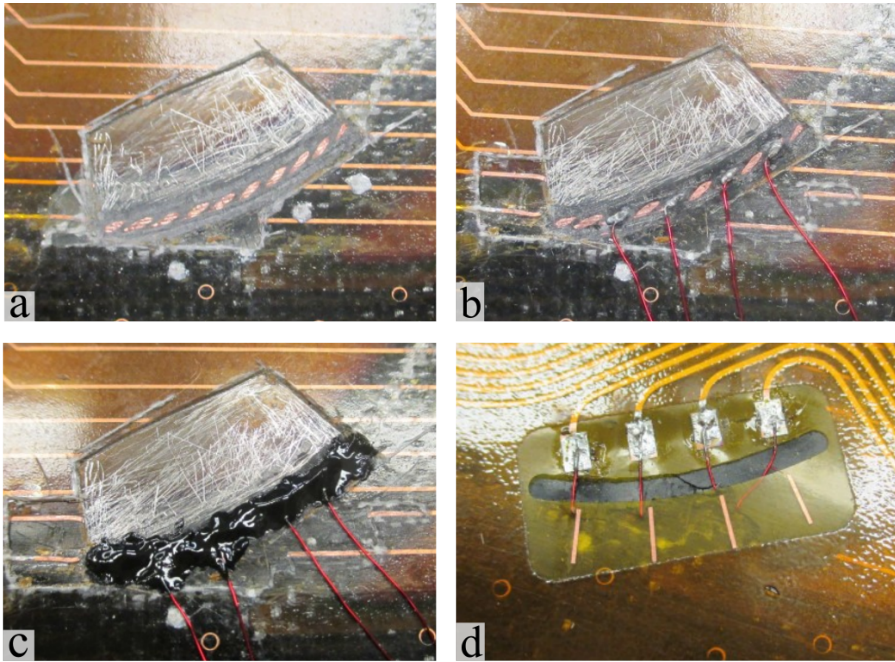


Figure 3.13: *Process of adding new voltage taps: a) Removing epoxy and insulation to expose the strands. b) Soldering wires to selected strands. c) Covering the exposed area with Stycast. d) Soldering the wires to copper traces.*

The voltage taps are connected to copper traces that connect to the instrumentation wires feed-through at the top of the cryostat. The steps of this process are illustrated in Figure 3.13. The traces are designed to minimize the exposed surface between the cable and the trace to reduce coupling with the magnetic field. Therefore, the trace closely follows the path of the cable to minimize flux enclosure with the windings, as can be seen in Figure 3.14.

Figure 3.14 shows the position of the voltage taps, as well as the introduced defect. In total, 12 new voltage taps are added to the magnet. They are grouped in sets of four. Groups B and C are set on either side of the defect at a distance of 1 twist pitch and group A is placed one twist pitch further away. The voltage

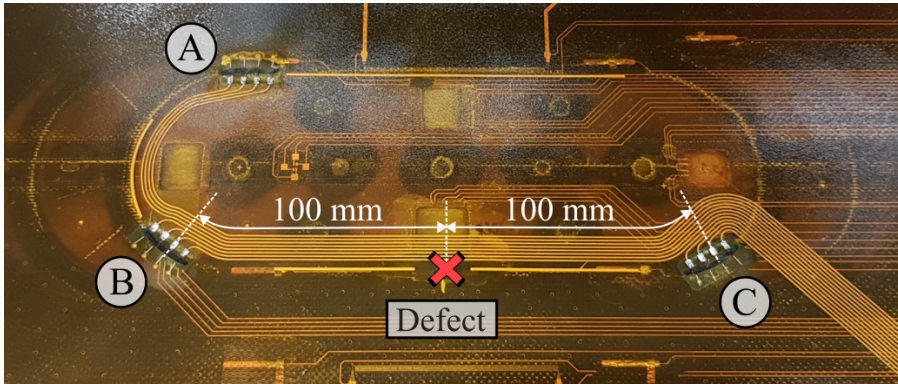


Figure 3.14: Locations of the defect and the voltage tap groups A, B, and C. Also visible are the copper traces that connect the new voltage taps to the instrumentation feed-through.

tap positions are chosen in such a way that they can be used to detect the main features of the current redistribution process predicted by the model. At each location A, B and C the voltage differences between strand 1 and strands 3, 5, and 7 are measured. In addition, the voltage drops over strands 1, 3, 5, and 7 are measured between locations A-B and B-C. The resulting voltage tap pairs are listed in Table 3.2. In Figure 3.15, the voltage tap positions are shown schematically with color-coded strands. For location A, it is more difficult to make sure that the voltage taps are located on the correct strands, since it is harder to follow the cable through the curved head. Due to the forces during earlier powering, tiny cracks have made the epoxy opaque near the interface of the winding pack and the pole pieces.

Table 3.2: Measured voltage tap combinations.

Between strands	Over strand sections
A1-A3	A1-B1
A1-A5	A3-B3
A1-A7	A5-B5
B1-B3	A7-B7
B1-B5	B1-C1
B1-B7	B3-C3
C1-C3	B5-C5
C1-C5	B7-C7
C1-C7	

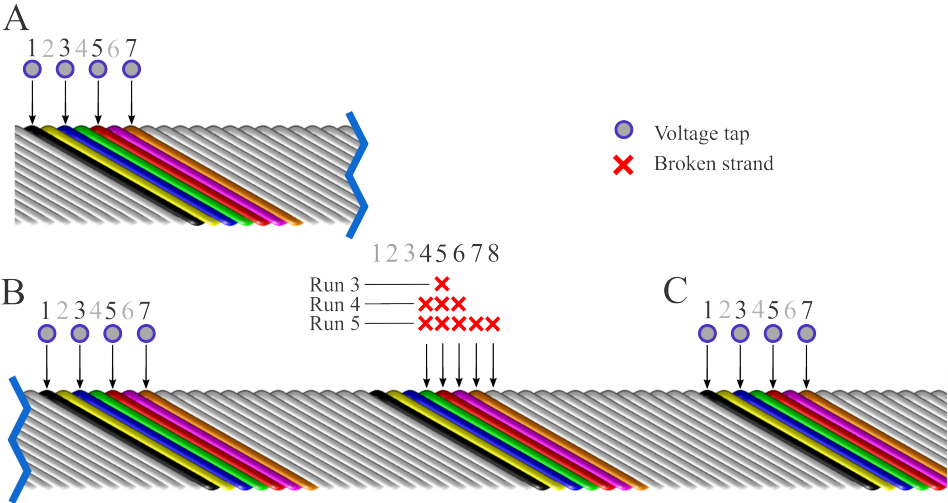


Figure 3.15: Schematic of the relative positions of the voltage taps and of the location where strands are interrupted. For clarity the number of strands in the figure is reduced to 20, as opposed to the real cable that features 40 strands.

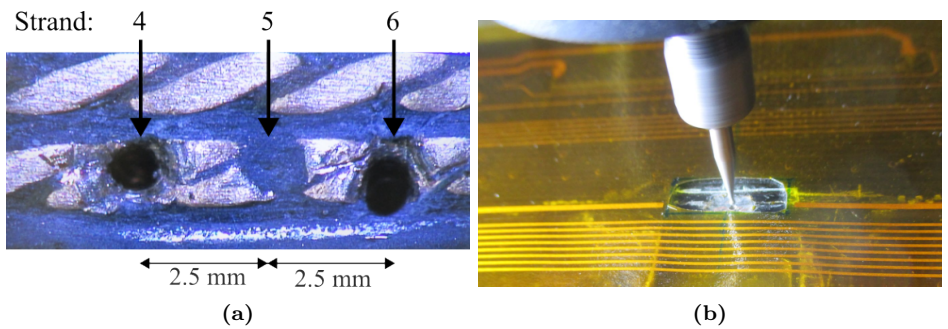


Figure 3.16: a) Close-up of the local defect. Strand 5 is visible in the centre and has a hole filled with Stycast. Strands 4 and 6 show an as-yet unfilled hole. b) The drill used to create the hole.

3.4.2 Local defect

In HL-LHC magnets, strands are most likely only partially degraded. A clean break traversing a whole strand is unlikely. For this controlled experiment however, the defect introduced consists of a locally fully interrupted strand. This is achieved by carefully drilling into one strand without damaging its neighbors. This way, the defect is well-defined. A partial degradation is more difficult to implement and hard to quantify accurately. The void that is left after drilling the hole is filled up with Stycast 2850FT to maintain mechanical integrity of the area. The number of interrupted strands is increased in each successive test of the magnet. In Figure 3.15, also the positions of the interrupted strands are shown. First, strand 5 is interrupted, then also strands 4 and 6 are interrupted, and finally in addition strands 7 and 8 are interrupted. In Figure 3.16a, the process of interrupting a strand is shown. Slight deformations are visible on the neighbors, which suggests that some minor critical current degradation might be expected in these strands as well.

3.5 Conclusion

The experimental tools and techniques used to study the physics phenomena pertaining to this thesis work were presented.

To study the performance of demonstration - and series magnets for the HL-LHC, two non-invasive methods are used; voltage taps and quench antennas. Depending on the type of magnet, voltage taps can be placed on conductors in the coil ends to measure the voltage over the full coil, or in the case of demonstration magnets, on internal conductor segments. The voltage probed by a voltage tap is equal to that of the last strand the copper contact strip touches.

Different types of quench antennas are used to locate and track quenches as they develop and the normal zones propagate in the magnet windings. The local quench antenna system with a high density of relatively small pick-up coils along the bore, is used to locate a quench along the length of the magnet and to study its propagation dynamics. A harmonic quench antenna, sensitive to specific magnetic field harmonics, is used to locate the origin of a quench in the transverse plane. Different transient phenomena during a magnet ramp, such as flux jumps and mechanical displacements, can also be studied with these tools.

The experimental aspects of a dedicated well-controlled experiment for studying current redistribution in Rutherford-type of cables in a representative environment were also introduced. A special coil in the SMC magnet was fitted with a set of additional voltage taps that are placed on selected individual strands. An artificial defect is then introduced in a controlled manner by interrupting an increasing number of selected strands in successive tests. The voltage differences between the strands and the voltage drops along them are then used to monitor the dynamics of the resulting current redistribution process.

Chapter 4

Current redistribution in damaged Rutherford cables

In this chapter, the current redistribution processes during magnet ramps and cycling are discussed that occur due to different types of defects in Rutherford cables. In the first part, a generic numerical study of the processes around local non-uniform defects is performed with the THEA software. It is shown how it is possible to distinguish between uniform and non-uniform defects. The influence of several parameters such as the inter-strand contact resistances, as well as ramp rate and temperature is discussed. Analytical solutions for current diffusion in a hypothetical two-strand cable are also provided. In the second part, the results of the controlled damage experiment on the Short Model Coil are presented. The measured voltages are compared to model predictions generated with RuNe for different numbers of intentionally broken strands.¹

4.1 Types of cable defects

Conductor damage or degradation may be classified in two groups: uniform or non-uniform. For uniform defects, all strands are affected equally, while for non-uniform defects, one or multiple strands are more affected than others. We will show how non-uniform damage manifests itself in V-I measurements on a current plateau by the presence of a decaying voltage over the affected section, while uniform defects give a stable voltage and are therefore easier described with the ‘classical’ power-law based V-I curve. In Figure 4.1 the difference in voltage response between the two types of defects is shown schematically. In the absence of defects, the voltage remains zero due to the high intrinsic n-value of the strands until the cable’s critical current is reached. In the case of a uniform defect with a relatively

¹This chapter is based, in part, on the following publication [118]:
R. Keijzer *et al.*, "Modelling V-I Measurements of Nb₃Sn Accelerator Magnets With Conductor Degradation," in *IEEE Transactions on Applied Superconductivity*, vol. 32, no. 6, pp. 1-5, Sept. 2022, Art no. 4001105, doi: 10.1109/TASC.2022.3153247.

low n -value, a voltage develops already below the critical current while the magnet current is ramping up, but it remains stable during the current plateaus. In the case of a non-uniform defect, the voltage also rises before the critical current is reached, but on the plateaus a quasi-exponential decay is visible, typically with time constants in the range of 10 s to 1000 s. This type of decay is indicative of an evolving current redistribution pattern in the cable. The dynamics of this process and how it depends on the cable properties and ramp parameters is the subject of study in this chapter.

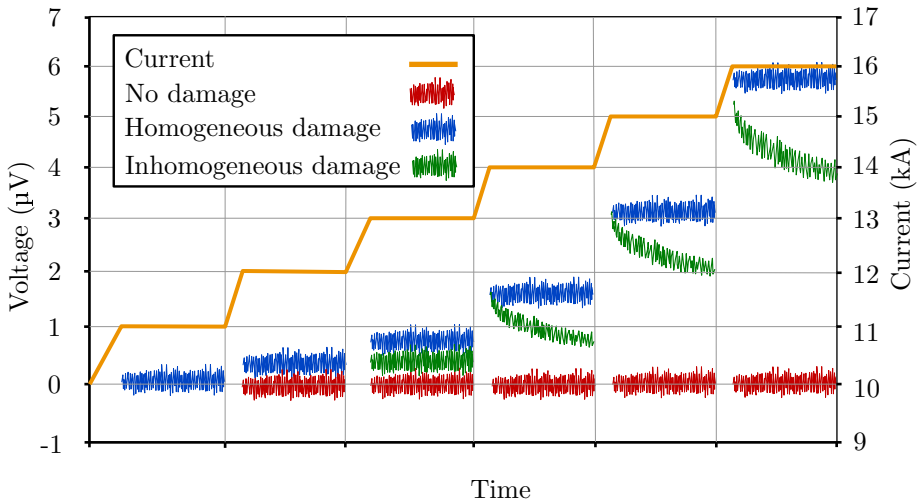


Figure 4.1: Schematic example of measured voltage tap signals during a ‘stepped’ ramp-up of the current in a Nb_3Sn magnet, with different colors corresponding to different types of damage. Note that the figure is illustrative and does not show actual measurement data.

Figure 4.2a shows a possible example of non-uniform damage, where a subset of strands in the cable is locally degraded or broken. In Figure 4.2b, an example of uniform damage is shown that could, for example, be induced by an excessive stress concentration at the inner edge of the Rutherford cable during coil production or magnet assembly, as for example in [120]. Note that the presence of a voltage well-below the quench current implies that the n -value is relatively low.

4.2 Simulation of a local defect in a Rutherford cable

In this section, the behaviour of a cable with a local non-uniform defect is described. The study focusses on a straightforward ramp sequence as shown in Figure 4.3. This sequence was applied for the 5.5 m long double-aperture 11 T dipole magnet MBHA002 at CERN. The current is ramped up to 10.5 kA with a rate of 10 A s^{-1} , after which it is kept at a plateau for 1800 s. Then the current is

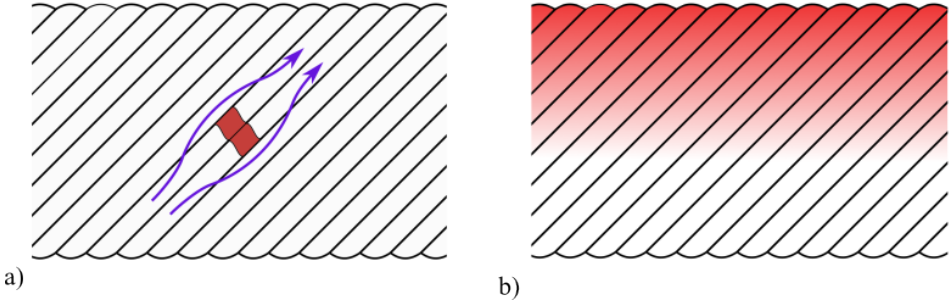


Figure 4.2: a) A local non-uniform defect in the Rutherford cable affecting two strands. Current is redistributed around this type of defect. b) A uniform defect that affects all strands equally, leading to no or little current redistribution.

ramped down to 6 kA, followed by a shorter plateau of 900 s. The displayed voltage is the voltage drop across a degraded coil in the magnet, from which the voltage of a non-degraded coil is subtracted in order to eliminate noise, as described in Chapter 3. The voltage tap pair covers the full cable length of about 600 m. On the 10.5 kA plateau, a positive voltage with a decaying component is visible, while on the 6 kA plateau a smaller negative decaying voltage is visible. The simulated voltage over a Rutherford cable with a local non-uniform defect is also shown. The predicted voltage has a lower value, but scaling it up by a factor 4 reveals that it has very similar time constants as the measured voltage. The voltage for a simulated uniform defect is also shown, which is steady on the current plateau. Below, the details of the simulation and the underlying physical mechanisms are discussed.

4.2.1 Simulation parameters

Table 4.1: Cable and strand properties used in the MBHA002 simulation.

Parameter	Value
Strand diameter	0.70 mm
Cu/non-Cu ratio	1.15
Cable twist pitch (40 or 16 strands)	100 or 40 mm
Cable width (40 or 16 strands)	15 or 6 mm
Cable thickness	1.25 mm
R_a	5 $\mu\Omega$
R_c	180 $\mu\Omega$
Temperature	4.5 K
Strand I_c at 12 T/4.5 K	490 A
n-value	40

In Table 4.1 the cable properties used in the simulation with THEA are shown.

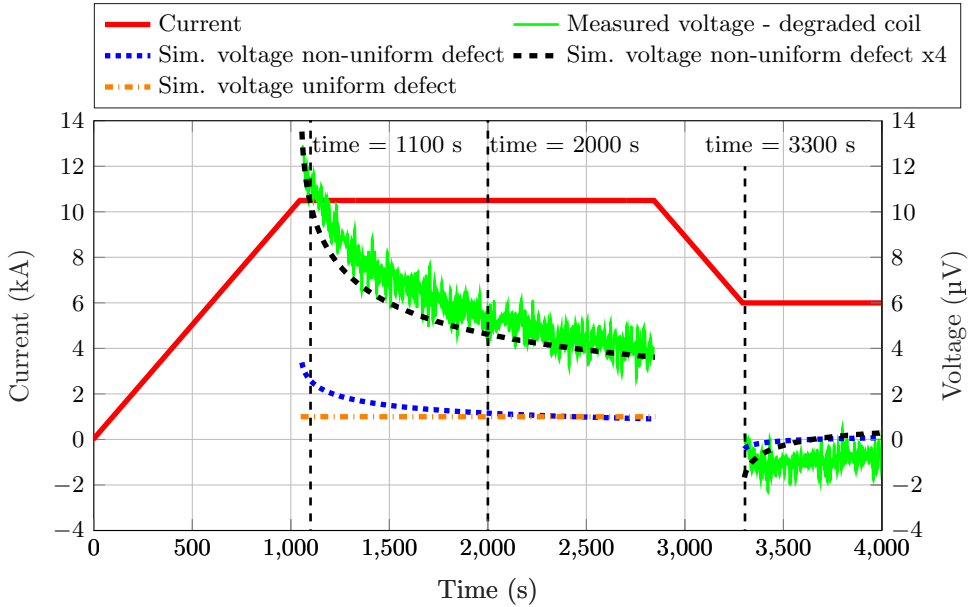


Figure 4.3: Comparison between measured and simulated cable voltage as a function of time. Both a uniform and a non-uniform damage case are shown. The measurement was performed on an 11 T dipole magnet (MBHA002) at 4.5 K.

The simulated Rutherford cable consists of only sixteen strands to speed up computation, but otherwise has the same specification as the cables used in the 11 T dipole magnets developed for the HL-LHC.

Nb_3Sn Rutherford cables typically have a stainless steel core between the two layers to prevent the establishment of large coupling currents. This means that the crossing contact resistance R_c of the cable is much larger than the adjacent contact resistance R_a . The characteristic values of the parameters used in Table 4.1 are based on measurements performed on cable samples reported in [121]. A distributed parameters model approach as described in Chapter 2 is used, so the values for the inter-strand conductances and the mutual inductances are averaged over the cable length.

A schematic of the model is shown in Figure 4.4. A subset of strands is assumed to be fully or partially degraded over a length of 1 mm at the center of the 100 m long model. The current redistributes around the degraded section through the neighboring strands.

For this simplified version of the model, the magnetic field is set to its peak value everywhere, since this is the limiting factor in the current distribution and for long cables the meshing is too coarse to implement the detailed field profile over the cable width.

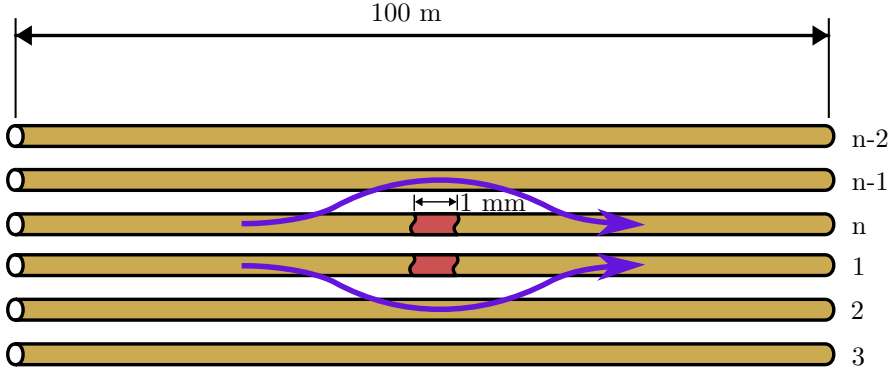


Figure 4.4: Schematic illustration of the model with a reduced number of strands. The modelled cable has a length of 100 m. At the center (at $z = 50$ m), a subset of adjacent strands are defected over a length of 1 mm.

During a current ramp several heat sources are present, such as ohmic loss due to coupling currents and magnetization loss, which may lead to some temperature increase, however such effects are assumed to be negligible here. In the model the temperature is fixed at 4.5 K unless stated otherwise. To describe the superconducting properties of the strands, the ITER parameterization of Nb_3Sn has been used [122, 123].

Defect parameters

The non-uniform defect is positioned in the middle of the model domain and involves six out of the sixteen strands. For this simulation, the copper matrix of the affected strands is left intact, but the superconducting area is reduced to zero over a length of 1 mm. In the controlled damage experiment presented later in this chapter, the strands are fully broken. In this simulation, this is not the case since for the HL-LHC magnets it is more likely that just the brittle Nb_3Sn filaments have degraded locally.

The uniform defect used in the simulation result of Figure 4.3, consists of a partial degradation that reduces the critical current in all strands to 33% of its original value and the n -value from 40 to 10, over a length of 1 mm.

4.2.2 Current redistribution during a current ramp

In the simulation, the magnet current follows the same ramp sequence as the one shown in Figure 4.3. The experimental ramp rate was 10 A s^{-1} for the full cable of 40 strands, but because the simulated cable has only sixteen strands this is scaled to 4 A s^{-1} . Figure 4.5 shows the current in each strand at the longitudinal loca-

tion of the non-uniform defect. From the sixteen simulated strands, only eight are shown because the model is symmetric. As expected, the defected strands do not carry current at the defect location. As the current is ramped up, the two strands adjacent to the defected strands take up most of the excess current, consistent with earlier findings of transient events in the case of sudden heating of a strand [54]. The strand critical current I_c is also shown in this figure. It decreases with overall cable current since the magnetic field increases during the current ramp. Once the adjacent strands reach their critical current, part of the excess current is transferred to the next-nearest neighbors, which eventually saturate as well leading to a cascade as more and more strands start to saturate, this cascade effect is also reported in [119]. The saturation events occur at an accelerating rate since the amount of current that needs to be redistributed increases each time while the critical current continues to decrease.

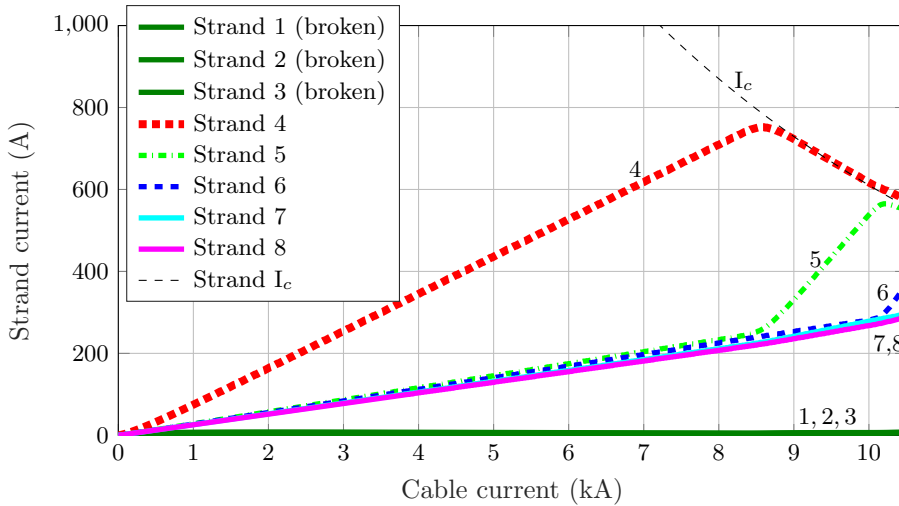


Figure 4.5: Calculated strand current at the position of the defect ($z = 50$ m) versus cable current. The critical current for a strand without degradation is indicated with the thin dashed line.

Figure 4.5 illustrates how initially almost no excess current is taken up by non-adjacent strands, since they have a much higher inter-strand resistance as well as lower mutual inductance with the degraded strands. A further important observation from Figure 4.5 is that at 8.5 kA, strand 4 reaches its highest possible current, well before the cable quench current is reached. From this moment on, this strand operates without current margin and from a thermal viewpoint can be considered to be in an un-stable regime. Quenches due to mechanical or electromagnetic transients may thus occur more frequently. Note that such instabilities can be even more frequent at 1.9 K compared to 4.5 K, since the critical current and therefore also the current in the fully saturated strand is higher.

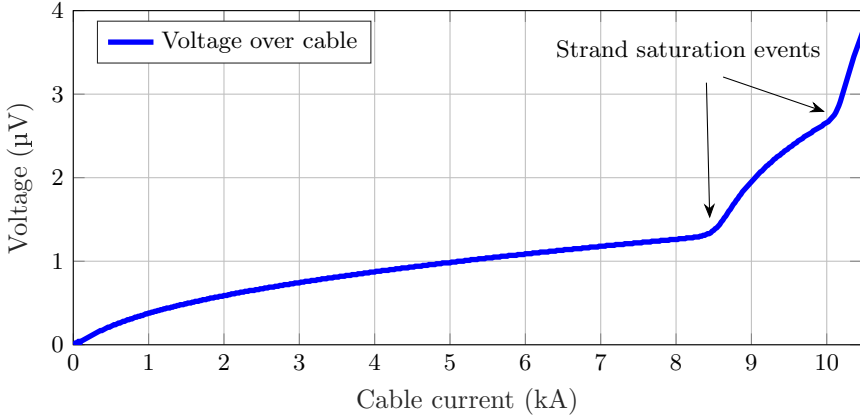


Figure 4.6: *Calculated voltage over the full cable length versus cable current, with the inductive component of the ramp removed. The kinks in the voltage coincide with the saturation of strands 4 and 5 as shown in Figure 4.5.*

In Figure 4.6, the simulated voltage over the full cable as a function of cable current is shown for the duration of the ramp. The inductive offset due to the current ramp itself is removed. The voltage contribution of the current redistribution process around the defect rises with a gradually decreasing rate as the current is ramped up. However, when the strands adjacent to the damaged ones reach their critical current and saturate, the voltage sharply increases. Such a kink in the voltage build-up occurs at each strand saturation event.

4.2.3 Current distribution during a current plateau

Figure 4.7a shows the profile of the current distribution in the strands along the length of the cable after 1100 s, as also indicated in Figure 4.3, and is 50 s into the 10.5 kA plateau. The current imbalance stretches out over a distance of about 15 m before and after the defect and mainly affects strands 4, 5 and 6. A relatively modest increase in the current around the defect for strands 7 and 8 shows that a small amount of current is also taken up by these strands further away from the defect through the crossing inter-strand contacts. Normally, due to transposition, the mutual inductance distributes the current evenly among the strands of a Rutherford cable. The local defect makes this impossible, but far away from it the current is still uniform, since it takes time for the current imbalance to diffuse through the cable.

In Figure 4.7b, the same profile is shown 900 s later. During the overall current plateau, the current imbalance propagates along the cable due to electro-magnetic diffusion. This diffusion process also explains why a decaying voltage is seen on the plateaus (as in Figures 4.1 and 4.3); since the current imbalance extends over

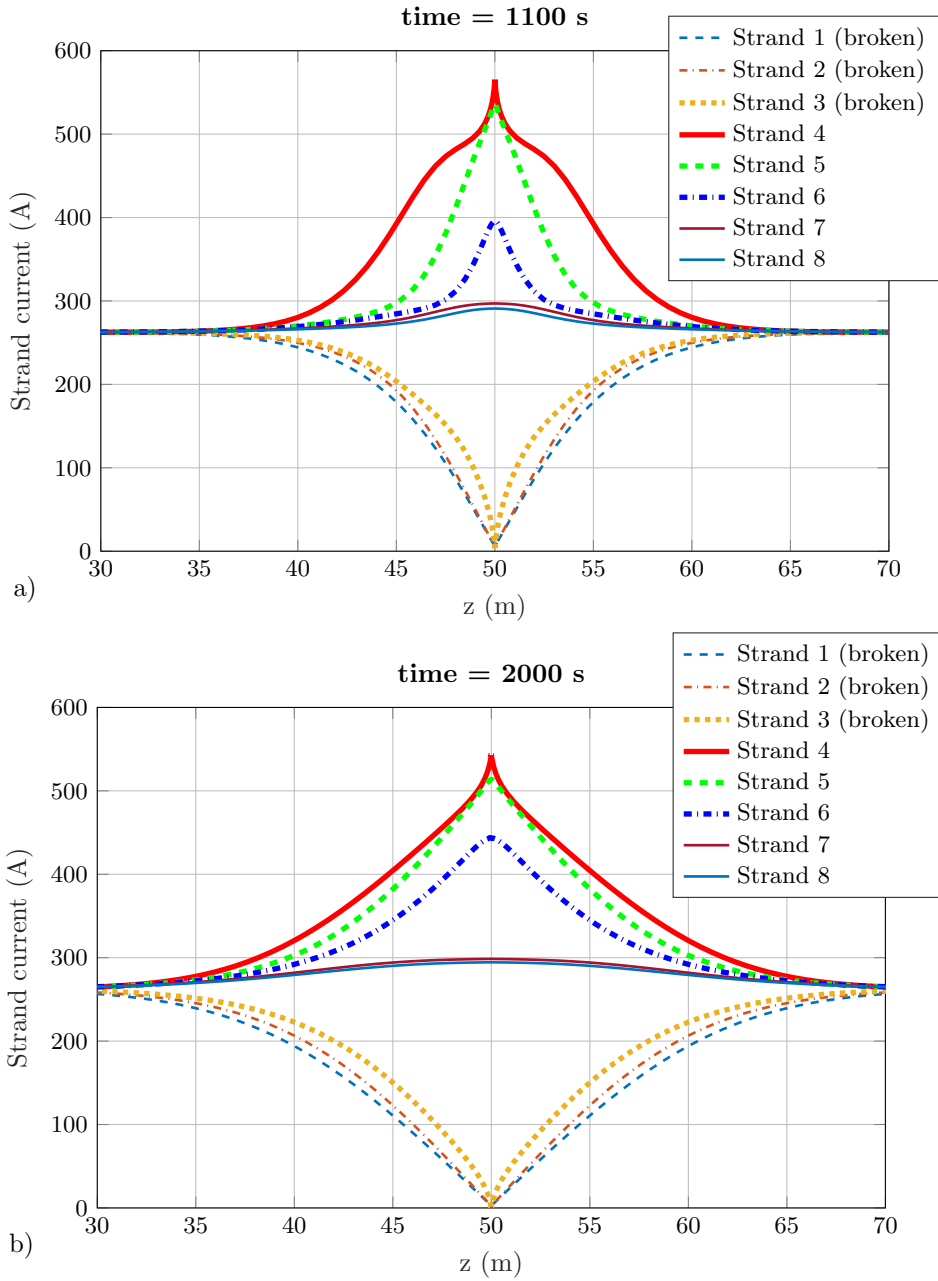


Figure 4.7: Calculated strand currents as a function of position along the 100 m long cable, with a defect at $z = 50$ m, shown 50 s (a) and 950 s (b) after the end of the ramp to 10.5 kA (see also Figure 4.3).

a longer distance, the current redistributes through more and more parallel resistances, which lowers the overall resistance of the redistributing current path and hence lowers the voltage. The diffusion speed, essentially determined by an L/R type of time constant, is higher when the contact resistance is higher, which also means that the current redistribution process will take place over a longer length.

In many ways, the propagation characteristics of a current redistribution zone around a local defect are very similar to those of Boundary Induced Coupling Currents (BICC), which in turn share similarities with electromagnetic waves in terms of dispersion and attenuation characteristics, as described in [60]. In fact, the current redistribution pattern can be seen as a type of BICC with the defect acting as a ‘boundary’.

4.2.4 Strand voltage during a current plateau

Figures 4.8 a and b show voltage profiles along individual strands over the cable length during a current plateau at $t = 1100$ s and $t = 2000$ s as indicated in Figure 4.3. The voltage shown is with respect to ‘ground’, which in the model is taken to be the first node of the first strand. There is a clear difference in the voltage trend between broken and intact strands. Due to current transfer between the strands, a voltage difference occurs between the strands with a defect and the intact strands, which increases in magnitude closer to the defect. For the broken strands, a sharp discontinuity in the voltage profile is visible at the defect position. The neighboring strands that are saturated also show a relatively sharp voltage decrease through the defect, but with a more gradual slope. The intact, unsaturated strands, show a smooth transition through the defect location.

The electric field along the strand direction, i.e. $\frac{dV}{dz}$, at any position other than the defect location, is purely inductive in nature. In steady-state conditions, for example when $t \rightarrow \infty$, all strands would have a constant voltage, except for a resistive voltage drop in damaged or saturated strands at $z = 50$ m. The magnitude of the voltage difference between the strands and therefore of $\frac{dV}{dz}$ is, however, also a function of the inter-strand contact resistance.

The large voltage imbalance between the strands implies that close to a non-uniform defect, measured V-I curves will depend strongly on exactly which strands are being probed. Consider for instance measuring the different possible combinations of voltage tap positions as indicated in Figure 4.8 by the vertical dashed lines. For the voltage drop between these longitudinal positions, one will obtain:

$$\begin{aligned} V_1(\text{intact/broken}) - V_2(\text{broken}) &< 0, \\ V_2(\text{any}) - V_3(\text{any}) &> 0. \end{aligned} \tag{4.1}$$

Note that the subscripts with the voltages here refer to the longitudinal position, not to the strand number. Closer inspection of Figure 4.8 shows how on a plateau after a ramp up, a negative voltage drop along a strand can only be measured

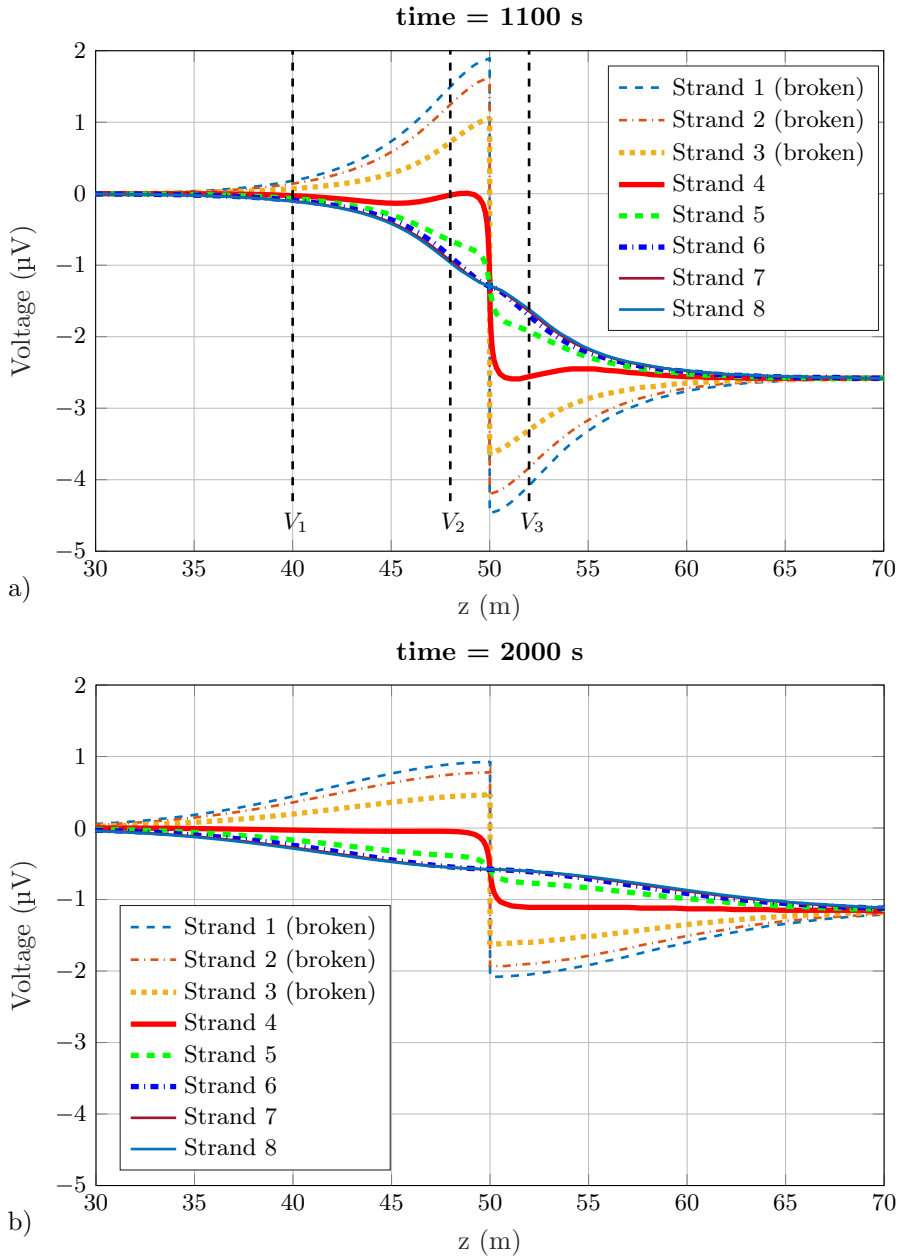


Figure 4.8: Calculated strand voltages to ground as a function of position along the cable, with a defect at $z = 50$ m, shown 50 s (a) and 950 s into the 10.5 kA plateau (see also Figure 4.3).

if *both* voltage taps are positioned either before or after the defect. Hence, if a negative voltage is measured over a probed segment then it can be ruled out that the damage location is situated within this segment.

In Figure 4.8b, the voltage profile is shown still on a current plateau but 900 s later. Similar to the current imbalance in Figure 4.7, the voltage profile has diffused further along the cable length. As a result of the longer current transfer length, the voltage difference between the strands has also become smaller. When measuring a full coil, i.e. when the voltage taps are ‘far away’ from the defect, the voltage drop generated by a defect is measured regardless of which strand the voltage tap is connected to.

Figure 4.9 shows an example of such a negative voltage experimentally observed over an internal conductor segment in the pole-turn of the MQXFBP1 quadrupole magnet after a ramp-up. The combination of the tell-tale time-decaying component and the negative sign of the voltage in one of the segments strongly points to a current redistribution process around a local defect as described above.

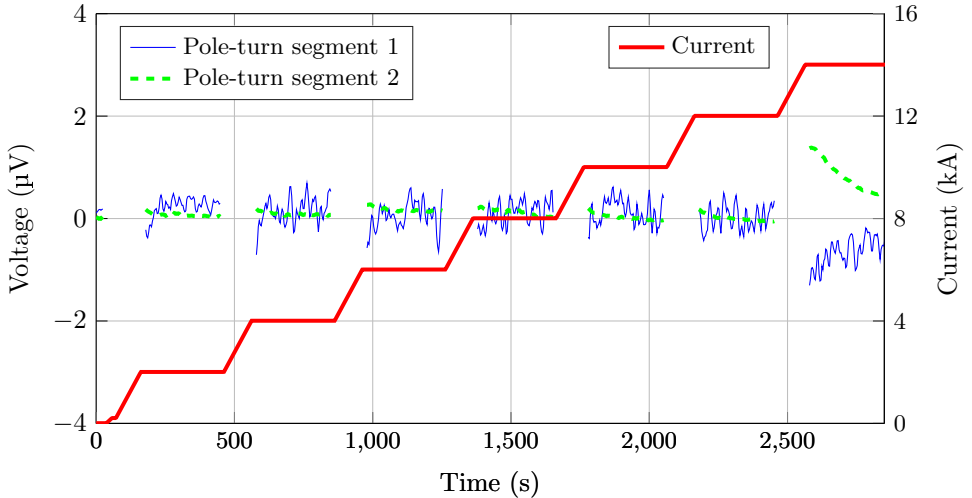


Figure 4.9: Voltage decay measured over two internal conductor segments in the pole-turn of the MQXFBP1 quadrupole magnet after a ramp-up [40].

4.2.5 Effect of a ramp-down in current

In Figure 4.10, the modeled strand current as a function of position is shown at the third time instance indicated in Figure 4.3, at 3300 s, just after the ramp-down to 6 kA has ended. As the total cable current ramps down, the induced voltages force the currents in all strands to decrease by a similar amount. However, around the defect the current in the locally broken strands was already close to zero, so for these strands there is now a reversal in the direction of the current flow. Due

to this effect, the adjacent strand (number 4) shows a significant current ‘dip’ in the immediate surroundings of the defect because also this reversed current flow in the broken strands has to bypass the defect.

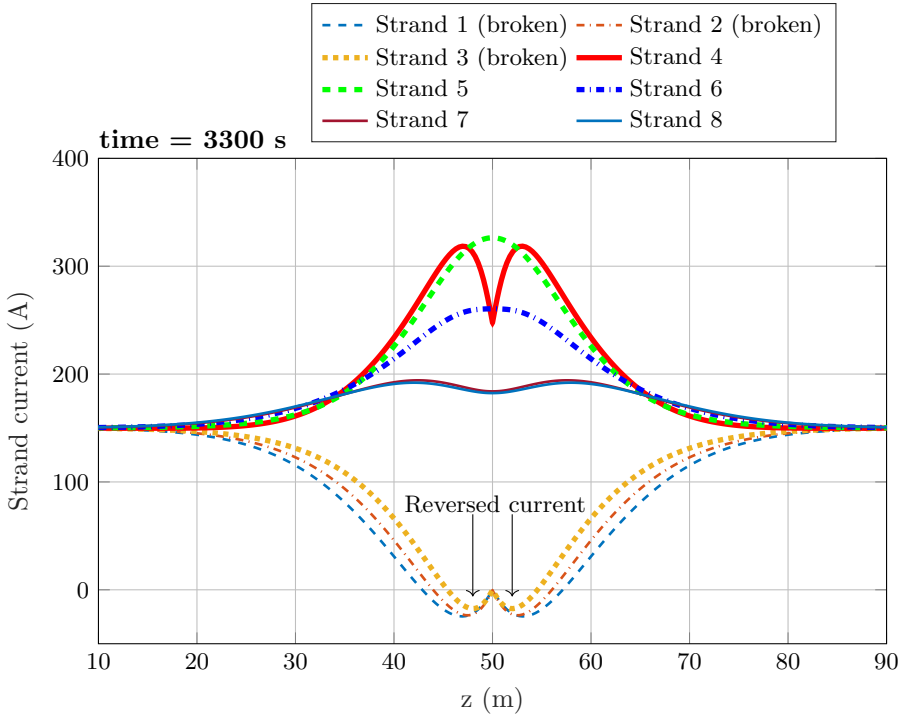


Figure 4.10: Strand current as a function of position along the cable immediately after a ramp down from 10.5 kA to an intermediate current plateau at 6 kA.

Due to the reversal of the current flow, the voltage drop also becomes inverted, as seen in Figure 4.11. This explains why a negative voltage can be measured after ramp-down in a cable with a local defect. The effect of the ramp-down has not yet had time to diffuse over a longer length of cable, so the current distribution in the cable is now determined by a superposition of the effects of the initial ramp-up, which still dominate far away from the defect, and the effects of the ramp-down, which dominate closer to the defect. As time progresses, the reversed current flow will die out and eventually disappear, since there is no external e.m.f. sustaining it. The voltage drop over the defect will then once again become positive.

4.2.6 Dependence on ramp rate and temperature

In Nb₃Sn magnets with performance issues, sometimes a higher quench current is observed for higher ramp rates. It is generally assumed that at lower ramp rates

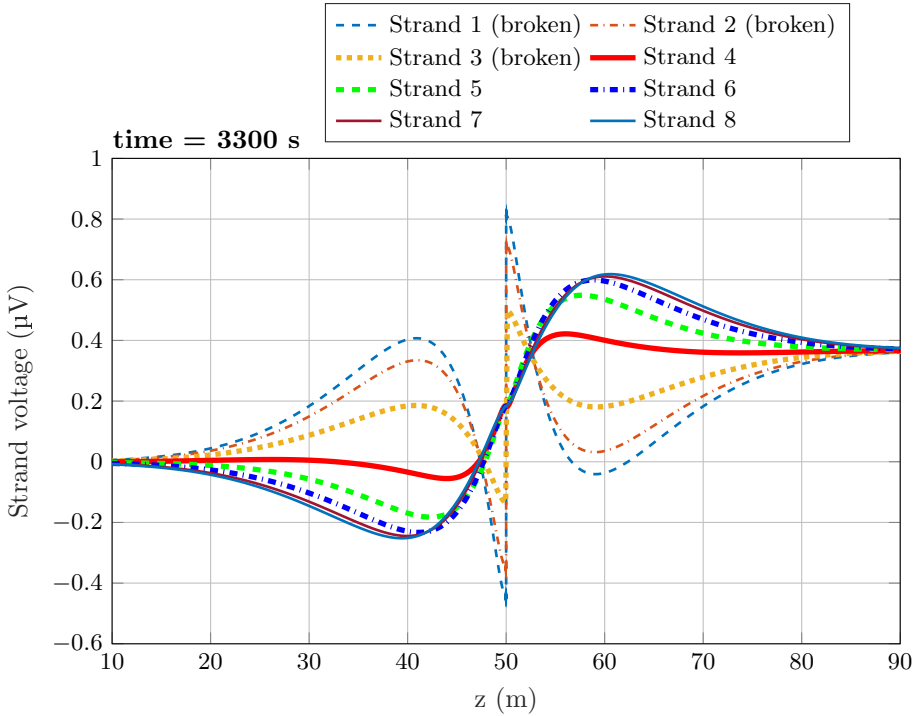


Figure 4.11: Strand voltage as a function of position along the cable after ramp down from 10.5 kA to an intermediate current plateau of 6 kA (see also Figure 4.3). Due to the reverse current flow around the defect the voltage drop has inverted.

a superconducting magnet can reach higher currents because the thermal load of the AC-loss is reduced. However, our simulations show that when a magnet is powered up with higher ramp rates, the inductive voltages will become relatively more important, which then influences the redistribution of current around a local defect, which in turn can cause this counter-intuitive relation between quench current and ramp rate.

In Figure 4.12, the current in the strand adjacent to the defect is shown for different ramp rates. The defect in this case consists of 6 partially degraded strands, with only 5% of their critical current remaining. The n -value has also been lowered to 5 for these degraded strand sections. With an overall ramp rate of 1 A s^{-1} , the adjacent strand carries up to 30% more current during a significant part of the ramp when compared to higher ramp rates. The peak in the current is also slightly higher and is reached earlier at lower ramp rates.

In Figure 4.13, the current in the adjacent strand as well as in its next neighbor is shown assuming two different temperatures. At 1.9 K the current imbalance can grow to a greater extent than at 4.5 K. At 4.5 K, the adjacent strand will saturate

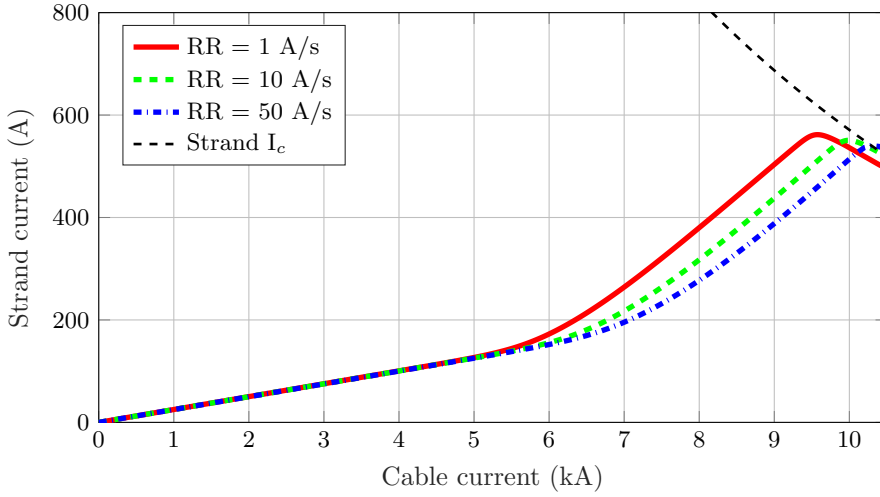


Figure 4.12: Calculated current at the longitudinal defect location in the most overloaded strand adjacent to the defect plotted versus cable current. At lower ramp rates, the initial current imbalance is higher.

sooner, but the greater current imbalance at 1.9 K later on in the ramp suggests why some magnets with a performance limitation are more prone to quenching at 1.9 K than at 4.5 K, especially when taking into account their reduced enthalpy margin at 1.9 K.

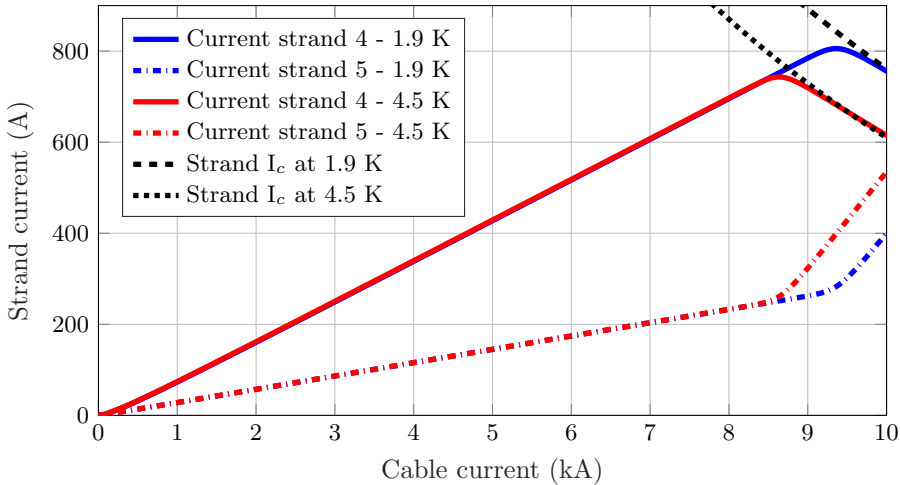


Figure 4.13: Calculated current at the longitudinal defect location in the most overloaded strand adjacent to the defect as well as the next-nearest neighbor strand, both plotted versus cable current. At lower temperatures, the current imbalance is higher.

4.3 Parametric studies

A set of parametric studies is performed to study the influence of various cable - and defect-related parameters on the current redistribution process.

4.3.1 Voltage over defect

To study the influence of the adjacent inter-strand contact resistance R_a on the generated voltage, a 12-strand model with 3 broken strands is used. In Figure 4.14a, the voltage decay over the coil during a current plateau is shown for several values of R_a . As expected, the voltage level rises with increasing R_a . In Figure 4.14b, the voltage at $t = 1050$ s and 2000 s is shown as a function of R_a . A square-root function scaled with the voltage calculated for $R_a = 1 \mu\Omega$ is also shown, demonstrating that there is a near square-root dependence between the voltage and R_a .

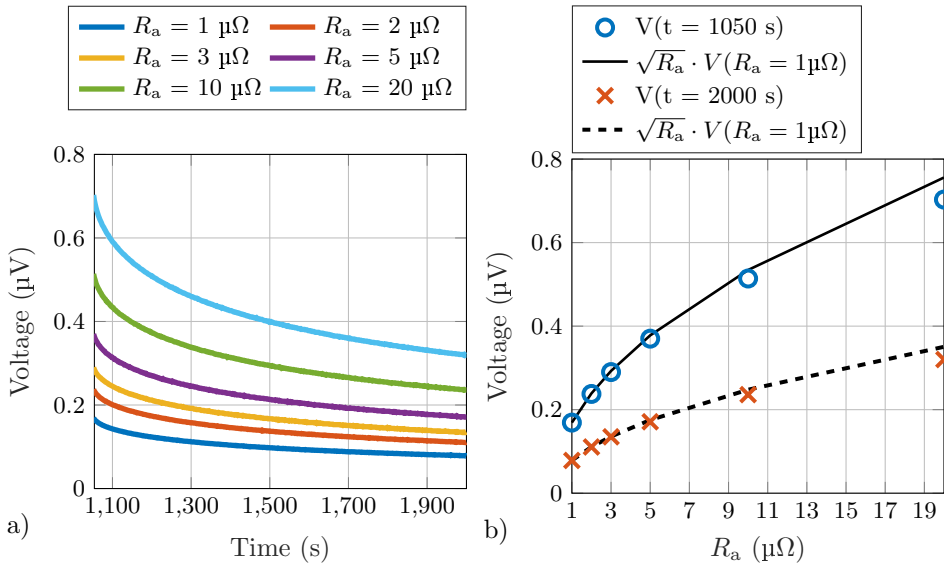


Figure 4.14: a) Calculated time-decay of the cable voltage during a current-ramp plateau probed far away from the defect assuming different values of inter-strand resistance R_a . b) Calculated cable voltage at different moments as a function of R_a .

A similar study investigating the effect of the defect size was performed as well, this time on a 40-strand cable. The adjacent inter-strand resistance R_a is this time kept constant at $5 \mu\Omega$. An increasing number of strands is broken and the resulting voltages are compared. The broken strands are always assumed to lie next to each other. In Figure 4.15, the voltage levels at the beginning and end of the current plateau are shown as a function of the number of broken strands. The relation

between the generated voltage and the defect size is very close to quadratic. Such a quadratic relation can be used to estimate how much damage occurs between different cool-downs, by comparing the voltage signals during identical ramp sequences in different cool-down cycles. However, even though extra broken strands immediately adjacent to already broken strands will yield a quadratic voltage increase, this is not the case for additional defects at different locations along the cable, which scale linearly. Nonetheless, this quadratic relation illustrates how an n times increase in the observed voltage between successive cool-downs does not necessarily indicate an n times increase in strand damage.

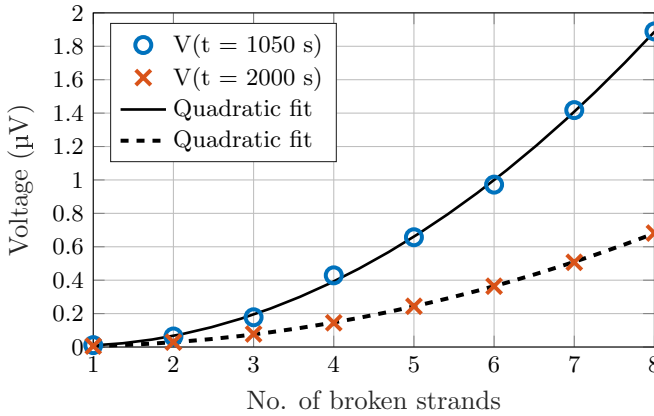


Figure 4.15: Calculated dependence of the cable voltage during a current-ramp plateau on the number of broken strands. A quadratic relation is found. The shown quadratic fitted line serves as a guide for the eye and is only valid for the integer number of broken strands.

4.3.2 Length-scale of the current redistribution

The current imbalance between strands extends up to a certain length around the defect position. The dependence of this length on the adjacent inter-strand contact resistance R_a and on the number of broken strands is shown in Figures 4.16 and 4.17. The profile length is defined as the length from the defect (at $z = 50$ m) to the positions where the current in the strand adjacent to the defect differs only 1 A from the uniform case. The profile length at 5 s after the end of the ramp is used. The R_a -dependence of the profile length once more is described well with a square-root function, as with the voltage in Figure 4.14. This also means that the propagation velocity of the current imbalance, which - as discussed below - is not constant and slows down with time, has a similar dependence on R_a . A similar relation is not found for the profile length dependence on the amount of broken strands: the length increases only by a negligible amount when more strands are broken.

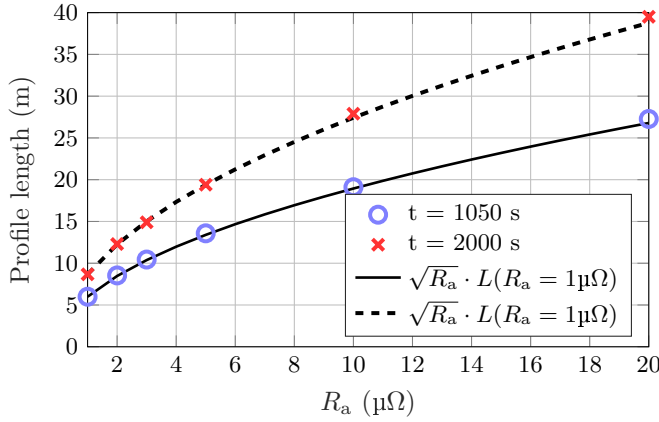


Figure 4.16: Calculated dependence of the current redistribution profile length on the adjacent inter-strand contact resistance R_a . A square-root dependence is observed.

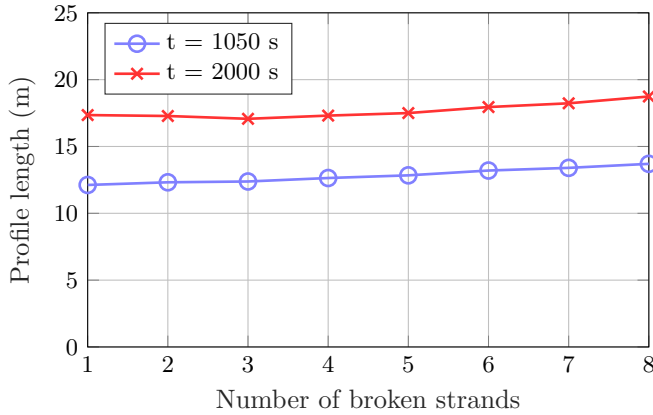


Figure 4.17: Calculated dependence of the current redistribution profile length on the number of broken strands. No significant variation is found with the number of broken strands. The lines serve as a guide for the eye and are only valid for an integer number of broken strands.

4.3.3 Effective time constants

In Figure 4.18, the simulated and normalized voltage decay versus time is shown for different values of R_a during the plateau in the ramp sequence of Figure 4.3. All decays match almost perfectly, indicating that the apparent time constant of the diffusion process is largely independent of R_a . Decreased resistance of the current redistribution path appears to be balanced by the increase in length of the current redistribution zone. Only minute differences between the voltage traces appear when approaching 2000 s.

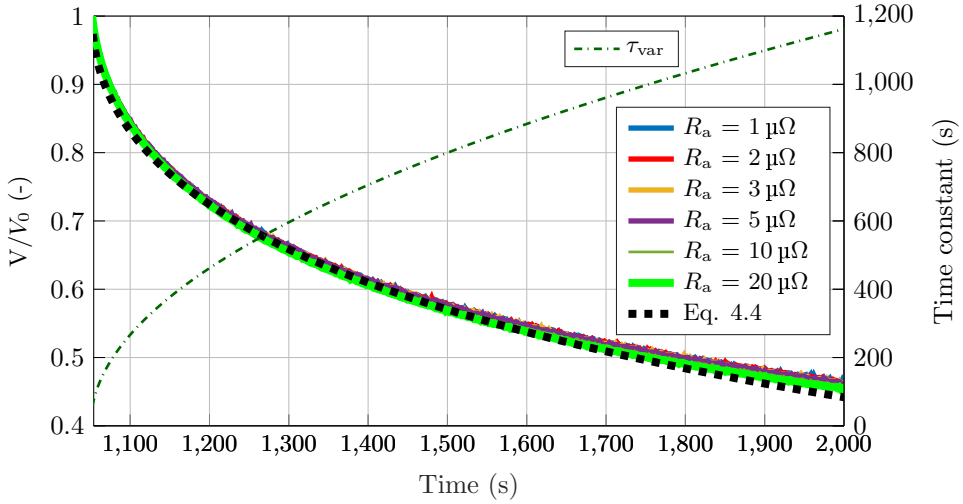


Figure 4.18: Calculated normalized voltage decay and time constant on the current plateau versus time for cables with different values of R_a . An exponential fit with a variable time constant τ_{var} (Equation 4.4) is plotted that describes the voltage decay accurately.

However, the voltage decay cannot be fitted with a ‘pure’ exponential decay assuming a fixed time constant. Instead, it seems that a range of time constants is involved in the current diffusion process. As the current imbalance profile expands, the length scale changes, which affects the apparent effective time constant of the voltage decay over the defect. For the voltage decays in Figure 4.18 we therefore may attempt to describe this behaviour with an empirical exponential fit with a variable time constant:

$$V_{\text{decay}}(t) = V_0 e^{\frac{-t}{\tau_{\text{var}}(t)}}. \quad (4.2)$$

Here V_0 is the voltage right after the ramp has ended. The variable time ‘constant’ τ_{var} is then empirically defined as:

$$\tau_{\text{var}} = d\tau\sqrt{t}, \quad (4.3)$$

which results in:

$$V_{\text{decay}}(t) = V_0 e^{\frac{-\sqrt{t}}{d\tau}}, \quad (4.4)$$

where $d\tau$ is a measure for how fast the time constant changes with time. It is of course peculiar to consider a time *constant* in this context, since τ_{var} has become a variable, but it accurately describes the time-evolution of the system’s characteristic time scale. In Figure 4.18, this fit is shown together with the evolution of the apparent time constant τ_{var} on the opposing y-axis. Because Figure 4.18 is normalized, V_0 is equal to 1, while the fitted value for $d\tau$ is $37.7\sqrt{\text{s}}$. This simple fit matches rather well with the simulation data, but because Equation 4.2 was

chosen empirically, this mathematical form may not fully describe the underlying physics. In the following Section 4.4, we therefore discuss a more a-priori mathematical description of the voltage decay in a simplified cable model that can be solved analytically.

From Figure 4.17 it follows that it is not possible to determine the number of broken strands based on the time constant of the decay. The apparent time constant early-on in the plateau will be shorter if a strand saturation event occurs just before the plateau is reached.

4.4 Analytical diffusion model

An increase of the apparent time constant of the system with time as seen in Figure 4.18 makes sense when regarding the current redistribution around a local defect as a classical diffusion problem. To investigate the underlying principles of the voltage decay in more detail, we derived analytical solutions for the simplified case of a cable consisting of two strands, with one of them broken. Similar diffusion-type problems for superconducting cables have previously been studied analytically in literature, such as for cables with non-uniform joints, cables with a developing normal zone and for cables under local voltage excitation [125, 126, 127, 128, 129, 130].

The analytical solution presented below follows from the diffusion equation, which is also the governing equation of the THEA model, see Equation 2.8, which, for ease of reference, is repeated here:

$$\mathbf{L} \frac{\partial \mathbf{i}}{\partial t} + \mathbf{R} \mathbf{i} - \frac{\partial}{\partial z} \left(\mathbf{G}^{-1} \frac{\partial \mathbf{i}}{\partial z} \right) = \Delta \mathbf{v}_{\text{ext}}. \quad (2.8)$$

Assuming no externally applied voltage ($\Delta \mathbf{v}_{\text{ext}} = 0$), no resistance in the strands ($\mathbf{R} = 0$) and a uniform adjacent inter-strand contact conductance, this equation reduces to:

$$\frac{\partial \mathbf{i}}{\partial t} = (\mathbf{G} \mathbf{L})^{-1} \frac{\partial^2 \mathbf{i}}{\partial z^2}. \quad (4.5)$$

For our analytical solution, we step away from the matrix notation:

$$\frac{\partial I}{\partial t} = D \frac{\partial^2 I}{\partial z^2}, \quad (4.6)$$

where the diffusivity D is defined as: $(g_a L_l)^{-1}$, with g_a the adjacent inter-strand contact conductance (S m^{-1} , see Equation 2.11) and L_l the leakage inductance (H m^{-1}) between the two strands, which is calculated from the strand self- and mutual inductance: $L_l = 2L_s - 2L_m$, see Equations 2.17 and 2.18.

4.4.1 Step-like ramps

When subjecting a two-strand cable with a broken strand to a fast ramp (approximating a step function) and assuming no saturation of the adjacent non-broken strand, the evolution of current in this adjacent strand (and thus also the broken strand, since $I_{\text{tot}} = I_{\text{adj}} + I_{\text{broken}}$) can be described by the well-known analytical solution of Equation 4.6 for an instantaneous constant source (in this case the ‘source’ is the current imbalance imposed by the defect, where $I_{\text{broken}} = 0$ and $I_{\text{adj}} = I_{\text{tot}}$)[91]:

$$I_{\text{adj}}(z, t) = \frac{1}{2} I_{\text{tot}} \left[1 + \operatorname{erfc} \left(\frac{z}{2\sqrt{Dt}} \right) \right], \quad (4.7)$$

where $2\sqrt{Dt}$ is the diffusion length (with a square-root dependence on t). Because of symmetry, we only regard half of the cable (Equation 4.7 is only valid for $z \geq 0$) and assume the defect is at $z = 0$. The strands’ coupling parameters used in this case study are listed in Table 4.2.

Table 4.2: *Coupling parameters between strands used in the analytical model.*

Parameter	Value	Unit
g_a	0.16	GS m ⁻¹
L_s	1.42	μH m ⁻¹
L_m	1.19	μH m ⁻¹

In Figure 4.19, the analytically calculated currents in the adjacent strand (Equation 4.7) are compared to a simulation of the same two-strand cable with THEA. In THEA the cable current is ramped to 500 A with a rate of 500 A s⁻¹ in order to approximate a step-like current ramp. At the end of the step-like ramp, the current in both strands is the same at each longitudinal position (except for $z = 0$ m) due to the induced voltage during the ramp. After the step-like ramp, the current imbalance forced by the defect starts to diffuse through the cable. The analytical calculation matches perfectly with the THEA simulation, as might be expected, since THEA’s governing equation is precisely the diffusion equation (Equation 2.8, see Section 2.3.1).

As illustrated in Figure 4.20, the voltage difference between two voltage taps placed on the adjacent strand can be calculated by integrating the electric field, since $\frac{dV}{dz}$ only has inductive components for this strand, see also Section 4.2.4. To calculate the electric field in the adjacent strand, we sum the contribution of self - and mutual inductances due to the current change in both strands:

$$\frac{dV_{\text{adj}}(z, t)}{dz} = L_s \frac{dI_{\text{adj}}(z, t)}{dt} + L_m \frac{dI_{\text{broken}}(z, t)}{dt} = \frac{1}{2} L_l \frac{dI_{\text{adj}}(z, t)}{dt}. \quad (4.8)$$

For the voltage over two voltage taps that are symmetrically placed before and after the defect we then obtain:

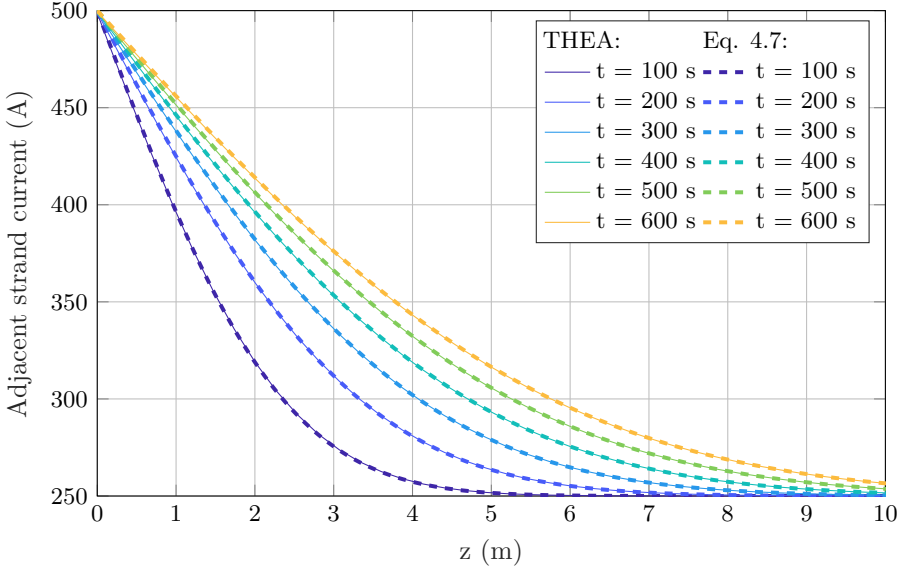


Figure 4.19: Simulated and analytically calculated (Equation 4.7) current in the adjacent strand as function of position for different time instances after a fast ramp (500 A s^{-1}) to 500 A for a two-strand cable, with a defect at $z = 0 \text{ m}$ in one of the strands.

$$V_{\text{taps}}(t) = L_1 \int_0^{z_{\text{vt}}} \frac{dI_{\text{adj}}(z, t)}{dz} dz, \quad (4.9)$$

where z_{vt} is the distance from zero (the defect position) to the voltage tap location.

The time derivative of Equation 4.7 is:

$$\frac{dI_{\text{adj}}(z, t)}{dt} = \frac{I_{\text{tot}}}{2} \frac{d}{dt} \text{erfc} \left(\frac{z}{2\sqrt{Dt}} \right) = \frac{I_{\text{tot}}}{2} \frac{ze^{-\frac{z^2}{4Dt}}}{2\sqrt{\pi Dt}^{\frac{3}{2}}} \quad (4.10)$$

Taking the integral of Equation 4.10 with respect to z from zero to z_{vt} then yields:

$$\frac{I_{\text{tot}}}{2} \int_0^{z_{\text{vt}}} \frac{ze^{-\frac{z^2}{4Dt}}}{2\sqrt{\pi Dt}^{\frac{3}{2}}} dz = -\frac{I_{\text{tot}}}{2} \sqrt{\frac{D}{\pi t}} e^{-\frac{z_{\text{vt}}^2}{4Dt}} \Big|_{z=0}^{z=z_{\text{vt}}} \quad (4.11)$$

Substituting this result into Equation 4.9 yields:

$$V_{\text{taps}}(t) = L_1 \frac{I_{\text{tot}}}{2} \sqrt{\frac{D}{\pi t}} \left(1 - e^{-\frac{z_{\text{vt}}^2}{4Dt}} \right). \quad (4.12)$$

This expression is similar to the one derived by Ries for a two-strand cable with a developing normal zone [128]. When the voltage taps are sufficiently far away for a given time ($z_{\text{vt}} \gg 2\sqrt{Dt}$), as indicated e.g. by the black dots in Figure 4.20,

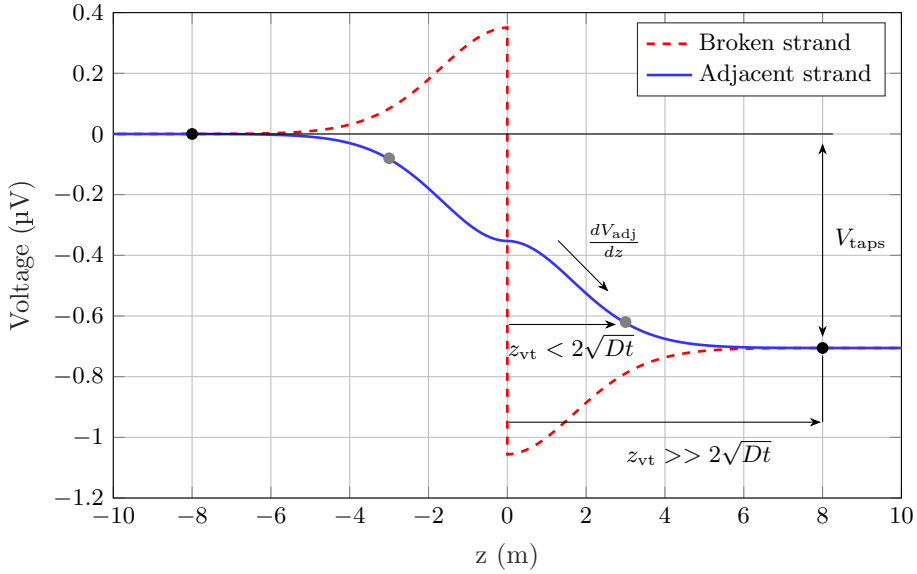


Figure 4.20: Schematic of the voltage with respect to ground along the broken and the adjacent strand. The voltage drop along a strand section V_{taps} can be calculated by integrating the electric field $\frac{dV_{\text{adj}}}{dz}$ of the adjacent strand along the cable length. Different voltage tap locations are indicated by the black and gray dots.

the exponential term in the round brackets has decayed and so the voltage over the taps simplifies to:

$$V_{\text{taps}}(t) = L_1 \frac{I_{\text{tot}}}{2} \sqrt{\frac{D}{\pi t}}. \quad (4.13)$$

In Figure 4.21, this result from Equation 4.13 is compared to the THEA simulation, which once more shows a perfect match. During a current plateau after a step-like ramp, the voltage over a cable with a defect thus decays as $\frac{1}{\sqrt{t}}$.

Also shown in Figure 4.21, is the result for a voltage tap pair close to the defect (at $z = 2$ m). At this distance and time range ($z_{\text{vt}} < 2\sqrt{Dt}$), the full solution of Equation 4.12 has to be used. After about 20 s, this voltage tap pair starts to diverge from the one that is placed far away and reads a lower voltage drop. In Figure 4.21b, the two normalized terms of Equation 4.12 are shown separately. After about 20 s, the second term starts to decay, indicating that at this moment the diffusion length starts to exceed the distance between the voltage tap and the defect (like with the gray dots in Figure 4.20).

For Equation 4.9 we assumed a symmetrically placed voltage tap pair on the adjacent strand, but in principle we can analytically obtain the voltage difference between two voltage taps placed at any location and also placed on any combina-

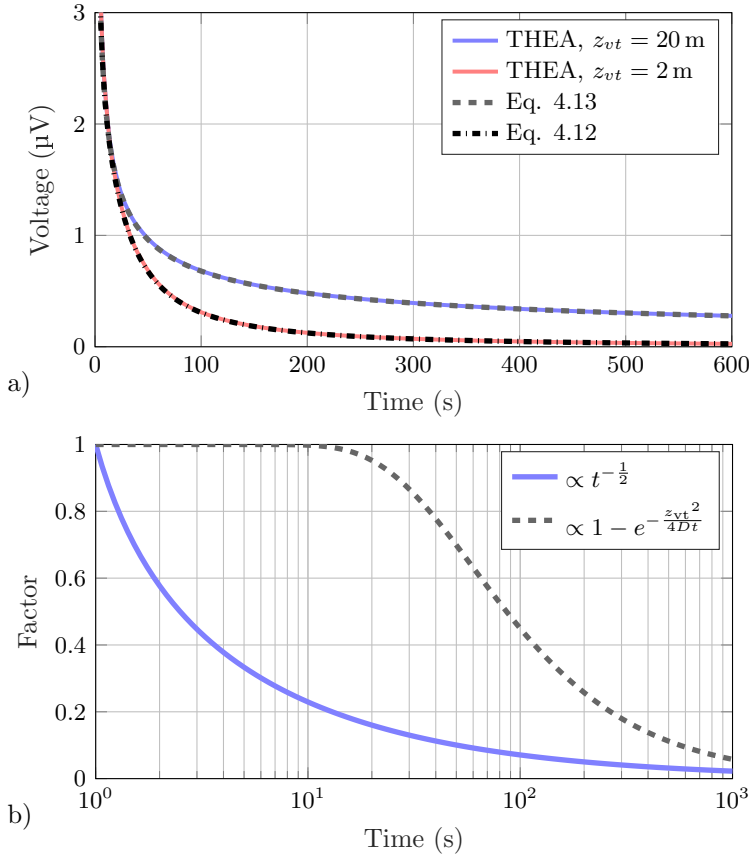


Figure 4.21: a) Simulated and analytically calculated (Equation 4.13) voltage over the cable as a function of time for a fast ramp of 500 As^{-1} to 500 A for a two-strand cable, with a defect in one of the strands. b) Time progression of the two time dependent terms in Equation 4.12. The exponential term (gray dashed line) starts to decay when the diffusion length exceeds the voltage tap position.

tion of strands by modifying Equation 4.9, for example, by splitting the integral in more terms for asymmetric voltage tap placement. By exploiting symmetry, this method can also easily be expanded to three strands, with one broken strand in the middle and two adjacent strands. Note also how the case of a step-like ramp is equivalent to the case of a cable at constant current with a sudden strand breakage.

4.4.2 Slow current ramps

The solutions above for step-like ramps can be expanded to calculate the behaviour for slower ramps with a finite duration. By taking the integral of the step-like solution Equation 4.7 over the duration of the ramp, we obtain the combined

contribution of the full ramp:

$$I_{\text{adj}}(z, t) = \frac{1}{2} \frac{dI_{\text{tot}}}{dt} \int_0^{t_{\text{ramp}}} \left(1 + \operatorname{erfc} \left(\frac{z}{2\sqrt{D(t-t')}} \right) \right) dt', \quad (4.14)$$

where t' is a time-delay defined for each infinitesimal time-step during the ramp, t_{ramp} is the duration of the ramp and $\frac{dI_{\text{tot}}}{dt}$ is the cable ramp rate (A s^{-1}). Solving this integral we obtain:

$$I_{\text{adj}}(z, t) = \frac{1}{2} \frac{dI_{\text{tot}}}{dt} \left[\left(\frac{2z^2}{4D} + t - t' \right) \operatorname{erf} \left(\frac{z}{2\sqrt{D(t-t')}} \right) + 2 \left(\frac{z\sqrt{t-t'} e^{\frac{-z^2}{4D(t-t')}}}{2\sqrt{\pi D}} + 1 \right) \right] \Bigg|_{t'=0}^{t'=t_{\text{ramp}}}. \quad (4.15)$$

In Figure 4.22, the thus calculated current in the adjacent strand is shown and compared to the THEA simulation for several time instances, this time also including one during the ramp (at $t = 50$ s). Once more there is a good agreement between the analytical and numerical models.

Similar to Equation 4.14, also the voltage over the taps of Equation 4.13 can be integrated to obtain the solution for a finite ramp with duration t_{ramp} :

$$V_{\text{taps}}(t) = \int_0^{t_{\text{ramp}}} L_l \frac{1}{2} \frac{dI_{\text{tot}}}{dt} \sqrt{\frac{D}{\pi(t-t')}} dt'. \quad (4.16)$$

This yields:

$$V_{\text{taps}}(t) = L_l \frac{dI_{\text{tot}}}{dt} \sqrt{\frac{D}{\pi}} \left(\sqrt{t} - \operatorname{Re} \left(\sqrt{t - t_{\text{ramp}}} \right) \right). \quad (4.17)$$

In Figure 4.23, the voltage over the cable (with $z_{\text{vt}} \gg 2\sqrt{Dt}$) during and after the slow ramp to 500 A is shown for both the THEA simulation and Equation 4.17, showing again a good agreement. During the ramp, the induced voltage due to the overall current change in the cable is subtracted from the THEA simulation for better comparison with the analytical solution, which only takes into account induced voltages due current exchange between strands. The mathematical form of the voltage decay due to a local defect on a current plateau is thus given by a square-root function from which a time-delayed square-root function is subtracted. The general shape of the voltage decay only depends on this time-delay i.e. on the ramp duration. During the ramp, the voltage increases proportional to $\sqrt{t}$. From the pre-factor of the time-dependent term in Equation 4.17, the square-root dependence of the voltage on R_a as seen in Figure 4.14 is also immediately obvious: $\sqrt{D} = \sqrt{(g_a L_l)^{-1}} \propto \sqrt{R_a}$. In addition, there is a square-root dependence of the voltage on the leakage inductance L_l as well as a linear dependence on the ramp

rate, which for a given t_{ramp} simply implies a linearity with current.

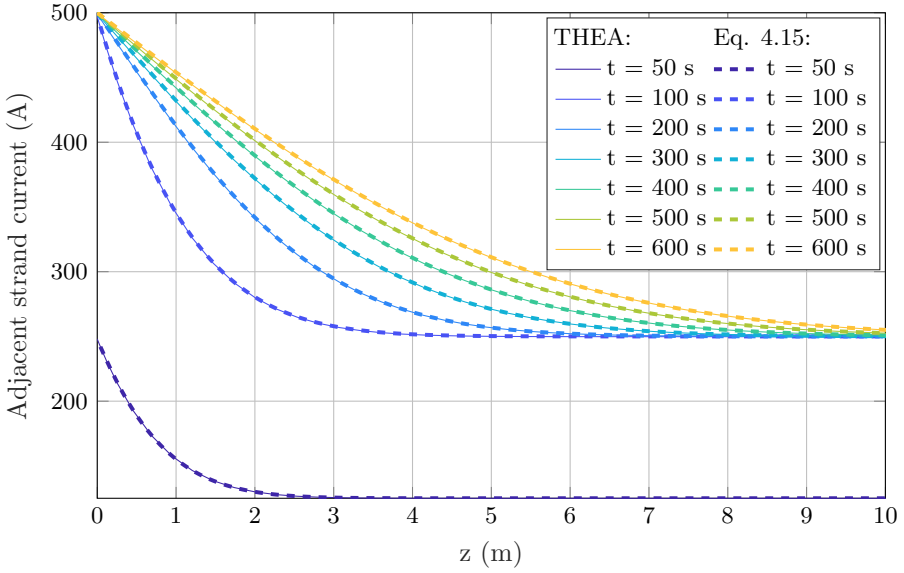


Figure 4.22: *THEA-simulated and analytically calculated (Equation 4.15) currents in the undamaged strand of a two-strand cable during and after a slow ramp (5 A s^{-1}) to 500 A, plotted as a function of position for different time instances.*

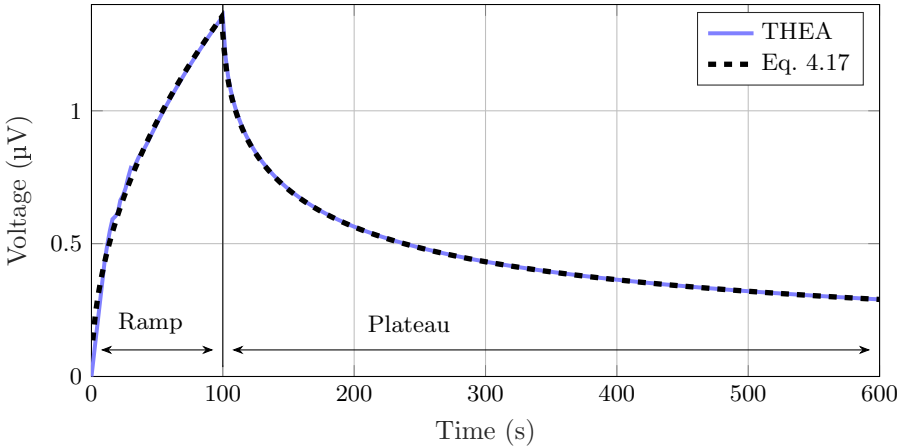


Figure 4.23: *THEA-simulated and analytically calculated (Equation 4.17) voltages over a two-strand cable as a function of time for a slow ramp of 5 A s^{-1} to 500 A, with a defect in one strand. During the ramp, the induced voltage offset due to the overall current change in the cable is subtracted from the THEA result for comparison with the analytical solution.*

4.5 Comparison between Network Model (RuNe) and Continuum Model (THEA)

In this section, the results of the older continuum model (THEA) are compared to the new Rutherford network model (RuNe, see Chapter 2). In RuNe, the Rutherford cable is simulated fully in 3D with the details of the magnetic field distribution over the cable width and length fully incorporated. For a 40 m long cable this leads to a system with almost 1.8 million degrees of freedom. It can nevertheless be solved in a practical time frame of several hours, due to the efficient solver sequence.

A comparison of the RuNe model with the THEA simulation is shown in Figures 4.24 and 4.25, which show the same current - and voltage profiles as in Figure 4.7 and 4.8, respectively. To make the comparison more relevant, the averaged circulant inductance matrix that is used in THEA is calculated by integration of the partial inductance matrix of RuNe instead of using the analytical expressions of Equations 2.15 and 2.16, or using the CID software [124].

The results of the two models match very well, with deviations between them in the order of 1 %. This difference might be due to numerical noise or it could be related to the fact that in RuNe the inter-strand contacts are included in the inductance matrix. In any case, for simulations on these relatively long length scales the inter-strand contacts can be neglected, they are only needed when modelling short length/time scale events such as the quenches studied in Chapter 5. After the current distribution during a ramp has been calculated, the final result can be used directly as the initial conditions for the quench modeling.

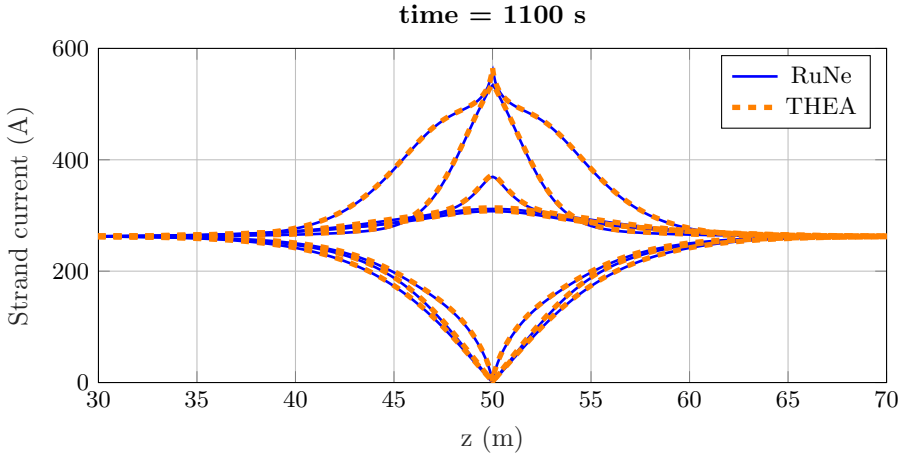


Figure 4.24: Strand current as a function of position as simulated with RuNe and THEA, showing a near-perfect agreement (see also Figure 4.7) and Section 4.2.3.

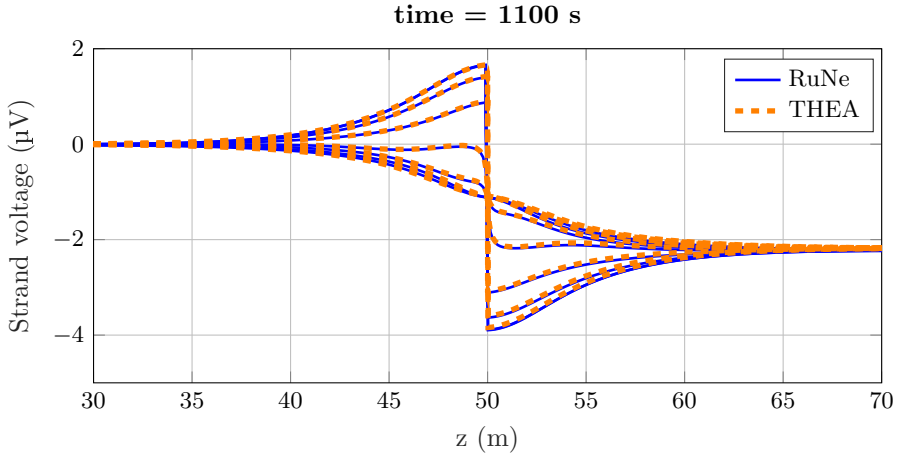


Figure 4.25: Strand voltage with respect to ground as a function of position as simulated with RuNe and THEA, showing once more a near-perfect agreement (see also Figure 4.8).

4.6 Effect of inter-strand contact resistances and damage distribution

Later on, in Section 4.7.3, the local values of the parameters R_a and R_c of the cable that was used in the SMC magnet are determined by comparing modeled and measured data of the voltage decay on a current plateau for different numbers of broken strands. There, we will show how, as expected, the current distribution in the strands strongly depends on the ratio R_a/R_c . When $R_a/R_c \ll 1$, the crossing strands contribute hardly to the current redistribution and almost all excess current is transferred to the adjacent strands. For $R_a/R_c > 1$, the adjacent strand still takes most of the current, but the crossing strands now also take a significant fraction of the excess current, i.e. in the latter case the current distribution in the cable is more uniform than in the former one.

Further analysis (see Section 4.7.3), revealed indications that certain strands in the cable, close to the damaged strands, also suffered a partial reduction in critical current. Realistically, most practical cases of cable damage in full-scale accelerator magnets are expected to consist of locally partially damaged strands, instead of fully broken strands. The case of fully broken strands must thus be seen as a limit. It is therefore useful also to study how a more distributed defect affects the cable performance.

In this section, the effect of these two variables: R_a/R_c ratio, and the spread of damage, are described using RuNe.

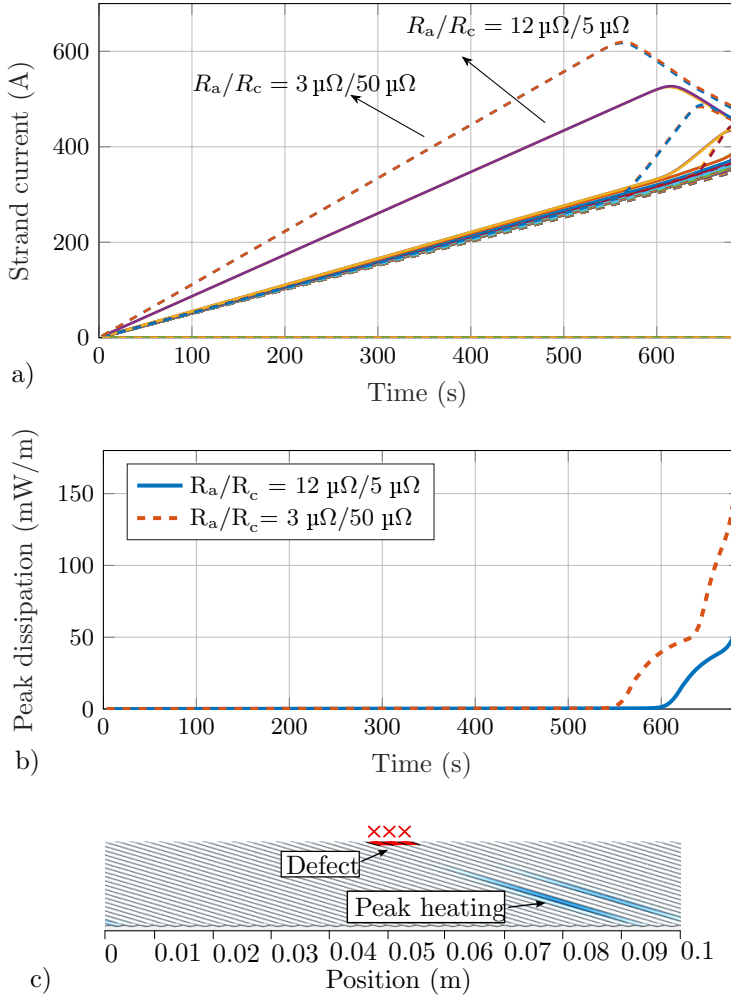


Figure 4.26: a) Simulated strand current as a function of time during the magnet ramp at the location of the introduced defect for two different ratios of the inter-strand contacts: $R_a/R_c = 12/5$ (solid line) and $R_a/R_c = 3/50$ (dashed line). b) Simulated peak power dissipation per meter versus time in the Rutherford cable. c) Schematic top-view of the cable indicating the locus of maximum heating.

4.6.1 Dependence of the current distribution and power dissipation on inter-strand contact resistances

In Figure 4.26a, the simulated strand currents in the cross-section at the defect location are shown for two cases of a cable with three fully broken strands. The first case has the same R_a/R_c ratio as the SMC magnet, that will be discussed in Section 4.7, i.e. $R_a/R_c = 12 \mu\Omega/5 \mu\Omega = 2.4$, (as determined in Section 4.7.3). The second case considers a cable with $R_a/R_c = 3 \mu\Omega/50 \mu\Omega = 0.06$. For the first

case, the current in the adjacent strand rises slower and therefore saturates at a later time. With this higher R_a/R_c ratio, only two strands are saturated at the end of the ramp, while for the second case already three strands are saturated.

For each unit cell in the model, the total power dissipation is calculated and the maximum value along the entire length of the cable is shown in Figure 4.26b. Note how the peak power dissipation is not located at the defect position, but about a quarter twist pitch further, at the position where the adjacent strand passes through the peak-field region of the cable cross-section, as shown in Figure 4.26c. When the adjacent strand saturates, a significant increase in heating power takes place and with each successive strand saturating the rate of heat dissipation accelerates further. When a strand saturates, not only does the surplus current need to circumvent the saturated spot in the strand, but some of the current that is already in the strand is also forced out due to the reduction of its critical current as the magnet keeps ramping to higher magnetic field.

Rather counter-intuitively, the behavior in Figure 4.26b illustrates how from a point of view of resilience against local damage, it can be beneficial to design a cable with a higher ratio of R_a/R_c . The current will then redistribute around a defect in a more homogeneous way, delaying strand saturation and thus as well the onset of excessive heating in the strands.

4.6.2 Effect of damage distribution and core design

The observations above are based on a cable model in which strands are fully broken, which is the case of the controlled damage experiment described in Section 4.6, but in practice is quite a severe limiting case of possible degradation. In a more realistic case of a cable that was accepted for magnet winding, i.e. with initially invisible damage (or a degradation occurring during the winding itself or even during magnet operation) it is more likely that strands are only partially degraded in a local spot.

An additional consideration is that, unlike the SMC magnet used for the experiment, state-of-the-art Nb₃Sn Rutherford cables for accelerator magnets normally have a stainless steel ribbon as a resistive core inserted between the two layers. This core prevents the formation of large Inter-Strand Coupling Currents (ISCC), thus limiting coupling loss in the cable and mitigating the effects of Boundary Induced Coupling Currents and the associated magnetic field errors in the magnets as well. However, such a desire for a high R_c value to reduce coupling loss poses a contradictory requirement to the need for a low R_a/R_c ratio for improved thermal stability, which means a compromise must be made.

There is some margin for optimization, since the transverse width of the inserted ribbon can be adjusted. The cable thus effectively has two values for the crossing contact resistance R_c , see Figure 4.27. The values are denoted as $R_{c,edge}$ at the edge of the cable, where in an idealized model no core is present, and as

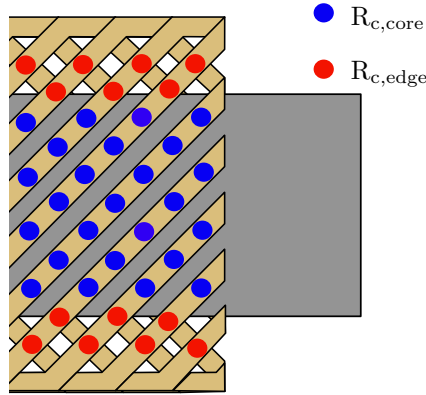


Figure 4.27: Illustration of the two types of crossing contact resistance between the two cable layers due to the presence of a stainless steel core. In real cables, the core is usually not perfectly centered.

$R_{c,core}$ in the center of the cable, where the core is situated. Note that in practice this core is not always well-centered and tends to dwell around. Also, the strand pairs that are in contact through $R_{c,edge}$ lie closer to each other than other crossing strands and thus have a higher mutual inductance, thus making them a more favorable path for current redistribution both in terms of resistance and inductance. This also implies that the dominant current redistribution paths all lead to strands that lie close to the degraded/broken strand(s), while only a negligible amount of current is redistributed to strands on the opposite side of the cable, e.g. from strand 1 to strand 20. The homogenization of redistributed current is thereby limited. In terms of AC losses, opposing strand pairs form the largest loops that couple with the magnetic background field and are therefore the biggest contributors to the ISCCs when no core is present [60]. Strand pairs that are close together form a smaller surface area for coupling with the background field and therefore contribute less to the ISCCs. We can therefore conclude that strand pairs that participate the most in current redistribution near a defect contribute the least to ISCCs. The requirement to keep coupling currents low and to increase the resilience of the cable to local damage are thus not fully in conflict.

We investigated numerically the effect of the cable core and of the inter-strand contact resistance ratio on the current redistribution process around a local defect with different types of damage distributions. Figures 4.28a - c schematically shows the different types of damage considered.

Characteristic literature values for R_a in a Nb_3Sn Rutherford cable vary over a wide range, in the order of $0.1 \mu\Omega$ to $100 \mu\Omega$ [131]-[145]. $R_{c,core}$ values are in the order of $1000 \mu\Omega$ or higher, which implies that these contacts hardly contribute to the current redistribution. For the bare copper-copper contact of $R_{c,edge}$, much lower values of $0.2 \mu\Omega$ to $10 \mu\Omega$ are observed, with some authors reporting values

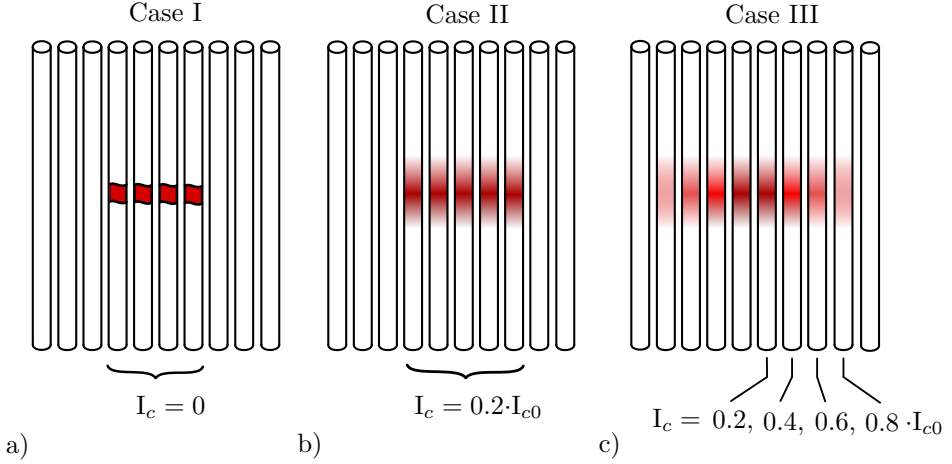


Figure 4.28: Different types of simulated damage distribution: fully broken strands (a), strands with equal partial damage (b), symmetric distributed damage (c). All cases exhibit the same total I_c reduction of the cable.

outside this range. As shown in Figure 4.27, the core overlaps only partly with some crossing contacts. Their fraction is calculated by using the ratio between the surface of the contact that is covered and that which is exposed. For this study, the core is assumed to be perfectly centered in the cable and covers 66 % of the available width. $R_{c,core}$ is set to 2000 $\mu\Omega$.

Table 4.3: Five different combinations of R_a and $R_{c,edge}$ (both in $\mu\Omega$) and the numerically calculated values of the corresponding inter-strand coupling loss.

$R_a/R_{c,edge}$	ISCL (mW m^{-1})
1/10	0.6
1/1	2.8
1/0.2	12.5
10/1	2.5
10/0.2	12

Cable with fully broken strands

A parametric study is performed to demonstrate how different contact resistances (and the ratio between them) influence current redistribution. The cable parameters and background magnetic field are taken to be equivalent to those of a cable in the pole-turn of an 11 T dipole magnet operated at 1.9 K. First we consider the damage scenario in Figure 4.28a, i.e. four adjacent strands that are fully broken at the high magnetic field edge of the cable. The magnetic field gradient over the cable width is equivalent to that of the pole-turn in an 11 T HL-LHC dipole

magnet. The cable current is ramped up with 10 A s^{-1} . The peak value of the heating along the cable length is shown in Figure 4.29a.

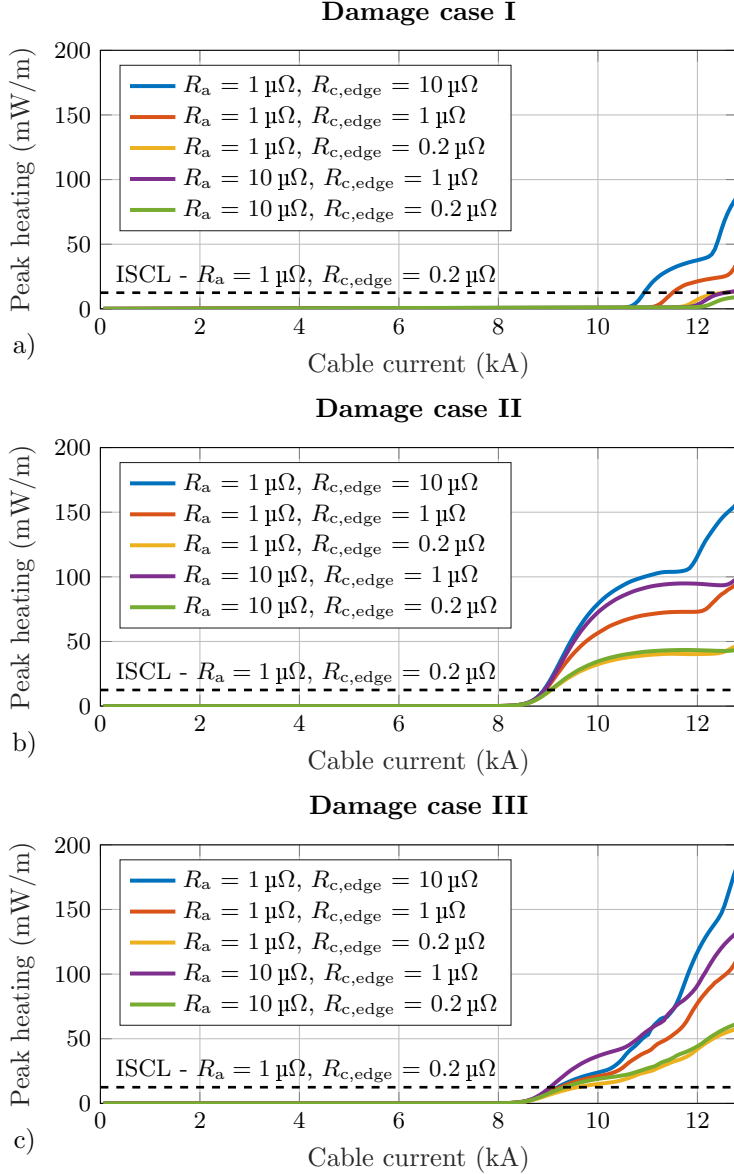


Figure 4.29: Calculated peak heating along the cable length per meter as a function of cable current for different combinations of R_a and $R_{c,edge}$, for fully broken strands (a), partially and equally degraded strands (b and c), corresponding to the cases in Figure 4.28.

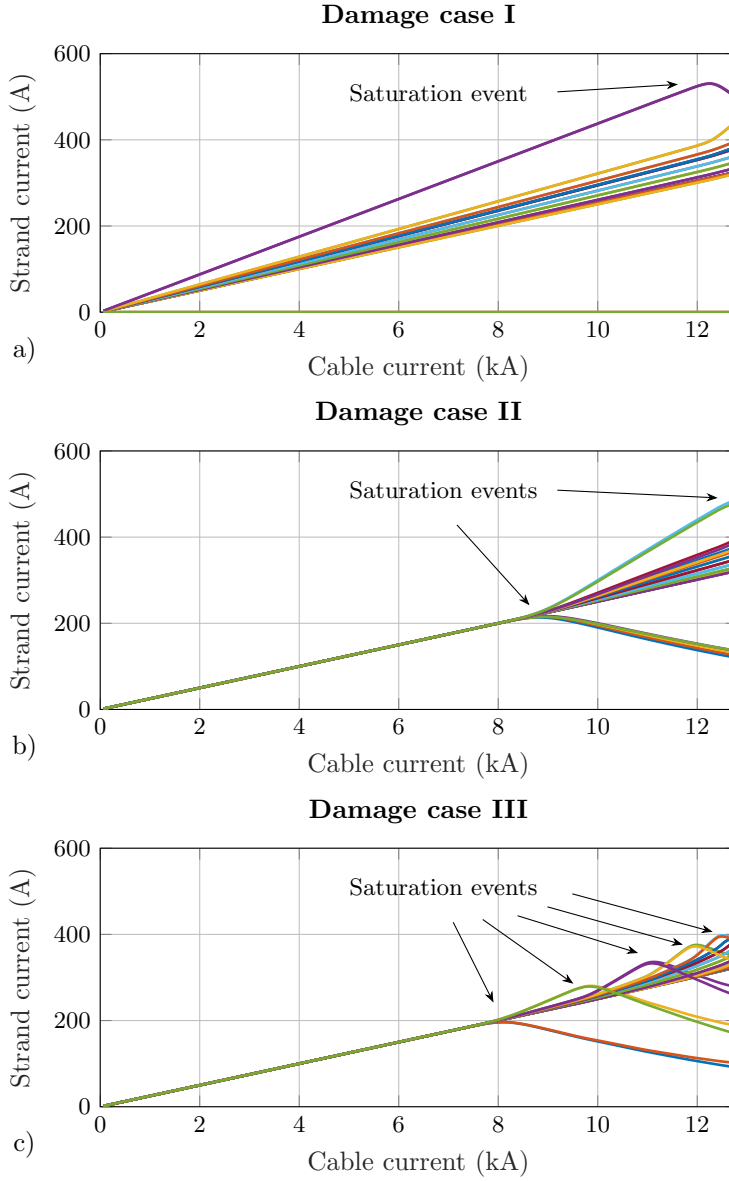


Figure 4.30: Calculated strand currents at the defect position during a cable current ramp to 12.8 kA with 10 As^{-1} (with $R_a = 10 \mu\Omega$ and $R_{c,edge} = 0.2 \mu\Omega$) for fully broken strands (a); partially and equally degraded strands (b); and partially but distributed degraded strands (c), corresponding to the damage types in Figure 4.28.

The power dissipation due to the generated inter-strand coupling loss (ISCL) associated with the chosen contact resistances for a magnetic field change rate

perpendicular to the broad face of the cable $\dot{B}_\perp$ of 0.01 T m^{-1} is listed in Table 4.3, with its highest value also shown in the Figures 4.29a - c for comparison with the peak heating due to current redistribution. The calculated ISCL values correspond to the steady-state case, i.e. when the ramp has been going on long enough for the ISCLs to fully establish. The time constants involved are in the order of several seconds. For a cored cable, the heat load associated with the ISCL is concentrated along the edges of the core [60]. Note also that the peak heating due to current redistribution is a local value while the ISCL is uniform along the cable length.

Note how for the two cases with $R_{c,\text{edge}}$ $1 \mu\Omega$ and $0.2 \mu\Omega$, a higher value of R_a leads to less power dissipation, because it delays the onset of strand saturation. This case is similar to the one discussed in Section 4.6.1 and presented in Figure 4.26.

Cable with partially degraded strands

In Figure 4.29b, the result of the same study is repeated but now for a cable with partially degraded strands. Five strands have a critical degradation of 80 %, leading to the same overall cable critical current decrease as in the previous case. For such partial degradation, the high ratio of R_a/R_c does not help with postponing the first strand saturation like it did in the case of fully broken strands. The heat generation at this event is much higher, because all five strands saturate simultaneously (see Figure 4.30b). Unlike the previous case, there is insufficient time for current to redistribute gradually around the defect since the start of the ramp.

Figure 4.29c shows the peak heating for a more distributed defect, still with the same overall cable critical current reduction. Eight strands are degraded in a symmetric distribution, with from the center outwards a critical current reduction of 80%, 60%, 40%, and 20%, respectively. A more gradual build-up for the heating is seen here as the strand saturation events are more spread out in time (see also Figure 4.30c). The initial heating is lower, but for all combinations of contact resistances a higher peak heating is reached at the end for damage case III than for damage case II (Figure 4.29b).

For both types of partial damage, when $R_{c,\text{edge}}$ is low enough, the precise value of R_a hardly has an influence on the peak heating.

4.7 Controlled damage experiment

This section presents the measurement results of the experiment on the cable in the SMC magnet with intentionally broken strands that was described in Chapter 3. The results of the measurements with 1, 3, and 5 broken strands are compared to the reference measurement on a cable without damage. A comparison with the model simulations is also made. In Section 4.2, a uniform field was assumed for simplicity's sake, which is sufficient to describe the general behaviour of the current redistribution process. In this section, for better comparison with the experiment, a detailed magnetic field map is used.

4.7.1 Ramp-rate and temperature variations

To compare the predicted voltage evolution with the measured one, different variations of the same ramp sequence are used. Specifically two parameters are varied: temperature and ramp rate. The comparison is then made for all three levels of damage.

Table 4.4: *Measured quench current I_q for the cases without intentional damage and with 1, 3 or 5 broken strands, at two different temperatures and for different ramp rates. All ramps include a 10 minute plateau at 2 kA, which is used for offset correction.*

Ramp rate (A/s)	I_q (kA) @ 1.9 K	I_q (kA) @ 4.5 K
No damage		
10	-	-
20	15.1	14.8
50	15.3	14.1
100	-	12.7
1 broken strand		
10	15.2	14.8
20	15.2	14.7
50	15.3	14.0
100	14.1	12.7
3 broken strands		
10	15.0	13.9
20	14.9	13.9
50	14.8	13.4
100	13.7	12.2
5 broken strands		
10	-	13.7
20	14.9	13.5
50	14.4	12.9
100	13.3	11.7

Table 4.4 provides an overview of the measured quench current at different

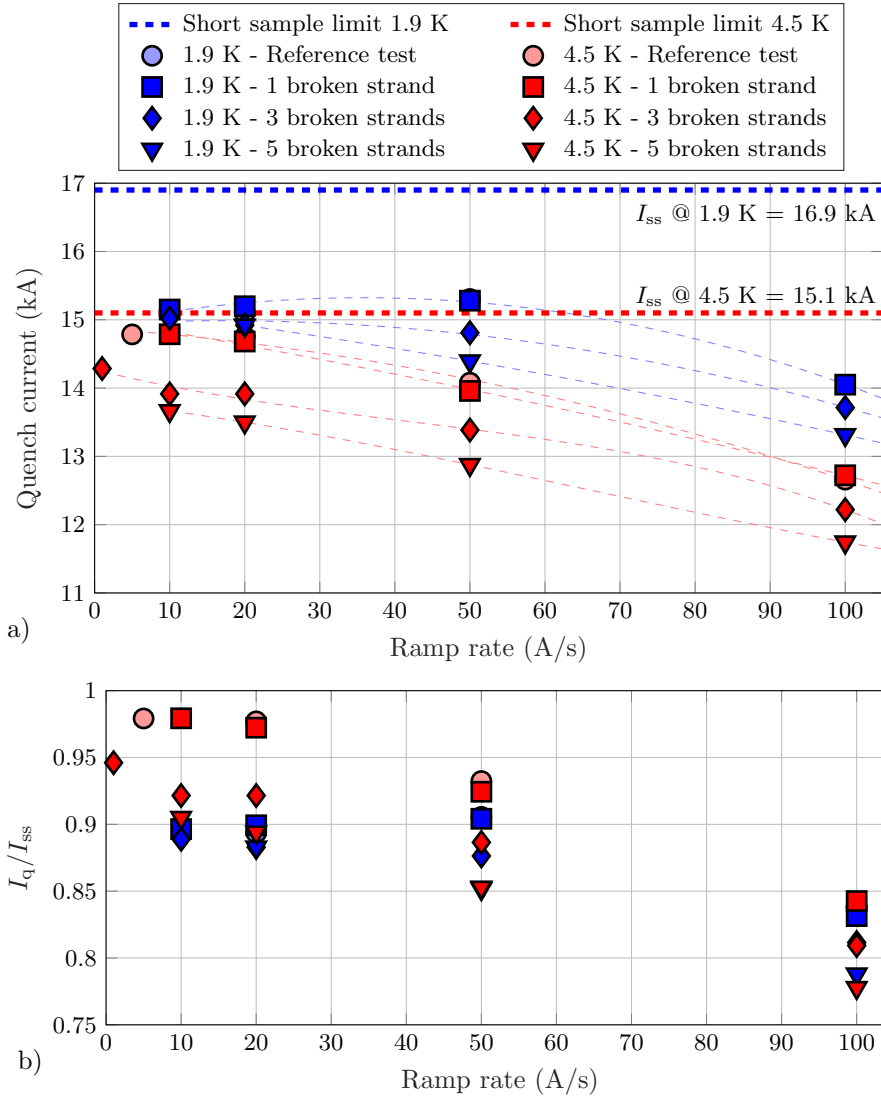


Figure 4.31: Absolute (a) and normalized measured quench current I_q (b) as a function of ramp rate for two temperatures and for three damage levels, as well as a reference measurement.

ramp rates and temperatures. The values are also reported in Figure 4.31. For the case of a single broken strand, the difference in quench current compared to the reference test for both temperatures and all ramp rates is negligible. The case of three broken strands shows a clearly lower quench current at 4.5 K, but at 1.9 K this decrease is very small at low ramp rates. For the case of 5 broken strands a further lowering of the quench current is seen, but at 1.9 K the difference is once

again minimal at lower ramp rates. At 4.5 K the difference between one and three broken strands is larger than the one between three and five broken strands. In Figure 4.31b the quench current is plotted as a fraction of the short sample limit, showing how for low ramp rates the magnet quenches at a lower current fraction at 1.9 K. This behavior is often observed in magnets with conductor damage (e.g. in [40]) and can be attributed to the lower enthalpy margin at 1.9 K compared to 4.5 K for a given I/I_{ss} .

By comparing the results with different ramp rates, temperatures, and damage levels to the RuNe model predictions, the parameters R_a and R_c can be estimated. Any thermal effects due to coupling current loss and magnetization loss are not taken into account. The Rutherford cable in this magnet does not include a core between the layers, so relatively large Inter-Strand Coupling Currents may arise. However, the voltage taps only span two twist pitches. The effect of ISCCs is expected to be small compared to the voltage stemming from the conductor damage. Moreover, as discussed above, the fact that the magnetic field orientation on the inner turns is largely parallel to the broad face also mitigates the formation of ISCCs in this region. Nonetheless, they lead to more uncertainty in fitting the results of measurements, especially at higher ramp rates.

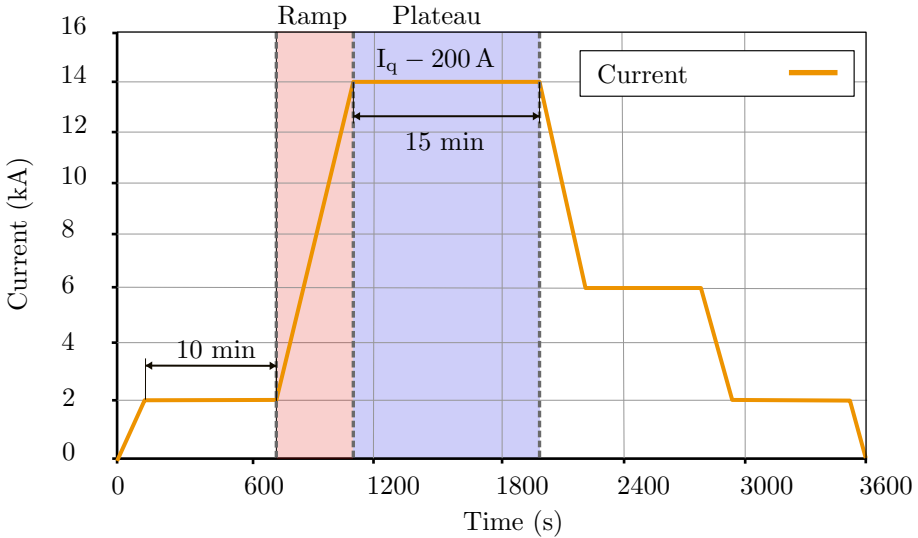


Figure 4.32: Cable current versus time, showing the ramp sequence used for the V-I measurements. A 2 kA plateau is included for offset correction. The longer plateau is set to the quench current minus 200 A.

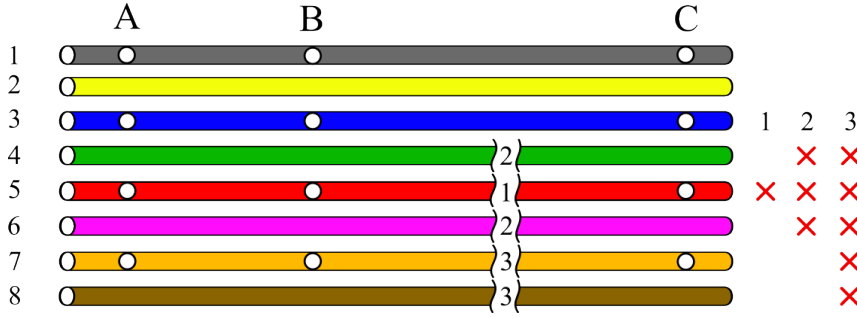


Figure 4.33: Schematic of the locations of the voltage taps and the broken strands. The numbers in the strand breakage position indicate the order in which they were broken. For more detail, see Figure 3.15.

4.7.2 Ramp sequence

The ramp sequence that is used for most measurements is shown in Figure 4.32. First, the current is ramped to a plateau at 2 kA where it is kept steady for 600 s, which serves to provide a voltage reference for offset removal. To avoid the influence of the initial voltage decay due to coupling currents, only the final 120 s of this first plateau are used. The magnet is then ramped further up to the quench current (corresponding to the used ramp rate) minus 200 A. By ramping the magnet all the way close to the quench current the maximum amount of signal for the voltage decay during the high-current plateau is achieved. The value of the quench current for a specific ramp rate was previously found during the ramp rate studies described in Section 4.7.1. This current is then held for 900 s, after which it is ramped back down to 6 kA for a plateau of 600 s. A final plateau at 2 kA is then used to check if no large voltage drift occurred over the course of the measurement, after which the magnet is ramped down to zero.

In the following sections, the ramp-up to the high current plateau, marked by the red area in Figure 4.32, and the plateau itself, marked by the blue area, are the focus areas for comparison with the model.

4.7.3 Comparison with RuNe

The main feature used to fit the model to the data is the voltage decay measured with voltage tap pair B5-C5 on the high current plateau at 4.5 K, with a ramp rate of 20 A s^{-1} . This comparison is made for all three levels of introduced damage. In Figure 4.33 a schematic overview of the voltage tap locations and the intentionally broken strands is shown. The order in which the strands are broken is also indicated.

Fit procedure

Various free parameters need to be evaluated, several of which can exhibit an unknown spatial or temporal variation, such as the inter-strand contact resistances or induced thermal loads. Unknown prior damage in the cable can be present too. Finding a single fit that matches all measured cases is thus an unlikely proposition. Therefore, the initial fit procedure assumes that the Rutherford cable has uniform R_a and R_c and that no degradation of the superconductor other than the intentionally introduced defect is present. Any strong deviations from the model predictions are then investigated separately.

Determination of inter-strand contact resistance

The values for R_a and R_c and the ratio between them largely determine the voltage difference between the measurements at the three levels of damage. In Figure 4.34, the measured voltage decay on the high-current plateau is shown along with the simulated voltage with the best fit for R_a and R_c .

The fit results indicate a ratio between R_a and R_c that is rather high compared to the values expected for HL-LHC magnets. A best fit was found for $R_a = 12 \mu\Omega$ and $R_c = 5 \mu\Omega$. The low value for R_c is likely caused by the absence of a stainless steel core in the Rutherford cable.

Measurements at 1.9 K

In Figure 4.35, the same voltage decay is shown but this time for the measurements performed at 1.9 K (using an identical ramp as the one used at 4.5 K). The model uses the same parameters as fitted from the data in Figure 4.34, which result in predictions that agree reasonably well with the data. The agreement between 1.9 K and 4.5 K results suggests that the superconducting properties are taken into account properly. Data for the test with 3 broken strands is not available at 1.9 K, because of test station limitations.

The voltage decay for a single broken strand starts at a slightly lower level at 1.9 K, but for the case of 5 broken strands the voltage level is almost the same as the one measured at 4.5 K. This is congruent with the observation that the quench current (and therefore also the plateau current $I_q - 200$ A) for 1 broken strand at 1.9 K was at a lower ratio of the short-sample limit compared to 4.5 K, whereas for 5 broken strands it was close to equal at both temperatures.

Ramp rate dependence

In Figure 4.36, the voltage decay over section B5-C5 is shown again, but now at higher ramp rates of 50 A s^{-1} and 100 A s^{-1} . The model predictions using the same R_a and R_c values are still reasonably close, with the voltage level of the cases of 3 and 5 broken strands being a bit closer to each other than the case of

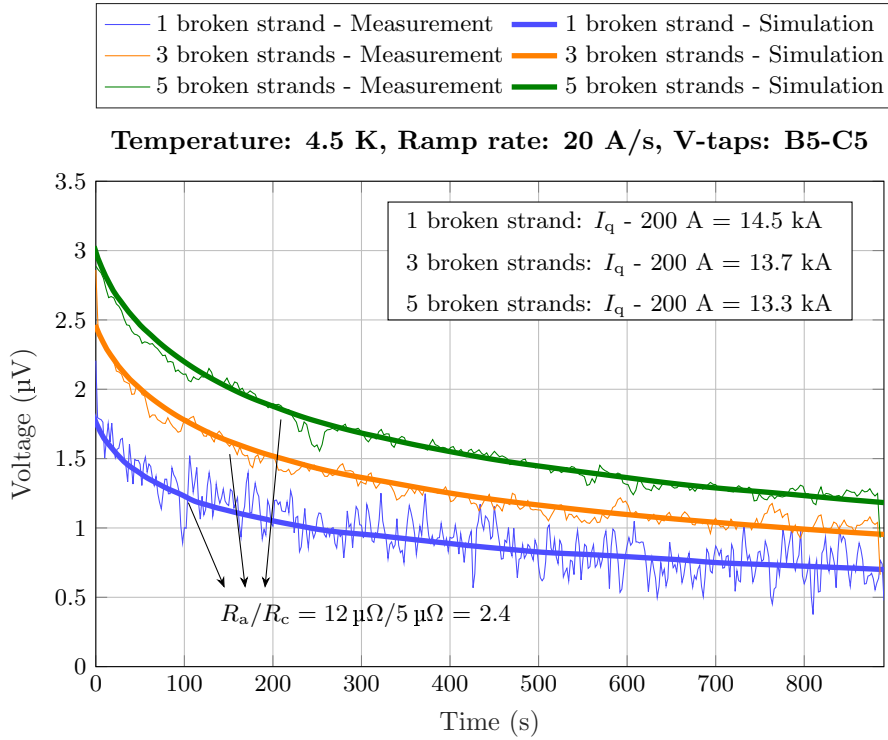


Figure 4.34: Measured strand voltage versus time on the high current plateau for three different levels of damage at 4.5 K (thin lines). The simulated voltage decay is shown as well (thicker lines). The best fit with the model is found for $R_a/R_c = 12\mu\Omega/5 \mu\Omega = 2.4$. A much smaller ratio of R_a/R_c does not fit well with the data, indicating that R_c is rather low.

1 broken strand. At higher ramp rates, the current redistribution length extends over a shorter distance than at 20 A s^{-1} . This implies that if a longitudinal spatial variance in R_a and R_c exists, these parameters might have a different average value than before. This will affect both time constants and voltage levels and may thus explain the observed deviations between model and measurements.

Time constants

The approximately quasi-exponential voltage decays are studied by extracting the dominant time constants from the data. As discussed in Section 4.4, in reality, the voltage decay is driven by a diffusion process over many inductive and resistive elements resulting in a spectrum of time constants. However, for the present comparison between data and model a double-exponential fit is taken to be acceptable. In practice, this allows for a sufficiently good description of the voltage decay considering that a fit with more time constants incurs loss of accuracy due

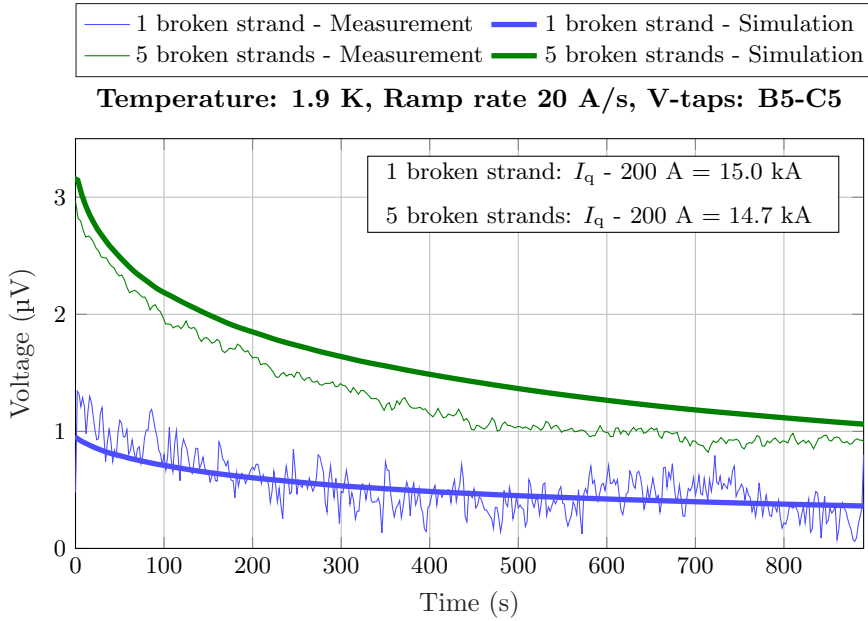


Figure 4.35: Measured strand voltage versus time on the high-current plateau for three different levels of damage at 1.9 K. The simulated voltage decay is shown as well.

to the noise level in the signals. The double exponential fit is given by:

$$V(t) = C_1 e^{-\frac{t}{\tau_1}} + C_2 e^{-\frac{t}{\tau_2}}, \quad (4.18)$$

where τ_1 and τ_2 are the two dominant time constants and C_1 and C_2 are the corresponding voltage coefficients. The extracted parameters using such a fit are shown in Table 4.5 and also in Figure 4.37. The shorter time constant τ_1 is in the range of 30 s to 120 s, while the longer one τ_2 lies in the range of 1200 s to 3000 s. No clear correlation between the time constants and the damage levels or ramp rate is observed. Note that the time constant of the voltage decay on a plateau depends strongly on how close to the end of the ramp a strand saturation event takes place. If it happens right before the end of the ramp, the time constant is smaller than when it is longer ago.

Voltage between strands

In Figure 4.38, the voltage difference between strands 1 and 5 is shown at both locations B and C (see Figure 4.33). Strand 1 is chosen because it is furthest away from the defect and carries at all times less than its critical current, while strands closer to the defect tend to saturate. Nonetheless, we start to see more divergence between the measurements and the model predictions. Qualitatively, the data still show the expected behaviour, e.g. the inter-strand voltages on opposite sides of the defect have opposite sign. For the case of 1 broken strand the voltage level

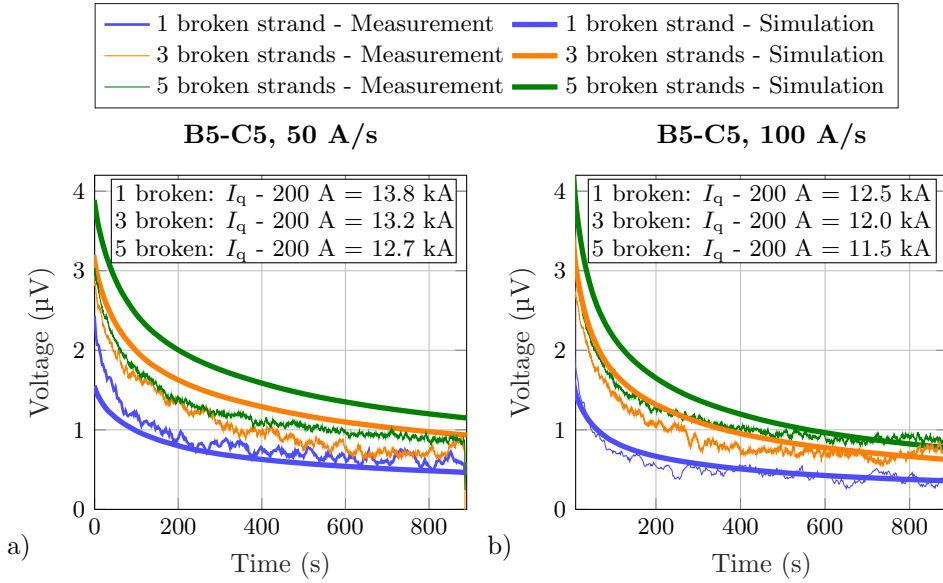


Figure 4.36: Measured and simulated strand voltage versus time on the high current plateau for different levels of damage at 4.5 K with a ramp rate of (a) 50 A/s^{-1} and (b) 100 A/s^{-1} for voltage tap pair B5-C5.

Table 4.5: Parameters of the double-exponential fit for the voltage decay over section B5-C5.

Parameter	1 broken strand	3 broken strands	5 broken strands
Ramp rate: 20 A/s			
C_1 (μV)	0.6	0.9	0.9
τ_1 (s)	66	100	113
C_2 (μV)	1.2	1.5	1.9
τ_2 (ks)	1.5	2.1	1.9
Ramp rate: 50 A/s			
C_1 (μV)	1.3	1.4	1.5
τ_1 (s)	62	122	84
C_2 (μV)	0.9	1.2	1.4
τ_2 (ks)	1.8	1.6	1.8
Ramp rate: 100 A/s			
C_1 (μV)	1.6	1.7	1.9
τ_1 (s)	35	86	65
C_2 (μV)	0.6	0.9	1.3
τ_2 (ks)	1.4	3.0	2.0

for location B matches very well, while for location C it is slightly higher than predicted. However, at location B there is no further voltage increase as more

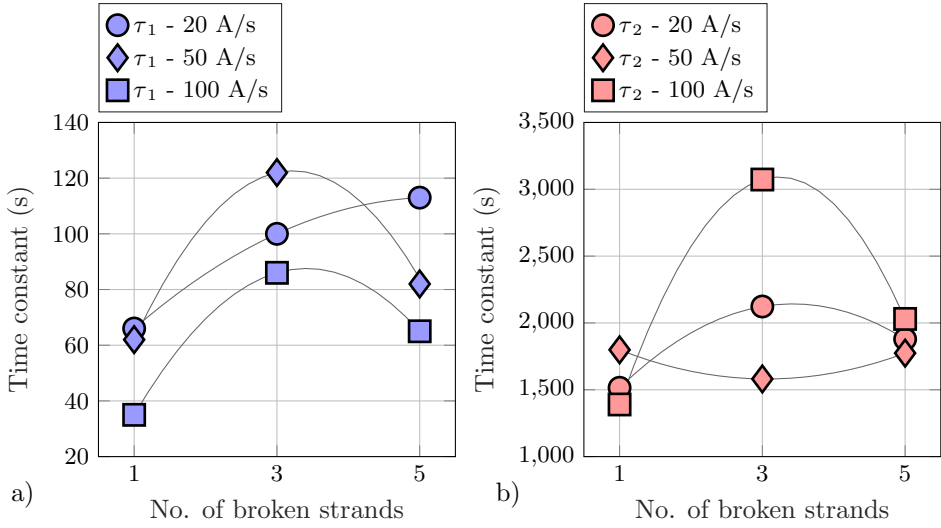


Figure 4.37: Time constants τ_1 (a) and τ_2 (b) of a double-exponential fit (Equation 4.18) for the voltage decay on the high current plateau for the three damage levels.

strands are broken. At location C, remarkably, the voltage difference between the strands drops to a lower level for 3 and 5 broken strands, and there is very little voltage decay visible here. Note that the voltage is measured between a supposedly *unsaturated* strand (number 1) and a *broken* strand (number 5). If some prior degradation is also present in strand number 1 for the case of 3 and 5 broken strands, it could explain the anomalous voltage readings.

Voltage over strand 1

To investigate further if this hypothesis might indeed explain the anomalous readings, the voltage drop measured along strand 1 is studied in more detail as shown in Figure 4.39. The model predicts no or a very low voltage drop along this strand, as expected. However, the measurements show a significant voltage drop, indicating that the strand has already saturated. This can also explain the voltage readings in Figure 4.38. As strand 1 saturates, its voltage will be closer to that of the broken strands since there is less current transferred from the broken strand to it. The fact that strand 1 is already saturated indicates that it is indeed likely that there is additional degradation present in the cable. This can be present either in strand 1 itself or in the strands adjacent to the broken strands. A more thorough analysis of such suspected extra damage will be presented in Section 4.7.6.

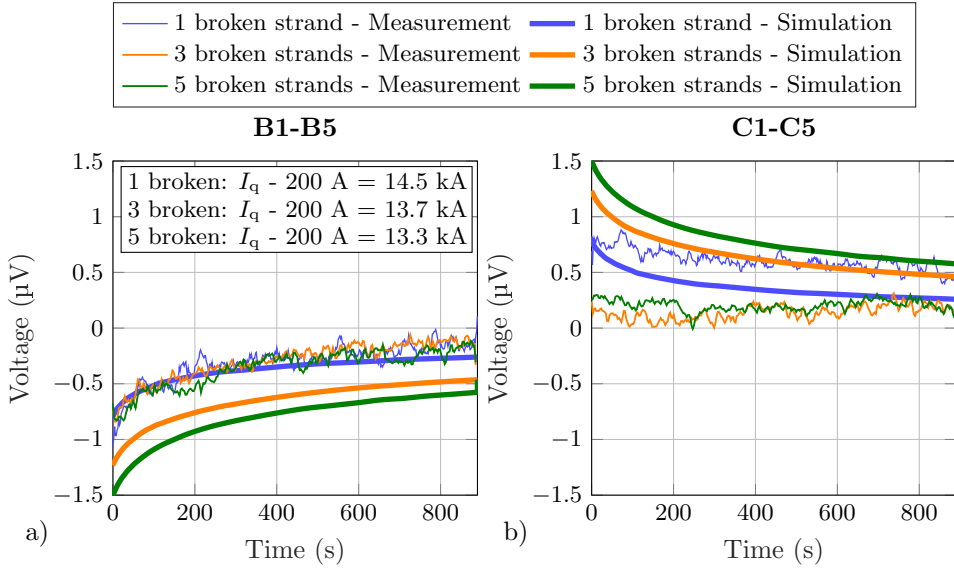


Figure 4.38: Strand-to-strand voltage difference versus time, measured between voltage taps B1-B5 (a) and C1-C5 (b) on the high current plateau for different levels of damage at 4.5 K.

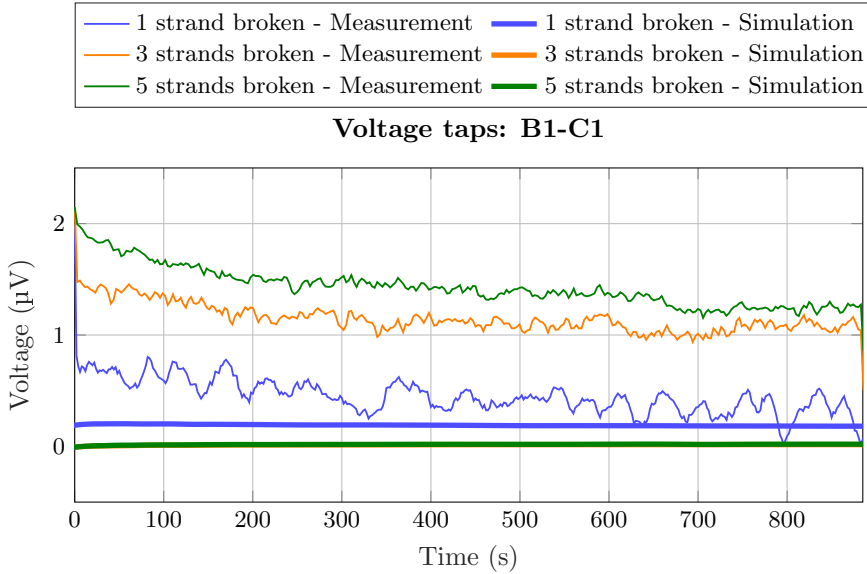


Figure 4.39: Voltage drop along strand 1 versus time as measured over voltage taps B1-C1 on the high current plateau for different levels of damage at 4.5 K.

V-I measurements after ramp-down

In Figure 4.40a, the voltage over taps B5-C5 is shown after the ramp-down from the high-current plateau. The measured value for a single broken strand does not show any significant voltage, but for three and five broken strands a clear negative decaying voltage component is visible as predicted by the model.

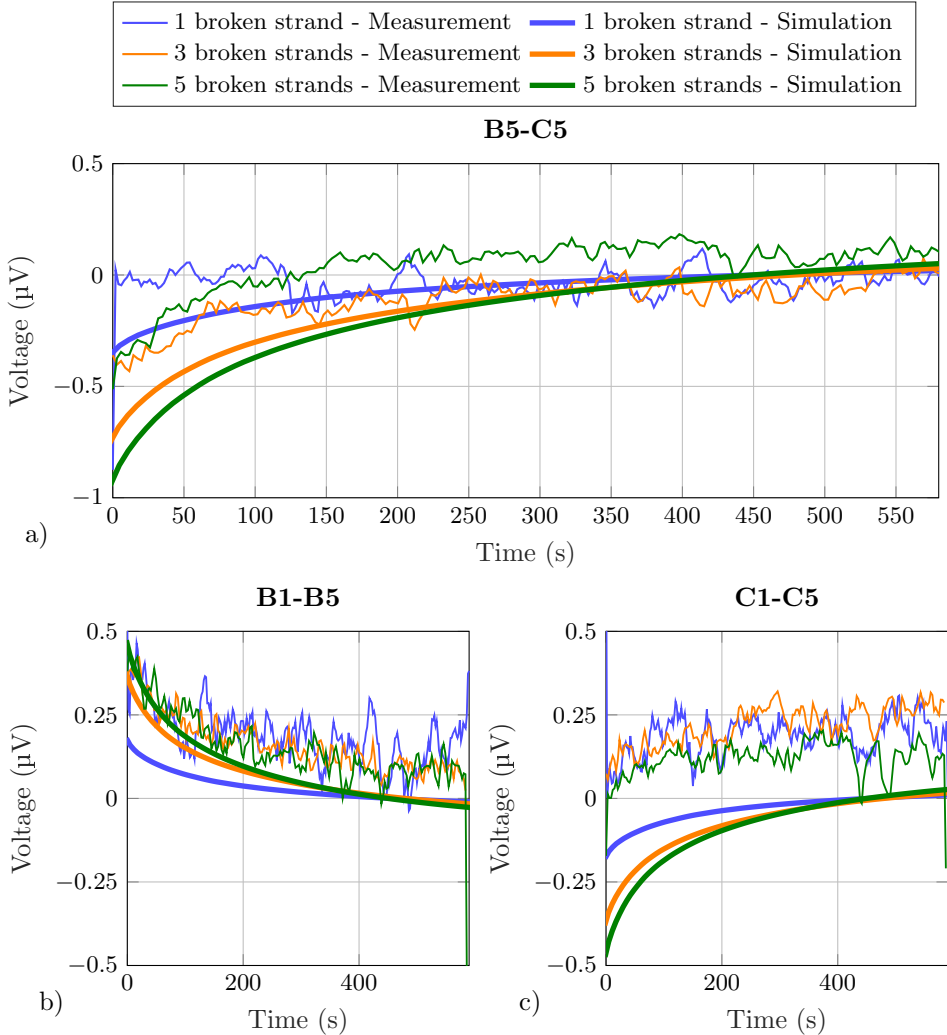


Figure 4.40: Voltage versus time on the 6 kA plateau after ramp down (see Figure 4.32). For the case of 1 broken strand over section B5-C5 (a) no significant voltage is visible, but for higher damage levels a negative decaying voltage is present, as also predicted by the model. The voltage differences between the strands at location B (b) and C (c) also show an inverted behavior.

In Figures 4.40b and c, the voltage decays measured between strands 1 and 5 before and after the defect are shown. The voltage decay switches polarity between locations B and C here too, but for location C there is a significant offset present. Due to the low voltage levels, the signal-to-noise ratio makes a more qualitative comparison with the model more difficult, but the qualitative behavior is in accordance with the model prediction.

4.7.4 V-I measurements during ramp

Compared to most voltage tap lay-outs in magnets, the voltage taps in this experiment are placed relatively close together. This results in an inductive voltage contribution that is sufficiently low to allow for analysis of voltage build-up during the ramp phase of magnet operation itself. The excitation sequence for this measurement is once more the one shown in Figure 4.32, but in this section current redistribution *during* the ramp is studied.

Inductive off-set removal

The inductive contribution to the voltage that is still present is removed by comparing it to results from the initial reference test without any intentionally broken strands. The steps in this correction process are shown in Figure 4.41. Due to iron saturation and superconductor magnetization the self-inductance of the magnet decreases during the magnet ramp. The decay of inductance during a 10 A s^{-1} ramp during the reference test without intentional damage is fitted with an empirical double-exponential function. This fit is then subtracted from the various data collected from the magnet after the different levels of intentional damage were introduced. The data early-on in the ramp retains a relatively high noise level, but this stage of the ramp is not as interesting for study as the later parts, which now clearly show several relevant features. Reference data for ramp rates higher than 10 A s^{-1} are not available, so the 10 A s^{-1} fit is simply scaled with the ramp-rate. This works less-well for the highest ramp rates, because dynamic effects, that do not scale linearly with ramp rates start to play a larger role. Therefore, only ramps at 20 A/s and below are studied with this method.

4.7.5 Observation of current cascade

In Figures 4.42 and 4.43, the voltage drop over the location of the intentional defect is shown for strands 1, 5 and 7, in this case with only strand 5 broken. The voltage evolution shows a clear kink for all three voltage tap pairs. Furthermore, the B5-C5 voltage directly along the broken strand shows the largest rise and the kink appears earliest here. This strand also shows a rising voltage from the very beginning, which is associated with the current redistribution around this broken strand. The two other voltage tap pairs show a smaller rise that also sets in at a later time, still with a discernible kink, albeit minimal for strand 1. Voltage tap pair B3-C3 is not included because no reference data for the inductive offset correction was available. For voltage tap pairs B1-C1 and B7-C7, there is also already

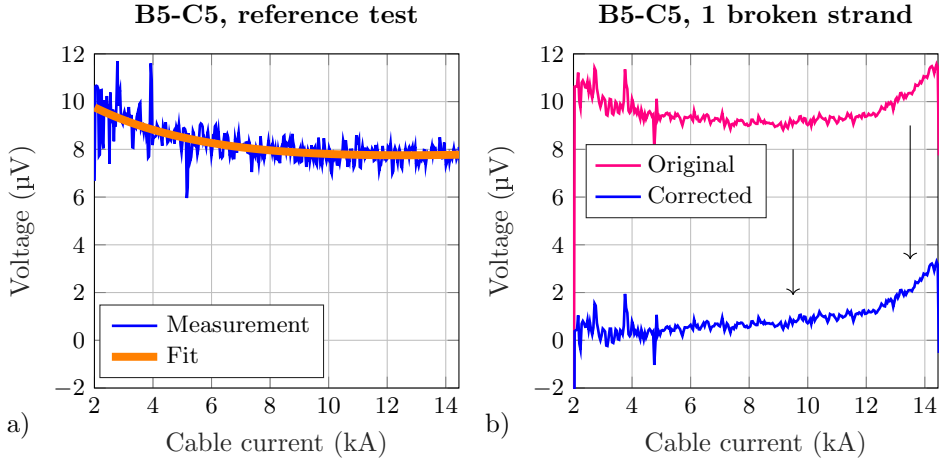


Figure 4.41: a) Inductive voltage versus cable current during the ramp as measured during the reference test over voltage taps B5-C5. A double exponential function is fitted to the data. b) The voltage fit to the reference data is subtracted from the voltage measured over B5-C5 during the test with 1 broken strand.

a slight slope visible before the kink, but this is likely an inductive component not properly filtered out.

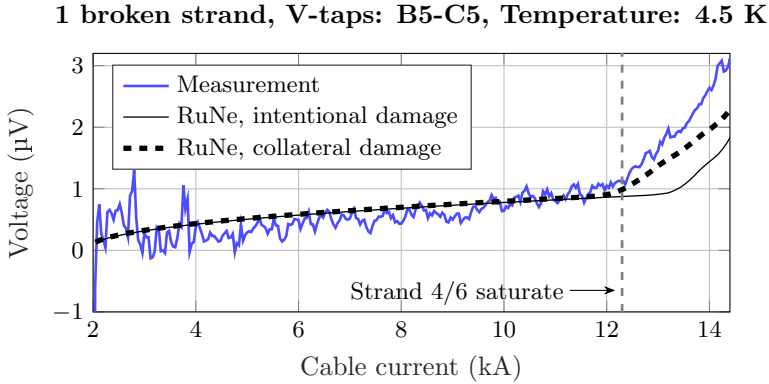


Figure 4.42: Voltage drop B5-C5 versus cable current in the case of 1 broken strand. The kink in the voltage build-up right after 12 kA indicates saturation of strands 4 and 6 adjacent to the broken strand.

An important feature to note while considering these figures is that the kink observed for B5-C5 in Figure 4.42a is associated with the *neighboring* strands saturating, whereas in Figures 4.43b and c, the strand over which the voltage is measured itself saturates. This also explains why the kinks in Figures 4.42 and

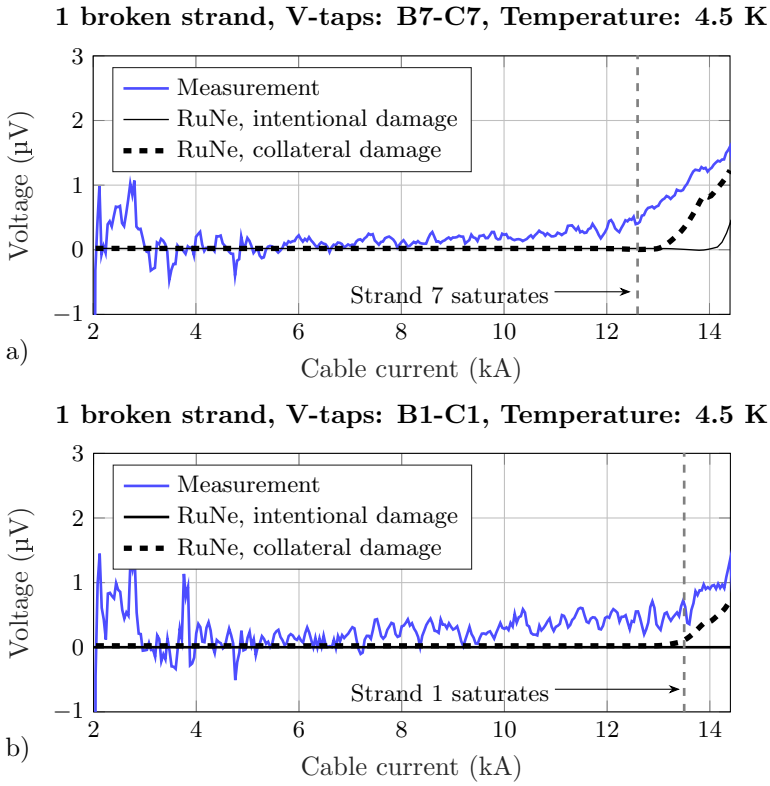


Figure 4.43: Voltage drops B7-C7 (a) and B1-C1 (b) versus cable current in the case of 1 broken strand.

4.43a occur close after each other as they represent adjacent strand saturations, whereas in-between the kinks of Figures 4.43a and b there is still another strand saturating (see also Figure 4.33).

The voltage drops simulated using RuNe are included in these figures as well. The thin solid line is a simulation that only assumes the intentionally introduced damage, whereas the thicker dashed line also includes collateral damage to the strands neighboring the drilled strand. Clearly, including the collateral damage results in a better correspondence between model and data. The likely location and severity of the collateral damage will be discussed in more detail in Section 4.7.6.

In Figure 4.44a, the voltage measured between strands 1 and 5 both before and after the defect, i.e. at positions B and C, are shown. As expected there is a mirror symmetry when measuring at the two positions. Similar to Figure 4.42, a clear kink occurs when the strands adjacent to strand 5 start to saturate. In Figure 4.44b the same is shown for position A. Here, the voltage behavior is very similar to position B, just with a slightly lower voltage level because it is further

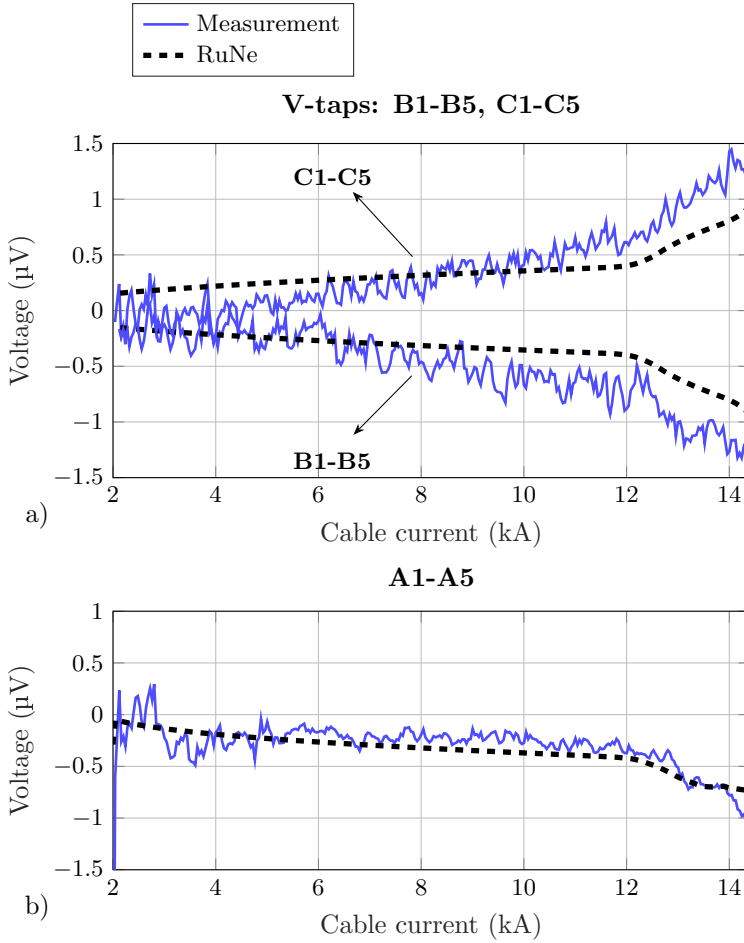


Figure 4.44: a) Measured (blue line) and simulated (dashed line) voltage between strands 1 and 5 at locations B and C versus cable current, measured one twist pitch before or after the defect (see also Figure 4.33). b) Voltage difference between strands 1 and 5 measured at location A two twist pitches before the defect.

away from the defect.

Finally, in Figures 4.45a - d the voltage drop along the strands between positions A and B is shown. Except for some slight drift observed over strand 7 (Figure 4.45d), no significant voltage rise is measured. This is expected since there is no defect between locations A and B and the strands are still fully superconducting between these two positions, so the little voltage that may arise has an inductive origin. The predicted voltage difference between the two positions is in the order of $0.1 \mu\text{V}$ to $0.2 \mu\text{V}$, which given the signal-to-noise ratio is too small to detect reliably.

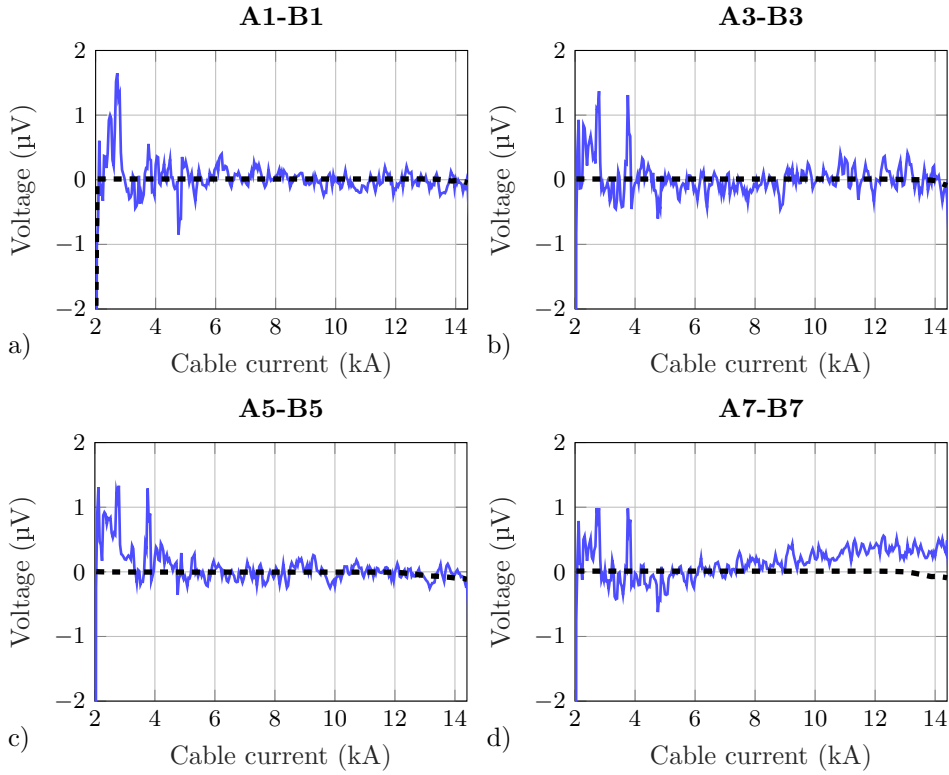


Figure 4.45: Voltage drop versus cable current, measured over strands 1 (a), 3 (b), 5 (c) and 7 (d) between positions A and B (see also Figure 4.33). Apart from some minor voltage drift over strand 7, no significant voltage is measured, as also expected from the model.

4.7.6 Collateral strand damage

The kink in the measured voltage drop B5-C5 along strand 5 in Figure 4.42 occurs earlier than the model predicts assuming only the intentional damage to be present. This likely indicates a lower than expected critical current of the adjacent strands, possibly due to some degradation caused by the drilling of the hole in strand 5. The close-up of the drilled strands in Figure 3.16a revealed some superficial damage to the neighboring strands surface, which may have affected some of its Nb₃Sn filaments as well. In Figures 4.42 and 4.43 a and b, a simulation that assumes some ‘collateral’ damage to these strands is also shown, which fits the measured voltage better. For this simulation, the adjacent strands were given a critical current reduction of 40 %. With this additional level of damage, the estimated values of R_a and R_c needed to be reduced to 10 $\mu\Omega$ and 4 $\mu\Omega$, respectively, in order to still match the data.

For the measurements with 3 and 5 broken strands, it is also necessary to in-

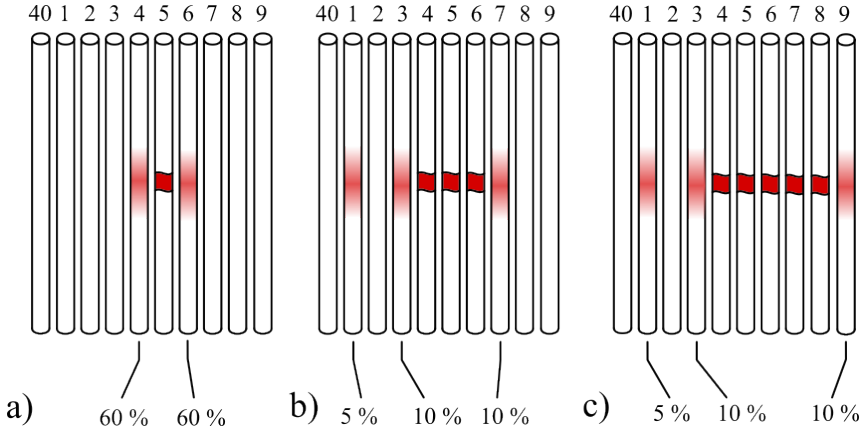


Figure 4.46: Overview of the position and severity of suspected collateral strand damage for the experiment with 1 (a), 3 (b) and 5 (c) broken strands. The %-values indicate the I_c value remaining in the strands that best corresponds with the measured data. Given the fact that strand 2 is surrounded by degraded strands, it is likely that it is also degraded, but no direct measurement took place to confirm it.

clude extra damage in the adjacent strands in the simulation in order to match the measurement results. The suspected levels and locations of collateral damage for all three cases are shown in Figure 4.46.

Several further clear observations pointing to the likely presence of collateral damage are shown in Figure 4.47. Figure 4.47c shows the voltage over strand 7 for the case of 3 broken strands. The model with only the intentional damage predicts a kink in the voltage around 12 kA, but in reality it appears much earlier, which leads to a suspected reduction of the critical current of 90 %. For both cases of 3 and 5 broken strands, the voltage drop along strand 1 (Figures 4.47e and f) starts to rise already at about 4 kA, whereas the initial model predicted no saturation (and hence no observable voltage) at all for this strand, since there are two strands between it and the defect. It is then likely that strand 2 is also damaged in these two cases of 3 and 5 intentionally broken strands, but there are no directly measured data for this strand available to proof this. The sharp kink around 11 kA for taps B1-C1 in Figures 4.47e and f likely corresponds to the saturation of strand 40 (next to strand 1).

For strands that are close to the lateral center of the defect zone, the kink associated with strand saturation is not as pronounced as it is for strands closer to the edge of the defect zone, since the former are further away from the saturating strands. For B5-C5 a sharper kink was predicted with the initial model, whereas the model with collateral damage predicts a smoother voltage rise, which corresponds better with the measured curve.

In Figure 4.48, the voltage difference between strands 1 and 5 for positions B

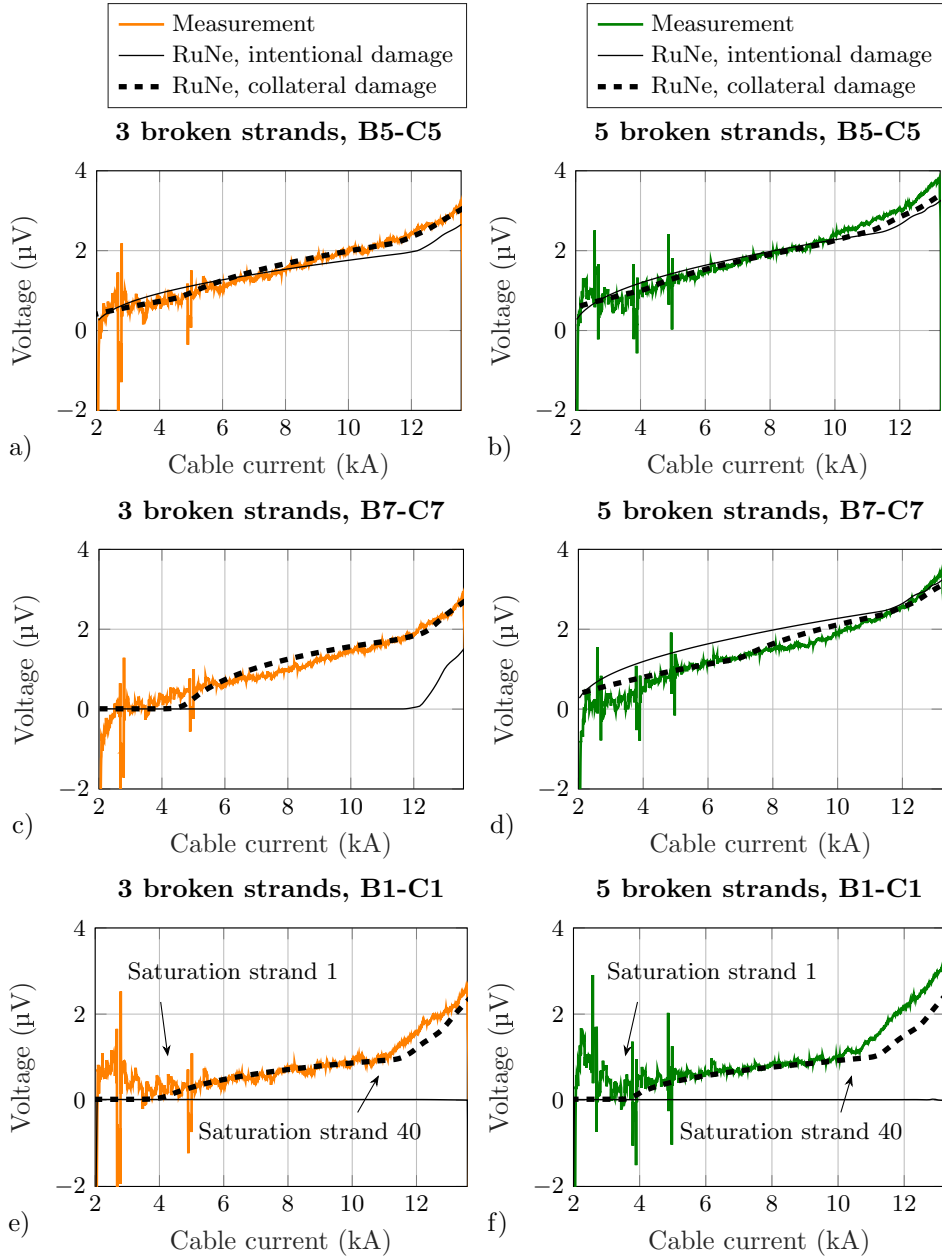


Figure 4.47: Voltage versus cable current measured over strands 1 (e,f), 5 (a,b) and 7 (c,d) between positions B and C, for the case of 3 and 5 broken strands. The inductive component is removed by aligning the measured voltage with the expected value from simulation between 2 and 12 kA.

and C (i.e. on either side of the defect zone) are shown for the cases of 3 and 5 intentionally broken strands. The mirror symmetry in the polarity is once again qualitatively present in the voltage curves, but at position B the voltage difference is twice as high. It is also interesting that the magnitude of the voltage difference now kinks *down* when a strand saturation occurs, which is different from the behaviour seen in Figure 4.44. The initial modeled predictions with only intentional damage also show this kink to occur upwards, but when collateral damage is taken into account the measured behaviour is reproduced. If collaterally damaged, strand 1 is already saturated (see Figures 4.47e and f) and as its neighbor also saturates, its voltage gets closer to that of strand 5, because it is now further removed from the still superconducting strands and less current is flowing transversely between strand 1 and strand 5. A more detailed explanation of this behaviour will be given in Section 4.7.8.

A possible reason for the higher voltage difference at position B compared to C could yet be additional damage that is not taken into account. It might also be related to a variation of the inter-strand contact resistance on either side of the defect, or to a combination of both. Even though no such asymmetry was observed in Figure 4.44 for the case of 1 broken strand, different pairs of strands are now dominant in the current redistribution process, which might also explain the appearance of the asymmetry.

4.7.7 Voltage build-up at 1.9 K

In Figure 4.49, the voltage build-up over voltage tap pair B5-C5 for the three levels of damage is shown at 1.9 K. The voltage rises in a very similar fashion as at 4.5 K, with the kink appearing at a cable current that is about 10% higher, which is expected due to the higher critical current. At the last part of the ramp the model slightly underestimates the voltage level, but the main features are reproduced.

For the higher damage levels this is more clearly visible over strand B1-C1 because of the more pronounced kink, as shown in Figure 4.50. Also in this case the ratio between the kink current and the short-sample limit is very similar to the one at 4.5 K.

Interestingly, the voltage increases to a higher level before quenching at the same level of I/I_{ss} for all measurements at 1.9 K, while the model predicts it should reach the same level. A possible explanation may reside in the lower enthalpy margin that is also suspected to be responsible for the different quench level of the I_q/I_{ss} at 1.9 K and 4.5 K (Section 4.7.1). The lower specific heat of the cable at 1.9 K versus 4.5 K means that a comparable heat load caused by the current redistribution at 1.9 K leads to a larger increase in temperature, thereby reducing the critical current.

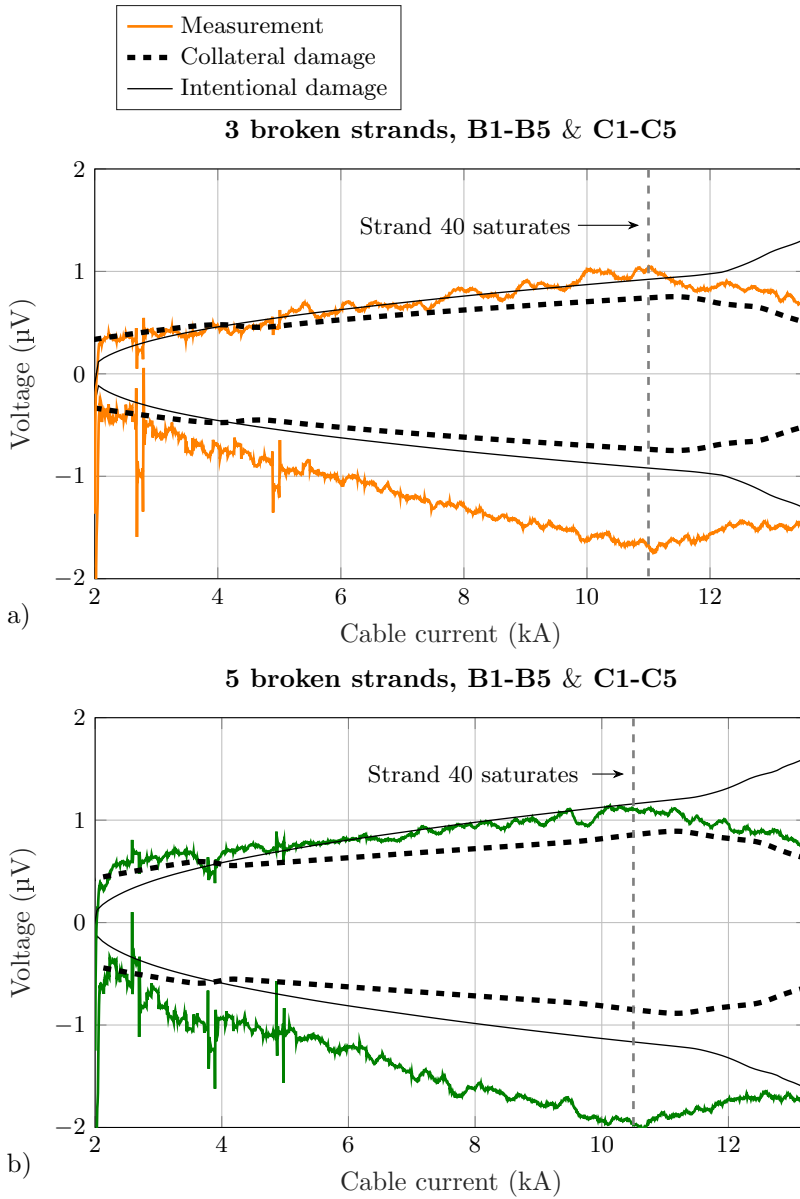


Figure 4.48: Voltage versus cable current between strands 1 and 5 before and after the defect, for the cases of 3 (a) and 5 (b) broken strands.

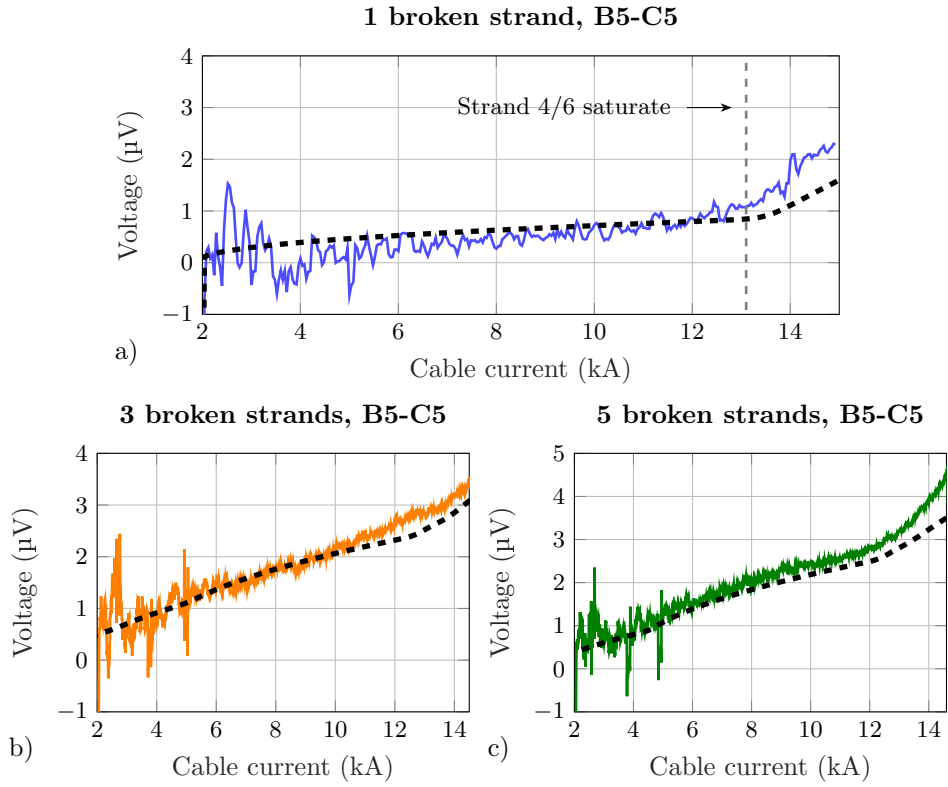


Figure 4.49: Voltage drop B5-C5 along strand 5 versus cable current during a ramp at 1.9 K for the cases of 1 broken strand(a), three broken strands (b), and 5 broken strands (c).

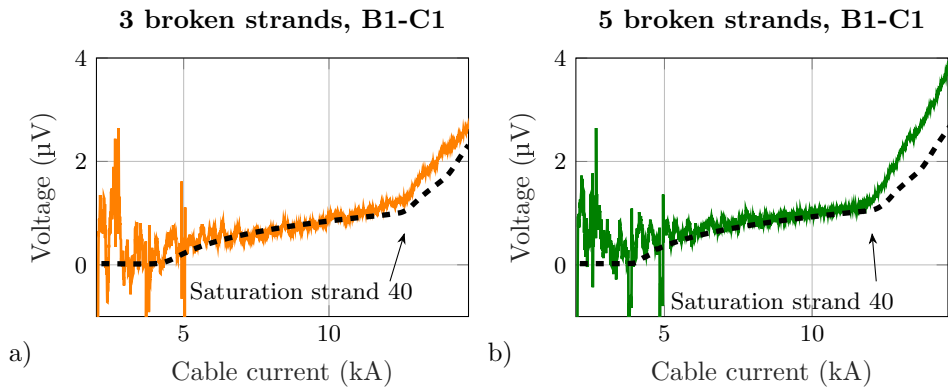


Figure 4.50: Voltage drop B1-C1 along strand 1 versus cable current, for the cases of 3 (a) and (5) broken strands, during a ramp at 1.9 K

4.7.8 Voltage landscape

For a more intuitive understanding of the voltage response of the cable, a 3D representation of the voltage ‘landscape’ around a local defect is shown in Figure 4.51, for the case of 3 broken strands. The voltage along the cable is shown for all strands (which are shown untwisted here for visualization purposes) with respect to ‘ground’, which is once more chosen to be the potential of strand 1 at longitudinal position $z = 0$ m. The strands that are broken for this visualization are strands 24, 25 and 26. The current is steady, so no inductive gradient is present in the voltage due to an overall current ramp. For this example $R_a = 12 \mu\Omega$ and $R_c = 5 \mu\Omega$. The voltage traces of three different strands are highlighted: a broken one, an intact but saturated one and an intact unsaturated strand. The voltage tap pairs in the SMC magnet probe voltage differences between different parts of such a voltage surface.

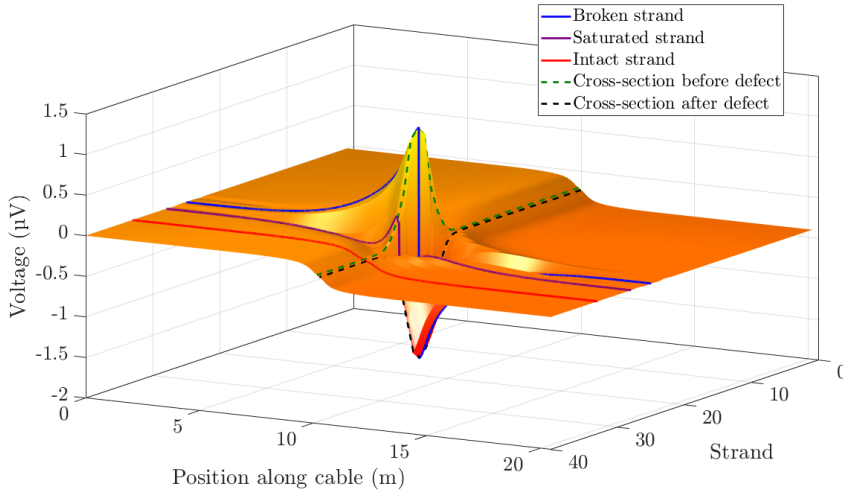


Figure 4.51: 3D representation of the voltage distribution with respect to ground in the cable. For visualization purpose, the strands are represented untwisted as if they are parallel on one of the axes.

The evolution of the voltage difference between strands in Figures 4.48a and b, can be more easily understood if we consider the change in the strand voltage as a strand saturates. In Figure 4.52, the voltage for two strands is schematically shown at two different time instances, before and after strand saturation. At time t_1 the reference strand (number 1) carries less than its critical current, while at the later time t_2 it has saturated and hence has developed a voltage drop, limiting its further role in the current redistribution process. The voltage difference between the two strands has then reduced. In Figure 4.53, this is shown in a cross-sectional view of the cable, before and after the saturation event. A slightly higher increase in voltage is visible at the edge of the bell-shaped profile around the defect. Note

that if R_c is much higher than R_a , this difference would be much smaller and all saturated/broken strands would show a sharp kink, not just the strands near the saturation event.

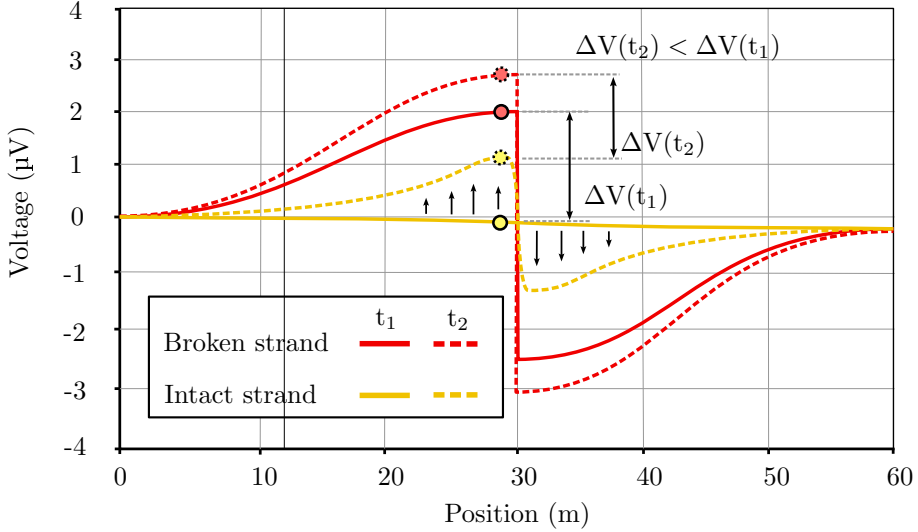


Figure 4.52: Strand voltage versus position along the cable. Schematically illustrating the effect of strand saturation on the voltage difference between strands. Once a strand saturates, its voltage will tend to get closer to that of the broken strands as it now develops a significant longitudinal voltage drop, which reduces transverse current transfer between the two strands. For clarity, the inductive contribution of the ramping overall transport current is not included here.

4.7.9 Strand saturation and stability

The observation of several successive kinks in the voltage measurements over different strands implies that, during operation, the Rutherford cable in the SMC magnet is able to withstand the saturation of several adjacent strands before it quenches. In this case, as many as four strand saturation events on either side of the defect are observed in Figures 4.42 and 4.43. Nonetheless, the magnet reliably reached its highest recorded current several times without suffering from premature quenches past the first saturation event. However, even though the cable might be able to sustain several saturated strands under ‘normal’ circumstances, such a saturated strand will still have a significantly lowered Minimum Quench Energy, resulting in a reduction of the overall cable stability [54, 146, 147, 148, 149, 150, 151]. The Short Model Coil is not subjected to the same disturbance spectrum as an operational accelerator magnet, which experience more transient heat loads, such as beam loss [152, 153]. Therefore, the ability of the Short Model Coil to reliably sustain multiple saturated strands might not translate directly to operational ac-

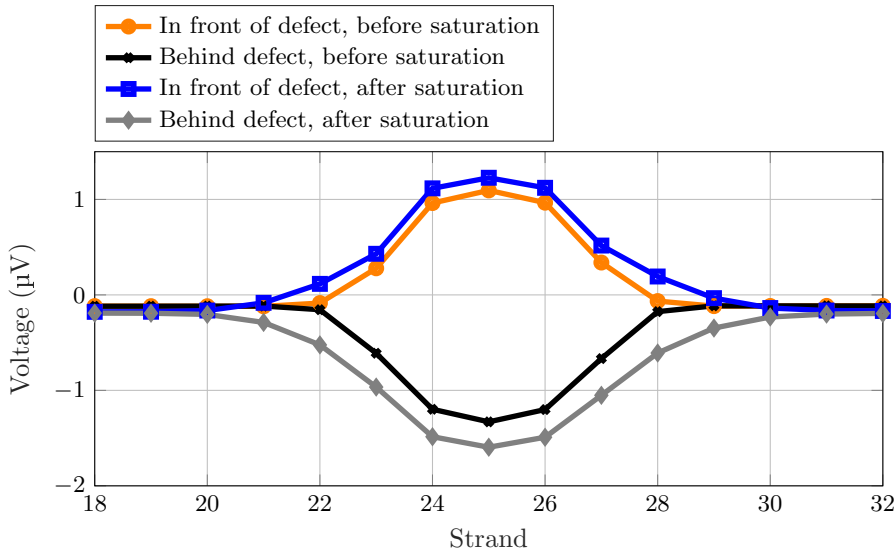


Figure 4.53: *The voltage with respect to ground of strands 18 through 32 before and after saturation of strand 22 and 28 in the cross-section around the defect.*

celerator magnets.

4.7.10 Effect on Magneto-Thermal Instabilities

One mechanism that can cause a magnet to quench is a magneto-thermal instability [154, 155, 156, 157]. A strand is not necessarily perfectly stable everywhere below its theoretical critical surface. At lower magnetic fields while carrying high currents, a strand is susceptible to magneto-thermal instability, caused by a high J_c resulting in potentially unstable intra-strand current distributions with high spatial gradients. Under normal operating conditions the strand's load line in a magnet stays far away from this unstable regime, but when several strands are damaged, the current in the adjacent strand can be pushed to unstable levels. Note also that even if the damage is located in the high-field region of a magnet, the current imbalance may well extend to the magneto-thermally less stable low-field region.

4.7.11 Detecting strand saturation

If, during operation, a single strand of a Rutherford cable within a magnet is in saturation, it may thus render the entire magnet unstable. During testing, strand saturation can not always be detectable if the underlying damage is relatively modest, and if the magnet is only tested up to its operational current. This is especially true for series-produced magnets, without internal voltage taps in performance-critical regions. A defect that generates a localized current redistribution may not

be detectable by measuring over the full coil, especially if the local value of R_a is low. A magnet that appears fine in the test station can thus nonetheless suffer premature quenches while operating in an actual accelerator due to the more extreme disturbance spectrum. To reduce the risk of such undetected conductor damage in the most critical high-field, high-stress areas, a magnet should always be tested close to its short sample limit. To reduce mechanical load on the magnet, such tests can be performed at 4.5 K instead of the final operational temperature of 1.9 K.

4.8 Conclusion

The performance of a Rutherford cable with local damage in some of its strands has been studied. Local non-uniform damage in the cable can be detected by the presence of anomalous voltages that will (partially) decay on a current plateau after a ramp. If the voltage does not decay but stays constant, the underlying damage has a more homogeneous nature, affecting all strands more or less equally. A general description of the current redistribution process around a local defect was presented with the aid of the THEA cable simulation software.

Current mainly redistributes through strands adjacent to the defect. In the course of this process, these adjacent strands saturate consecutively, each time creating a kink in the voltage build-up during the ramp. A large current imbalance between the strands thus arises that extends over several meters around the defect zone and whose length has a square-root dependence on the inter-strand contact resistance but does not depend on the number of broken strands. The time-decay of the voltage drop over such a defect during a current plateau can be empirically approximated quasi-exponentially, with time constants ranging from 10 to 5000 s for practical cases. An analytical diffusion model of the voltage build-up during the ramp as well as the voltage decay on the plateau has been derived for a simplified two-strand model cable.

A negative decaying voltage after ramp-up may arise, when directly probing a damaged strand with voltage taps. A negative decaying voltage can also arise after ramp-down, due to the induced reversed current flow around the defect. Simulations of the power dissipation show that only in specific localized damage cases, such as the fully interrupted strands in the SMC experiment, an increase of R_a may be beneficial to delay the onset of strand saturation. However, for more realistic degradation distributions expected in actual HL-LHC magnets this is likely not the case.

To verify these findings and the corresponding model descriptions further, an experiment on an SMC magnet with a representative Nb₃Sn Rutherford cable was performed. An increasing number of strands was intentionally broken and the magnet was each time tested in detail, with the measured voltages compared to

RuNe predictions. By placing voltage taps on selected strands, the current redistribution around the defect was monitored. The clear progression of the current cascade predicted by the models, in which subsequent neighboring strands saturate in sequence, was indeed observed. Remarkably, the magnet could still operate with up to 4 adjacent saturated strands on either side of the defect.

The results of the experiment are in agreement with the modeled predictions, with the voltage decays showing the expected time constants and voltage levels. By matching the voltage levels at different damage levels it was found that the ratio R_a/R_c was greater than 1, which is consistent with the fact that the Rutherford cable used in the SMC magnet does not contain a stainless steel core. When tracking the cascade of saturating strands during the ramp, it was necessary to include a significant amount of extra ‘collateral’ damage in strands adjacent to the intentionally damaged ones to get satisfactory quantitative agreement with the model. All these features were successfully reproduced with the RuNe cable simulation model, that allows to extract numerical estimates for the cable-relevant parameters involved.

Chapter 5

Quench development and normal zone propagation

5.1 Introduction

In this chapter, the initial development and the propagation of a quench in Nb₃Sn Rutherford cables is treated. The influence is studied of cable parameters, such as the inter-strand contact resistances, but also of external factors, such as the applied background magnetic field. This is done for short time scales up to 10 ms, for single cables with strand level detail. Transverse turn-to-turn propagation of the normal zone through the magnet's winding pack is not considered. Special attention is given to the effect that pre-existing cable damage has on the evolution of a quench, and how this may be used to diagnose problems with the conductor. The phenomena are studied by comparing model predictions obtained with RuNe to several case studies of magnets from the HL-LHC program and to quench tests performed on the SMC magnet with controlled levels of cable damage.¹

5.1.1 Phases of a quench: development and propagation

Generally, a quench on cable level can be divided into two phases: its initial development and its subsequent propagation, schematically illustrated in Figure 5.1. During the development phase, part of the transverse cross-section of the cable is still superconducting. This phase is characterized by large current redistribution effects as the current is pushed from the resistive strands into the still

¹This chapter is based, in part, on the following publications [158, 159]:

R. Keijzer *et al.*, "Effect of Strand Damage in Nb₃Sn Rutherford Cables on the Quench Propagation in Accelerator Magnets," in IEEE Transactions on Applied Superconductivity, vol. 33, no. 5, pp. 1-5, Aug. 2023, Art no. 4700605, doi: 10.1109/TASC.2023.3244140.

R. Keijzer *et al.*, "Measurement and Modeling of Initial Quench Development in Nb₃Sn Accelerator Magnets," in IEEE Transactions on Applied Superconductivity, vol. 35, no. 5, pp. 1-7, Aug. 2025, Art no. 4003007, doi: 10.1109/TASC.2025.3540833.

superconducting ones. The details of this current redistribution process are highly dependent on the inter-strand contact resistances.

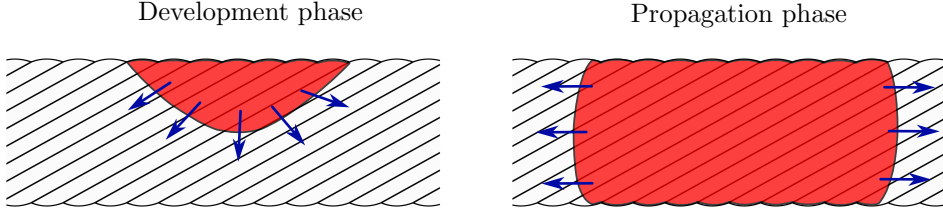


Figure 5.1: *Development (left) and propagation phases (right) of a normal zone.*

During the the propagation phase a normal zone has been established over the full cross-section of the cable and is now propagating longitudinally along the cable in both directions. Under normal circumstances, current redistribution during this phase is driven mostly by electric field variations due the magneto-resistance of the copper matrix and to a lesser extent by the value of the inter-strand contact resistance [106].

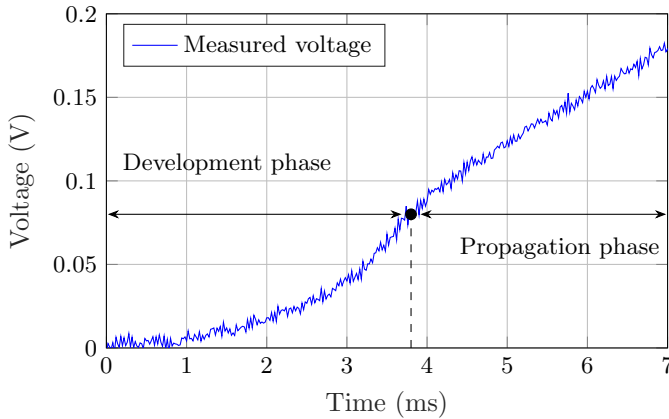


Figure 5.2: *Voltage versus time as measured over a coil of a Nb_3Sn quadrupole magnet (MQXFBP2). The voltage build-up initially accelerates (i.e. $\frac{d^2V}{dt^2} > 0$) until the normal zone has expanded over the entire cable cross-section. Then the voltage continues to rise linearly as the fully-formed normal zone front propagates along the cable in both directions.*

In Figure 5.2, the voltage build-up during a normal zone formation is shown for both phases of the quench. During the development phase, the voltage build-up accelerates until the cable is in normal state over its entire cross-section. Then the normal zone reaches its propagation phase and the voltage rises linearly. The shape of the development phase depends strongly on parameters such as the inter-

strand contact resistances, the initial current - and magnetic field distribution, and the quench's start position.

In this chapter, these two phases of the normal zone development are studied in more detail in Sections 5.2 and 5.3, respectively. Specifically, we try to reveal information on the quench's start location, as well as on the influence of the inter-strand contact resistances, magnetic field distribution, and initial current distribution, based on the signals from the voltage taps and on quench antenna measurements. These insights are then illustrated comparing modeling results with experimental SMC data in Section 5.4.

5.2 Quench development

To study how the cable properties and quench start conditions affect the quench development, a set of case studies using HL-LHC inner-triplet quadrupole magnets as examples, is performed. A phenomenological description of the time evolution of the quench is given. The studies focus on the effect that the initial conditions, such as the location and size of the initial heat pulse, have on the signals captured by the diagnostic equipment. In this section two different type of quenches are studied: conductor-limited quenches and training quenches. The conductor-limited quenches occur at the magnet's performance limit, i.e. its maximum achieved current for a given temperature and ramp rate. The cable is then assumed to quench at some weakest spot where a normal zone starts to develop in one of the strands. A training quench has a mechanical origin that can instigate a normal zone in many strands at once.

5.2.1 Quench diagnostic measurement results (in MQXF)

In Figure 5.3, the measured voltage build-up and the measured quench antenna signals are shown for a quench occurring at a current level of 16.41 kA and at a temperature of 4.5 K. The harmonic quench antenna described in Chapter 3 is used to capture the signals. The characteristic acceleration in the initial voltage build-up is clearly visible. During this development phase, the quench antenna signals slowly build up after which they flip polarity, followed by a prominent peak that coincides with the kink in the voltage build-up. As the quench enters the propagation phase, the voltage rise becomes linear and the quench antenna signals become more stable.

Figures 5.3c and d show the reconstructed positions and directions, using Equations 3.6 and 3.12, of the dipole moment caused by the changing current pattern during the quench - as explained in Section 3.2.3 - for the two time windows indicated as 1 and 2 in Figure 5.3b. The reconstruction shows that the quench occurs in the pole-turn region of the magnet. For these two reconstructed snapshots, the magnetic dipole moments are nearly perpendicular to the broad face of the cable, since the current redistribution mostly occurs within this plane due to the high

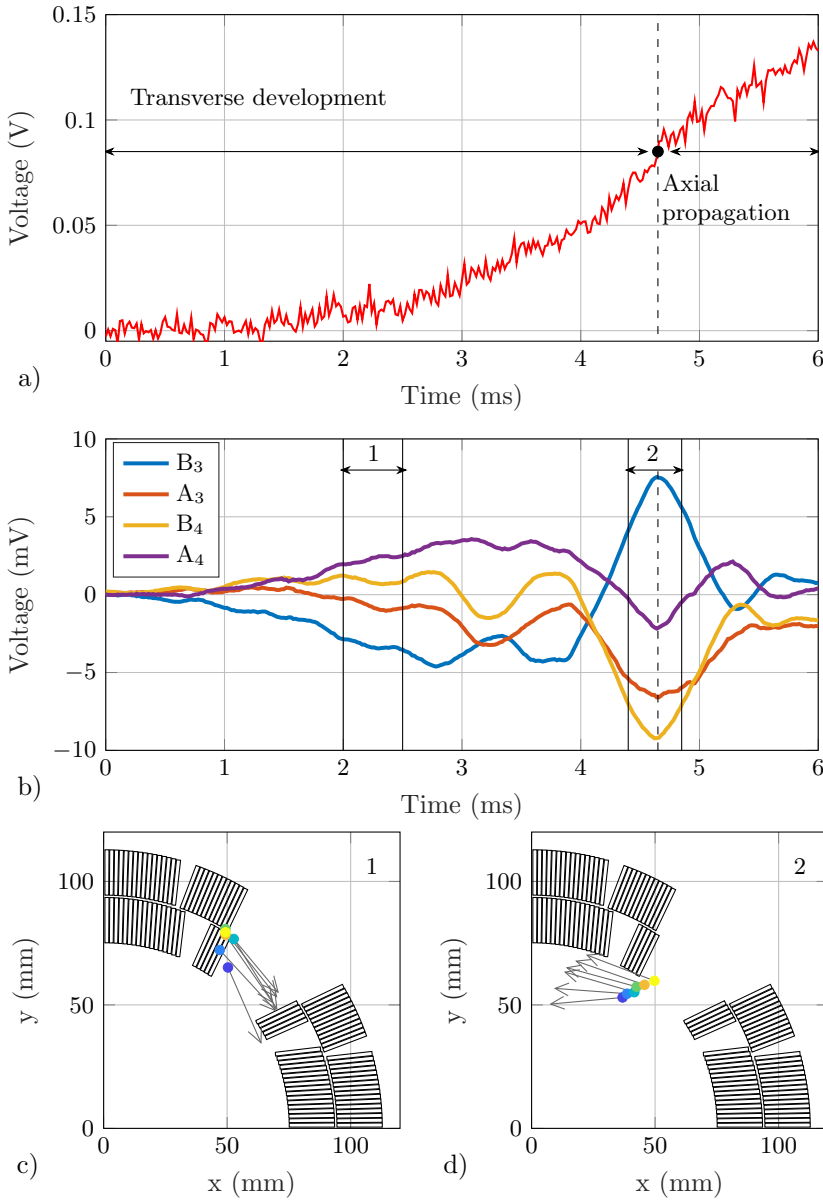


Figure 5.3: a) Measured resistive voltage build-up during the first few milliseconds of a quench of magnet MQXFB2 at 4.5 K and a cable current of 16.4 kA. b) Measured quench antenna signals corresponding to the normal B_n and skew A_n 3rd and 4th order harmonics. c/d) Reconstructed quench position and moment vector for the two time instances indicated in (b).

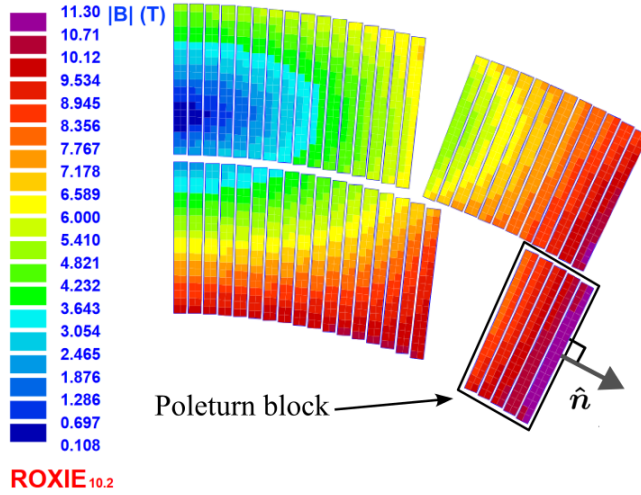


Figure 5.4: Magnetic field map of one octant of the cross-section of an MQXF magnet operating at 16.23 kA. The pole turn block, the region with the peak field is indicated with the black box. $\hat{n}$ is the normal vector to the pole turn used in Equation 5.1.

aspect ratio of the cable cross-section [106]. The direction of the dipole moment flips over between the two cases.

5.2.2 Comparison with RuNe simulation results

The quench described above initiated in the pole-turn region of the MQXF magnet, so the magnetic field profile on the pole turn is used for the simulation. The magnetic field profile is calculated with the ROXIE software [89], see Figure 5.4 and 5.5. In Table 5.1 the simulation parameters are listed. Unless stated otherwise, the possible effects of superconductor magnetization on the induced voltage in the pick-up coils are not included in the simulation.

To find a good match between the simulation - and measurement results, a wide parameter space was probed. There are uncertainties in both the electric and thermal inter-strand contact resistances, and in the quench start position in the cable cross-section. The best qualitative and quantitative matches between data and simulation are shown in Figures 5.6a and b, corresponding to the measurement results in Figure 5.3. In this simulation, the quench initiates in strand position 33, close to the center of the cable. The voltage build-up shows a similar acceleration, while the simulated quench antenna signals show a similar progression as the measured ones. In the simulated results, the successive quenching of individual strands is visible as a modulation on the signal with a period of about 200 μ s. In the measurement this modulation is not as clearly observed.

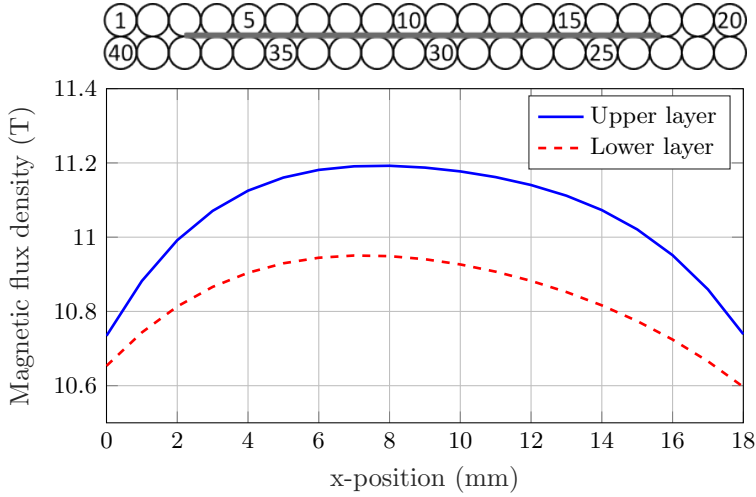


Figure 5.5: Magnetic flux density as function of the lateral position in the cross-section for the two layers of the pole turn cable. The numbering scheme for the strand positions is shown above.

Table 5.1: Cable parameters used in the simulation of MQXF-type Rutherford cables.

Parameter	Value
Number of strands	40
Cable twist pitch	109 mm
Cable width	18.15 mm
Cable mid thickness	1.53 mm
Strand diameter	0.85 mm
Cu/non-Cu ratio	1.2
Copper RRR	300
R_a & R_c	40 & 2000 $\mu\Omega$
α_a & α_c	0.02 & 0.002 $\text{W m}^{-1} \text{K}^{-(1+b)}$
b	1.54
Temperature	4.5 K
Strand I_c at 12 T, 4.5 K	610 A

The reconstructed position and direction of the magnetic dipole moment for the same two time windows as in Figures 5.3c and d are shown in Figures 5.6c and d. Even though the signals are simulated, and thus contain no noise, offset or drift, the reconstruction is still not perfect, due to the fact that the quench antenna, in its current design iteration, is still sensitive to odd higher-order multipoles, as explained in Section 3.2.

In Figure 5.7, the evolution of the strand current at the longitudinal position of the quench start is plotted for all strands. As a strand develops a normal zone,

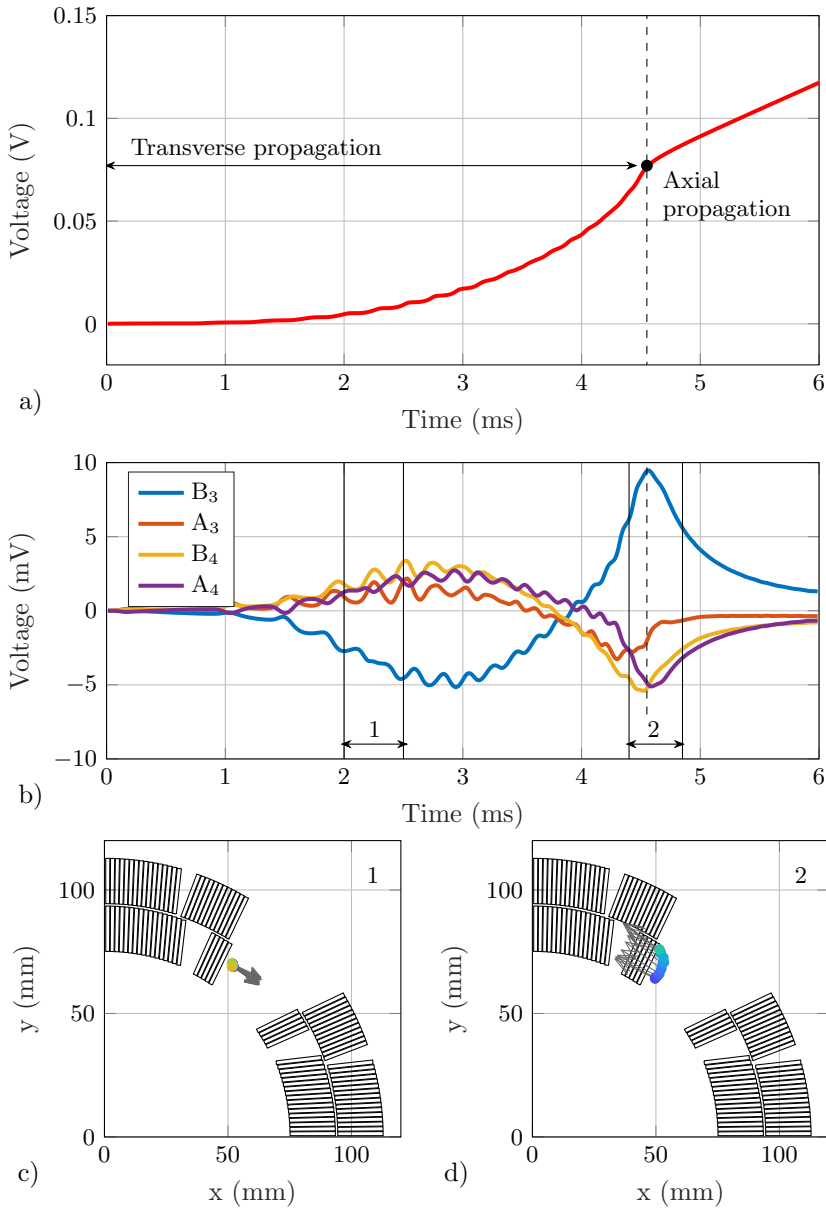


Figure 5.6: a) Simulated voltage versus time during a quench, with the model parameters adapted to yield as good as possible correspondence with the experimental data in Figure 5.3. b) Simulated voltage induced in the pick-up coils of the quench antenna. c/d) Reconstructed quench position and moment vector from the simulated quench for the two time windows indicated in (b).

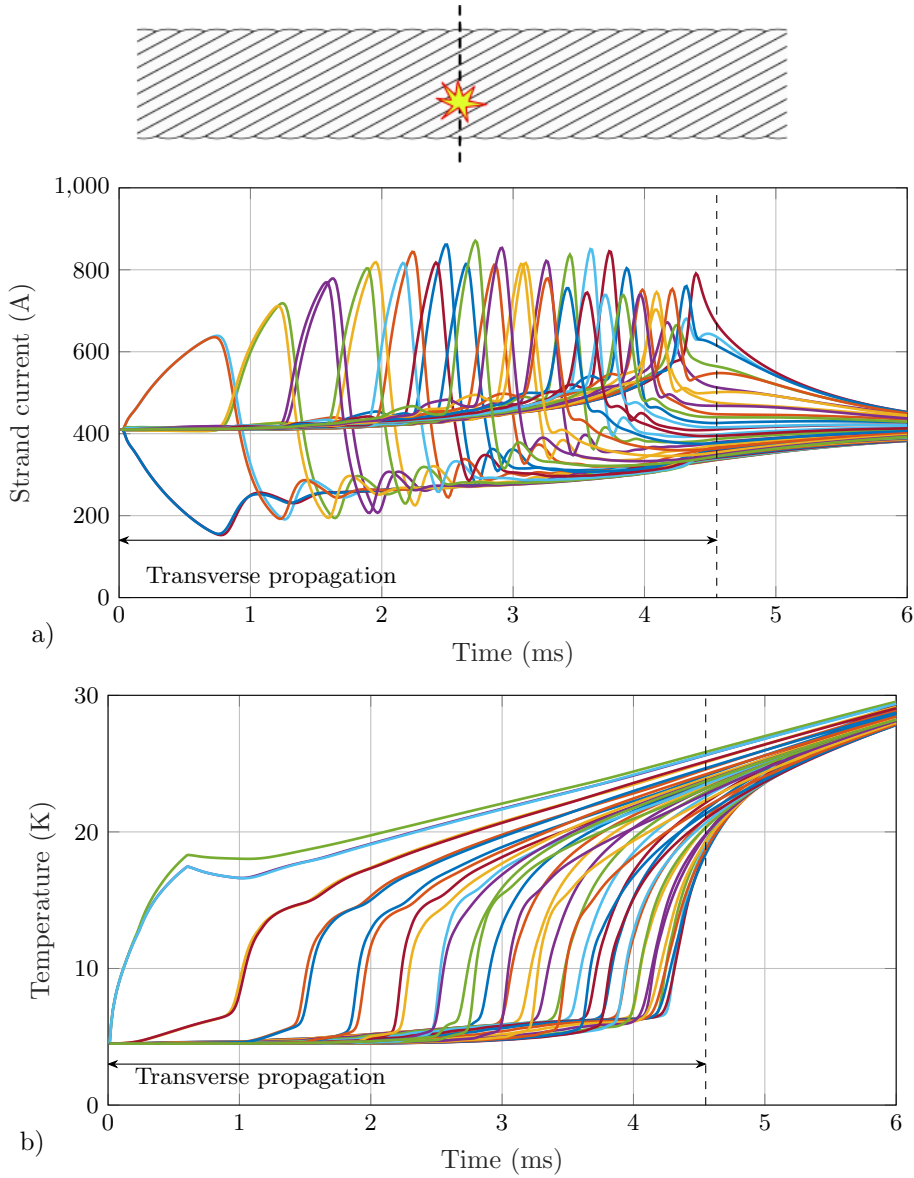


Figure 5.7: Simulated strand currents (a) and temperatures (b) at the quench location as a function of time.

its current is partially redistributed to the neighboring strands that are still superconducting. As more and more strands in the cross-section become normal, an increasing amount of excess current accumulates in the remaining ones, which has the double effect of lowering the current-sharing temperature and increasing the generated heat, thereby accelerating the transverse normal zone propagation.

This is visible by the decreasing time between successive strand quenches. Once the normal zone has spread over the full cross-section, the current redistributes mainly according to the magneto-resistance state of the copper.

Another comparison between measured and simulated quench antenna signals is shown in Figure 5.8. The initial rise in the induced voltage now first levels off on B_3 and A_4 before it flips polarity. In this case the quench is initiated in strand 28. Comparison between Figures 5.6b and 5.8b reveals how the quench antenna signal details are indeed sensitive to small variations in the start position of the quench, as discussed further in Section 5.2.4

In Figure 5.9, another such comparison is shown for a quench in the MQXFBP3 magnet. This time the magnitude on the quench antenna signals is less than half of the two previously shown quenches. The best model fit, shown in Figure 5.9b, is when the quench starts near the center of the cable in strands 30 and 31. The simulation predicts a slight inversion of the signals after the maximum peak, but this is not observed in the measurement.

Sensitivity analysis of inter-strand contact parameters

Both the thermal and the electric inter-strand contact resistances are expected to affect the way the normal zone develops, so a parametric study on the influence of these parameters was performed. In Figure 5.10, the voltage build-up is shown for several values of the parameter R_a , where the thermal contact resistance parameter α_a is kept constant at $0.01 \text{ W m}^{-1} \text{ K}^{-(1+b)}$. Current distribution through the crossing contact resistances R_c is negligible for cored cables and is not further considered here. For higher values of R_a , the kink occurs at a later time. Also, the voltage level at which the kink occurs shifts upwards as R_a increases. This can be interpreted as an increase in the ratio between the longitudinal and transverse propagation velocities, since in the ‘propagation’ phase of the quench a higher voltage corresponds to a longer normal zone.

In Figure 5.11 a similar study is repeated for adjacent inter-strand thermal conductance α_a , varying it with the same factor 50, this time keeping the inter-strand resistance R_a constant at $20 \mu\Omega$. The thermal contact conductance has a similar qualitative effect on the transverse quench propagation but quantitatively less pronounced. Note also how the thermal and electric contact resistances do not affect the longitudinal normal zone propagation velocity, since for both cases all voltage traces become parallel to each other once the normal zone has developed over the full cross-section.

During most pole-turn quenches of the MQXF magnet near nominal current, the kink in the voltage build-up occurs between 0.07 V to 0.1 V. Based on the parametric study, it is estimated that this implies a local value of R_a in the range of $20 \mu\Omega$ to $50 \mu\Omega$, and a local value of α_a between $0.01 \text{ W m}^{-1} \text{ K}^{-(1+b)}$

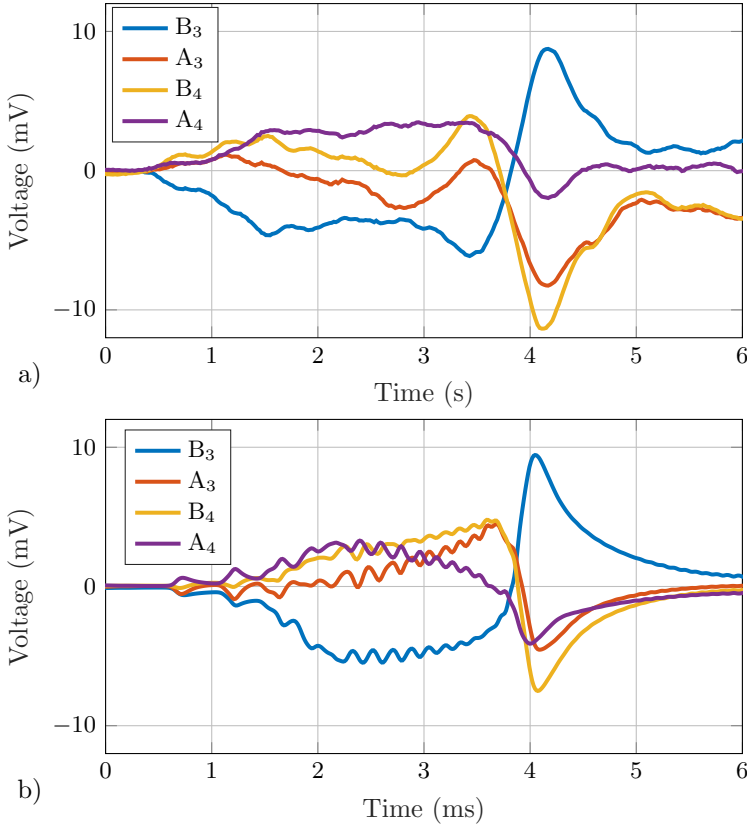


Figure 5.8: a) Measured quench antenna signals during a quench of the MQXFB2 quadrupole magnet at 16.4 kA, at a temperature of 4.5 K. b) Corresponding best match of simulated quench antenna signals.

and $0.05 \text{ W m}^{-1} \text{ K}^{-(1+b)}$.

Current and temperature evolution over time

In Figures 5.12 and 5.13, the current distribution (a) and temperature distribution (b) in a MQXF-type of Rutherford cable are shown for several time instances following the initiation of a quench. In Figure 5.12, this is shown for the case of weak coupling between the strands, while Figure 5.13 shows the same evolution in the case of strongly-coupled strands. ‘Weak’ and ‘strong’ are defined here as:

- **Weak coupling:** $R_a = 20 \mu\Omega$, and $\alpha_a = 0.01 \text{ W m}^{-1} \text{ K}^{-(1+b)}$.
- **Strong coupling:** $R_a = 2 \mu\Omega$, and $\alpha_a = 0.1 \text{ W m}^{-1} \text{ K}^{-(1+b)}$.

In the case of weak electric strand coupling, the current transfer length is longer, as can be seen when comparing Figures 5.12a and 5.13a. In combination with

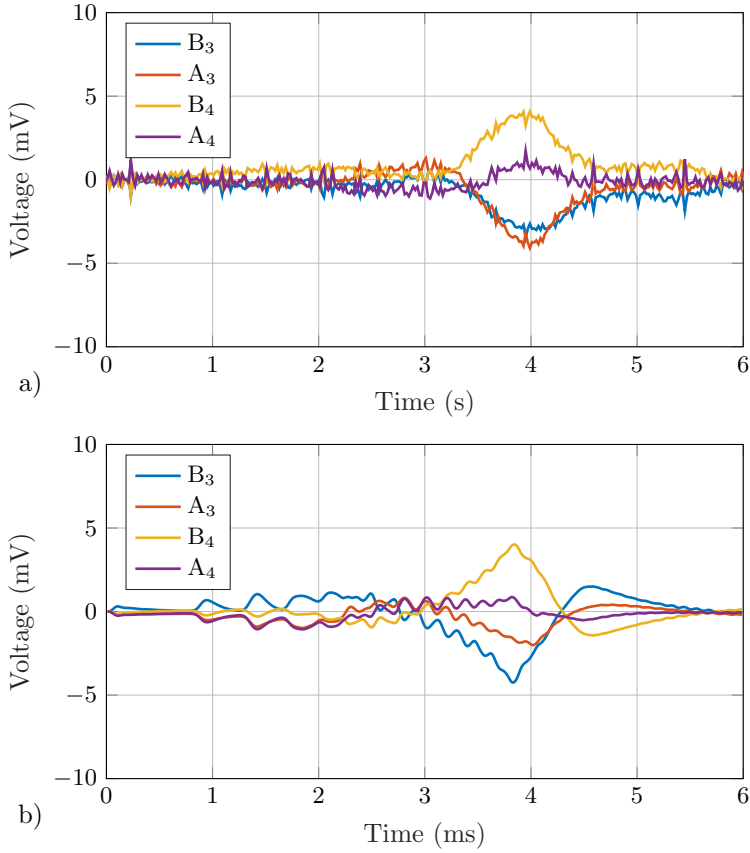


Figure 5.9: a) Measured quench antenna signals during a conductor limited quench of the MQXFBP3 quadrupole magnet at 15.9 kA. b) Simulated quench antenna signals of a quench initiated in strands 30 and 31 of the pole turn of an MQXF magnet.

the twist pitch of the cable, this will also influence the strength of the generated magnetic dipole moment. As the strands move around the edge of the Rutherford cable, the magnetic moment generated by the current imbalance on the other face of the cable will oppose the moment generated at the quench location. Cables with weakly coupled strands are therefore expected to generate a magnetic moment with a lower magnitude during a quench. Note that, as discussed above, based on the voltage level of the kink in the voltage traces, the strands in the MQXF magnet cables tend towards this weak-coupling regime.

Figures 5.12b and 5.13b show the temperature profiles corresponding to these two regimes. For higher inter-strand contact resistances (i.e. weaker coupling) the normal zone propagation occurs more along the direction of the strands, at an angle with respect to the cable's longitudinal direction. For lower inter-strand contact resistances (stronger coupling), transverse propagation becomes faster and

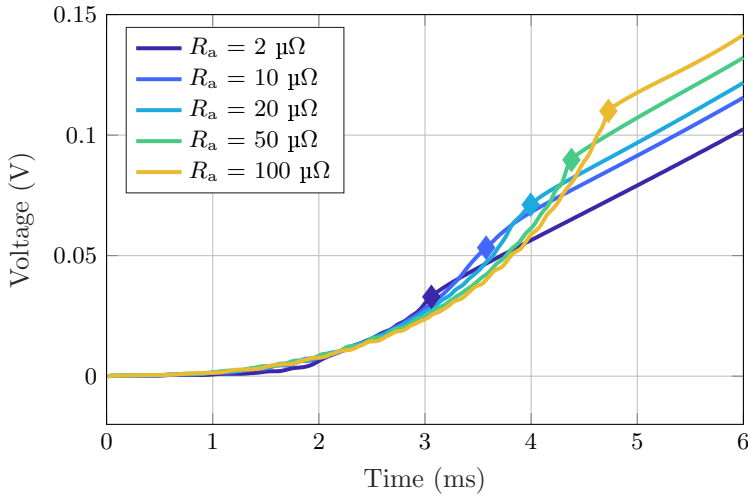


Figure 5.10: Simulated voltage versus time during quench development for different values of the electric inter-strand contact resistance.

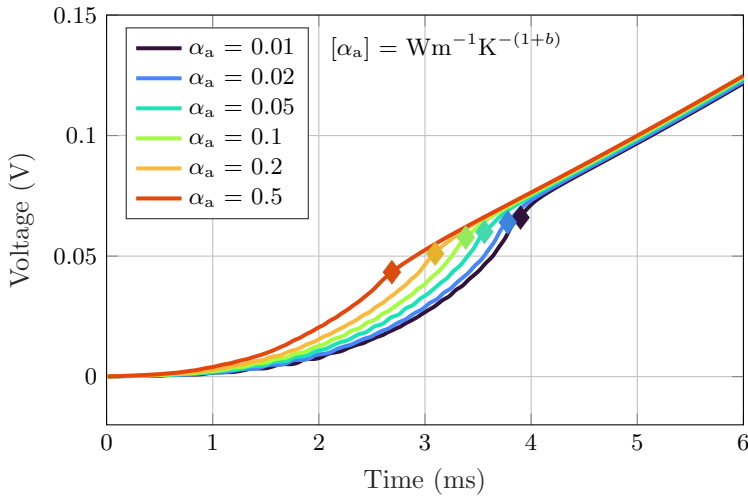


Figure 5.11: Simulated voltage versus time during quench development for different values of the scaling factor α_a of the thermal inter-strand contact conductance.

the normal zone front is more uniform across the cable width.

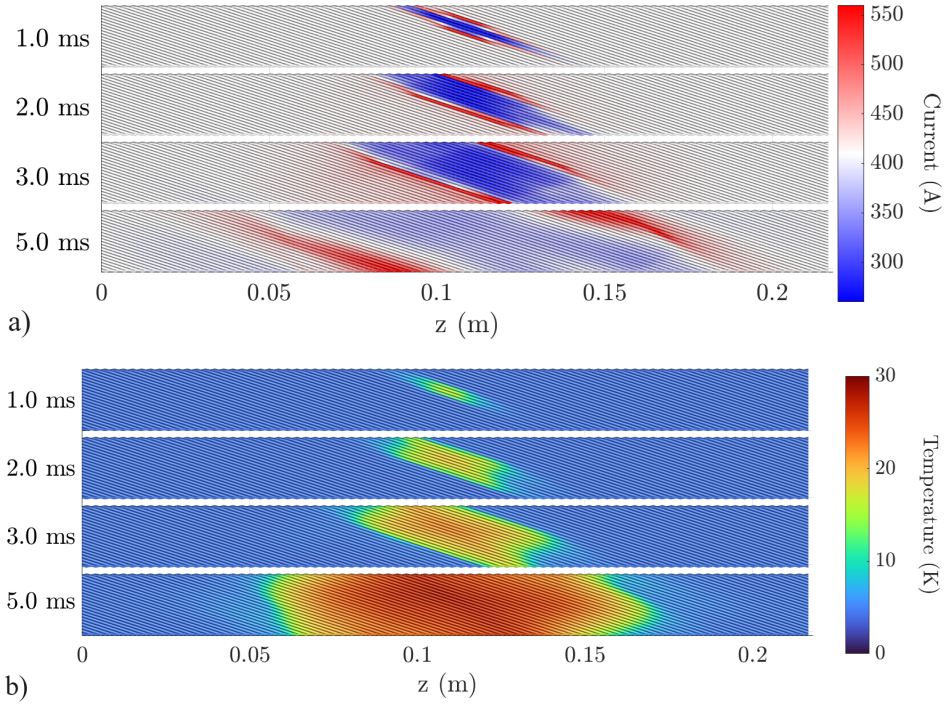


Figure 5.12: Time evolution of simulated strand currents (a) and temperatures (b) shown in a top-down view of the cable for the case of weak coupling between strands.

5.2.3 Influence of the quench's start position on magnetic dipole moment

In Section 5.2.2, we illustrated for a few cases how the quench's start position in the cable cross-section can be estimated by matching the shape and magnitude of the quench antenna signals with the simulation. The shape of the curves clearly does not just depend on the inter-strand contact parameters, but also on the start position of the quench. However, this type of trial-and-error reconstruction by fine-tuning the model is labour-intensive and case-specific. A more general way of finding the quench origin is by studying its influence on the time-integral of the magnetic dipole moment. Figures 5.14a through d illustrate this conceptually, comparing two extreme cases. A quench initiated near the center of the cable will divert current to both sides of the hot-spot (a) and (b). This results in two opposing magnetic dipole moments, whose far-fields cancel out relatively rapidly. On the other hand, a quench initiated at the cable edge can only divert current to one side, generating a single strong magnetic dipole moment (c) and (d). The stainless steel core strongly reduces coupling between the two layers of the cable, thus the two transverse quench fronts travel around the edges of the cable until they meet at the opposite side, see Figure 5.14e. The longer the two transverse

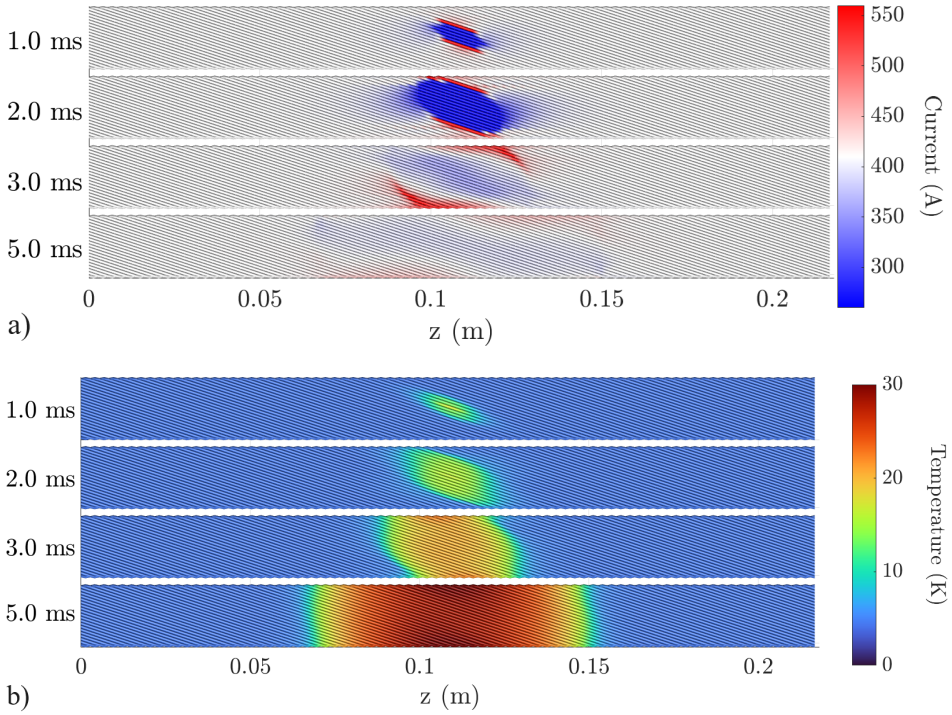


Figure 5.13: Time evolution of simulated strand currents (a) and temperatures (b) shown in a top-down view of the cable for the case of strong coupling between strands.

quench fronts propagate in the same direction, the larger the integrated dipole moment M_{int} will be:

$$M_{\text{int}} = \int_{t_{\text{start}}}^{t_{\text{end}}} \mathbf{m} \cdot \hat{\mathbf{n}} dt, \quad (5.1)$$

where $\mathbf{m}$ is the generated magnetic dipole moment and $\hat{\mathbf{n}}$ is a perpendicular unit vector, as indicated in Figure 5.4. t_{start} coincides with the start of the quench and t_{end} coincides with the transition from the quench development to its propagation phase, i.e. when the kink in the voltage build-up occurs (see for example Figure 5.3a), since we want to probe only the influence of the quench start position and not that of the magneto-resistance. To obtain the direction and magnitude of $\mathbf{m}$, Equations 3.12 and 3.14 are used. Strictly speaking, this mathematical formulation is valid in 2D only, describing an ‘infinitely long’ dipole moment generated by two parallel current lines, whereas the current redistribution zone of a real quench has a finite length, as illustrated in Figures 5.12a and 5.13a. Measured voltages $B_{3/4}$ and $A_{3/4}$ are proportional to the harmonics and depend on specific design features of the quench antenna that is used, such as its length or number of turns. The obtained value for M_{int} is therefore specific to a given quench antenna, which is accounted for by including a dimensionless pre-factor λ in the units of

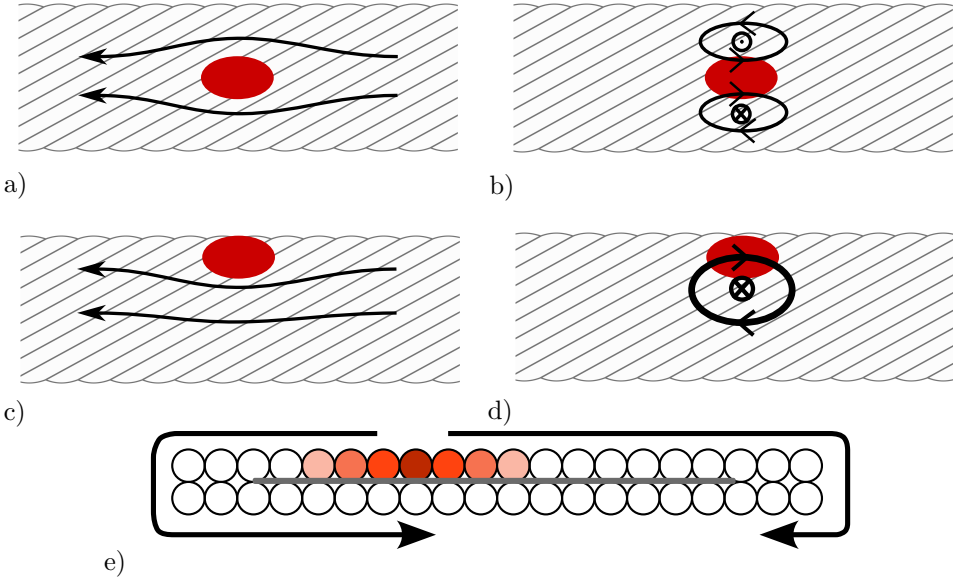


Figure 5.14: a) Quench initiation in the center of the cable, current diverts on either side. b) Current redistribution creates two opposing dipole moments on either side. c) Quench initiation at the cable's edge, current diverts towards the center of the cable. d) Current redistribution creates a single strong dipole moment. e) Two transversely propagating quench fronts meet at the opposite side of the cable.

the figures below.

Figure 5.15 shows the integrated dipole moment M_{int} for all initial quench positions in the cable for the case of either weak or strong strand coupling. As above, ‘weak’ is defined as $R_a = 20 \mu\Omega$; $\alpha_a = 0.01 \text{ W m}^{-1} \text{ K}^{-(1+b)}$, while ‘strong’ is taken as $R_a = 2 \mu\Omega$; $\alpha_a = 0.1 \text{ W m}^{-1} \text{ K}^{-(1+b)}$. A clear relation between the lateral start position of the quench and M_{int} is observed. As discussed in Section 5.2.2 in the context of Figure 5.12, the weaker coupling generally leads to a lower magnitude of $\mathbf{m}$ and hence also of M_{int} . For this specific simulation, with relatively uniform magnetic field and current distributions, there is only a relatively small difference ($\sim 10\%$) in M_{int} for quenches occurring in the upper and lower layer of the Rutherford cable. The dashed horizontal line indicates the value for M_{int} derived from the measurements in Figure 5.6b. It crosses the lines of the weakly coupled curve around strand 33 or strand 7, depending on the layer.

In Figure 5.16, the result of a similar simulation is shown, but now a linear magnetic field gradient across the width of the cable is present. As opposed to quadrupole magnets, dipole magnets typically experience such a linear magnetic field gradient in the pole turn, rather than the broad maximum shown in Figure 5.5. This difference in operational environment affects the early quench development. In Figure 5.16, the integrated dipole moment M_{int} is shown for the

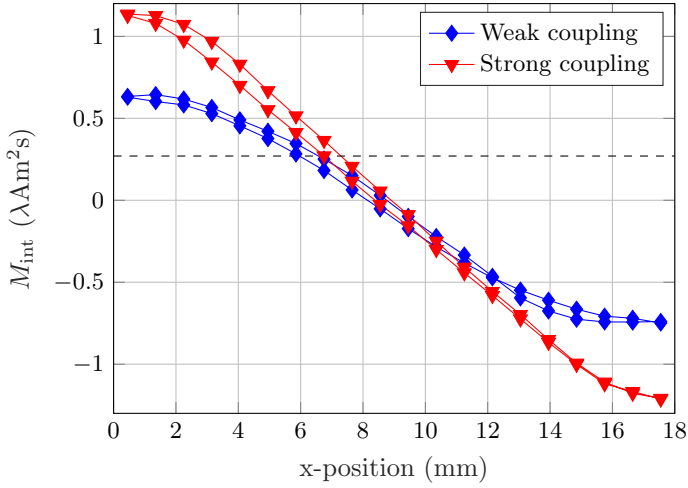


Figure 5.15: Integrated dipole moment as a function of lateral quench start position in the cable. Different symbols correspond to different coupling strengths between the strands. The two ‘branches’ within each curve correspond to quenches initiated in the top layer or bottom layer of the cable.

case of a magnetic field B (with linear gradient) ranging from 10.2 T to 11.2 T (left) and from 9.2 T to 11.2 T (right). The clear relation between M_{int} and the quench start position still exists, but the linear magnetic field gradient adds an offset. If the inter-strand contact parameters are known, e.g. from the accelerating resistive voltage build-up discussed above, the method can still be used to determine the quench’s start position.

A second type of non-uniformity relates to the cable properties themselves. In the case of local damage in the conductor, a non-uniform current distribution may be already present in the cable as current bypasses the degraded strands. In Figure 5.17, M_{int} is shown for the case of such a non-uniform current distribution. The total cable current is assumed to be 16 kA, which corresponds to an average strand current of 400 A. Eight adjacent strands are assumed to carry a reduced current of 175 A, while four adjacent strands on either side carry the excess current at 625 A. For this case, the quench is always assumed to start in the overloaded strands. The transverse normal zone propagation will accelerate or slow down depending on whether it crosses the ‘damaged’ zone or moves away from it, respectively. This affects the value of M_{int} and also causes a larger difference between the quenches initiated in the upper layer and lower layer.

5.2.4 Influence of superconductor magnetization

Besides redistribution of transport current, also the collapse of the critical-state-type superconductor magnetization induces a voltage in the pick-up coils of the

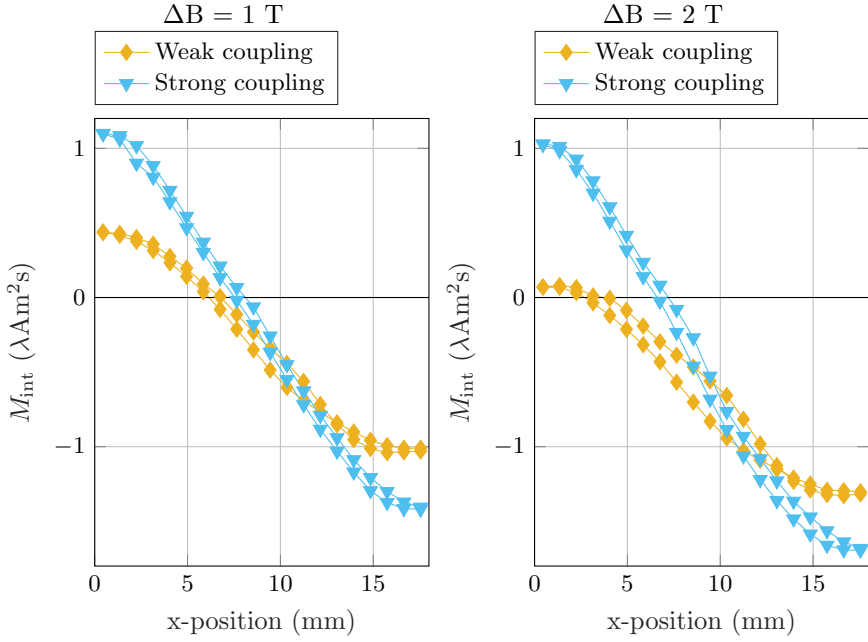


Figure 5.16: Integrated magnitude of the dipole moment as a function of transverse position in the cable for the case of a linear field gradient of 10.2 to 11.2 T (left) and 9.2 to 11.2 T (right).

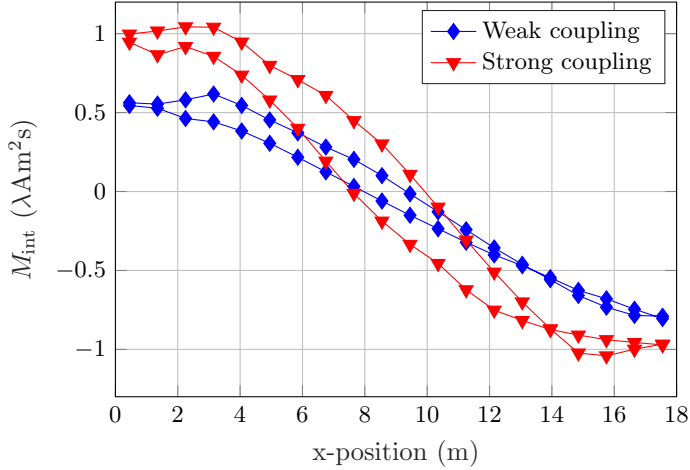


Figure 5.17: Calculated integrated magnitude of the dipole moment as a function of transverse position in the cable for the case of a non-uniform current distribution.

quench antenna. In Figure 5.18, its relative contribution to the induced voltages is shown by comparing two simulations, either with or without magnetization. The

magnetization in the strands was calculated with Equation 2.59 and its direction was assumed to be opposite to the background magnetic field. The difference between the two simulated cases is relatively small, the redistribution of transport current is clearly the dominant effect. This is not too surprising, since the critical state magnetization is associated with current distribution on filamentary scale ($\sim 50 \mu\text{m}$), whereas the developing quench rearranges currents on a strand - to cable scale ($\sim 1 \text{ mm}$ to 10 mm).

However, the dipole moment of the magnetization mostly points in the same direction throughout the quench, so it might have a larger contribution to the *integrated* signal. On the other hand, in the pole turn of dipole or quadrupole magnets, the background magnetic field and thus also the magnetization is aligned close to parallel ($< 25^\circ$) with the broad face of the cable. This means that the magnetization contribution to the moment integral of Equation 5.1 is reduced because of the dot product with the normal vector. For the example presented in Figure 5.18, the magnetization's contribution to M_{int} is only 5%. In this case study the quench is assumed to start in strand 32, close to the cable's center. As discussed above, quenches that occur close to the edge of the Rutherford cable give rise to a larger netto 'cable-level' moment and the proportion of the signal from the transport current will be larger still.

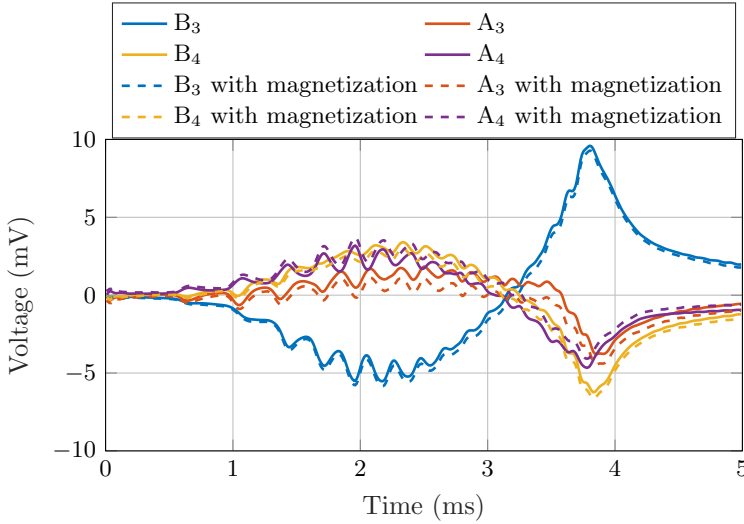


Figure 5.18: Calculated induced voltage in the quench antenna versus time with and without the effect of the collapse of filament magnetization.

5.2.5 Training quench development

During training, a magnet quenches due to the settling of the conductor, accompanied by epoxy cracking and/or slip-stick motion between interfaces in the winding

pack [18]. This often constitutes a distributed heat load that may drive a relatively large part of the cable into the normal state all at once. A quench can thus be initiated in multiple strands simultaneously, affecting the magnitude of the generated magnetic dipole moment. When the quench does not start in a single strand, the expanding normal zone accumulates less redistributing current at the quench front as in the single strand case (see Figures 5.12 and 5.13) and hence typically generates a smaller magnetic moment. Here, we study quench development of training quenches with a few case studies, once more by comparing experimental and modeling results.

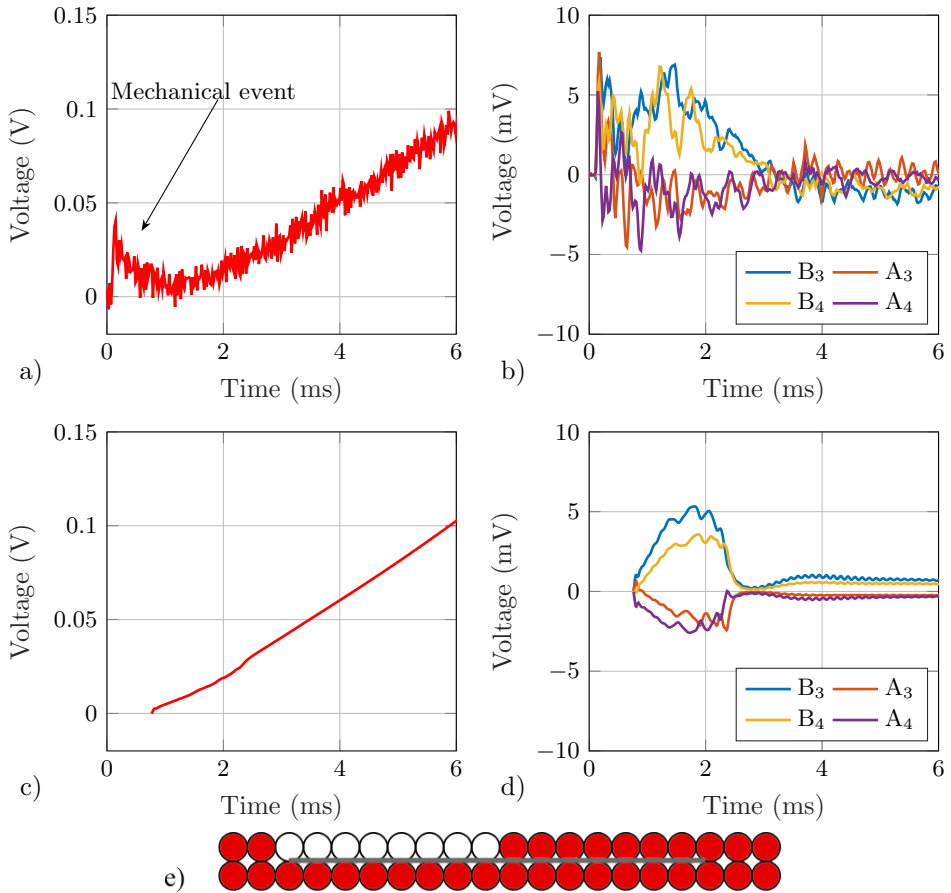


Figure 5.19: *a) Measured resistive voltage build-up during a training quench of the MQXFBP3 quadrupole magnet at 16.1 kA and (b) the corresponding experimental quench antenna signals. The best match of the simulated voltage build-up (c) and quench antenna signals (d). e) Cross-sectional view of the location of the heat pulse used in the simulation, which is applied to the red strands.*

In Figure 5.19a, the measured voltage build-up during a training quench of the

MQXF3 magnet is shown. A clear precursor feature, a sudden jump followed by an exponential decay, is associated with a mechanical event. About 1 ms after this feature, the voltage starts to rise linearly, and continues to do so without an accelerating voltage build-up. The development phase of the quench with the cascade of quenching strand is cut short, suggesting that the mechanical event has quenched most of the cross-section all at once. In Figure 5.19b, the corresponding quench antenna signals are shown. Apart from a noisy event at the start, almost no signal is picked up during this quench after the first few millisecond. Nonetheless, during the first 3 ms a large modulation is visible on the quench antenna signals, with a timescale that is compatible with individual strands quenching.

Attempting to find a good match with simulations is even harder than the ‘single-strand’ cases described in Section 5.2.1. There are significantly more unknown variables, such as the number of strands that are initially quenched by the heat pulse, the shape of this initial normal zone, its size and the pulse’s duration. The often lower quench antenna signals of training quenches also make it harder to localize the turn of the quench accurately, which means there is uncertainty which magnetic field map to use. Despite these uncertainties, the main features of the measurement can still be reproduced by the model. A best match between measurement and simulation is found by cycling through all possible combinations of heat pulse width and location. To reduce the parameter space, the only type of heat pulses considered were those that quench a set of adjacent strands and are deposited over a length spanning one node of the thermal FEM calculation, which corresponds to 1.36 mm. The heat pulse is assumed to have a block waveform with a duration of 100 μ s. In principle, this way we only test which strands quench first. Any time delay between strand quenching due to the mechanical event is not considered. Also, only a uniform initial current distribution is assumed. The cable parameters are the same as the ones listed in Table 5.1.

In Figures 5.19c and d, the simulated voltage build-up and quench antenna signals are shown that closest resemble the measurements, with the strands that received a heat pulse highlighted with red in Figure 5.21e. An interesting quench antenna feature that the simulation reproduces, is the modulation caused by individually quenching strands. However, in the measurement the amplitude of this modulation is larger. The initial spike in the quench antenna signals during the first 0.5 ms caused by the mechanical event itself is not visible in the simulation. The start of the simulation coincides with the start of the voltage rise after the mechanical event in the measurement.

In Figure 5.20, the strand currents at the quench location are shown as a function of time. Since many strands now already have quenched at the same time due to the initial heat pulse, the development phase of the quench lasts shorter. The current redistribution mainly occurs in the few strands that were not quenched by the heat pulse, as the current is pushed from the normal strands into the still superconducting ones.

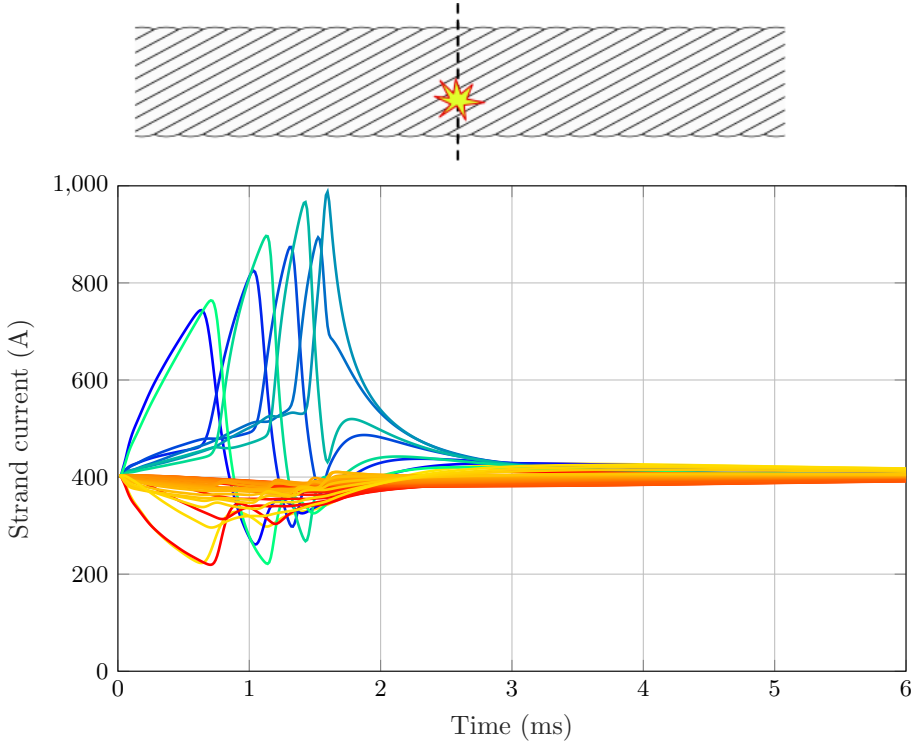


Figure 5.20: *Simulated strand currents at the quench location as a function of time, corresponding to the simulation in Figures 5.19 c and d. The red/orange lines correspond to strands that were quenched initially by the heat pulse, the blue/green lines to strands that are driven normal at a later stage.*

In Figures 5.21a and b, the measured voltage build-up and quench antenna signals during the first training quench of the MQXFB2 magnet are shown. There is only a single change of slope visible in the voltage build-up. The amplitude of the quench antenna signals is again much lower than the two single-strand cases in Figures 5.3 and 5.8. About 2 ms after the induced voltage starts to rise, it reaches a steady-state behaviour with a nearly constant non-zero voltage. This indicates a stable quench front that is propagating in the cable. The magnitude of this signal depends on the local magnetic field gradient and is mainly dependent on the magneto-resistance of the copper, as discussed also in Section 5.3.1. The initial heat pulse that best fits the measured signals initiates a normal zone in 34 strands simultaneously.

Finally, in Figures 5.22a - d, the measured and simulated voltage build-up and quench antenna signals for the second training quench of this magnet MQXFB2 are shown. During the first 2 ms after the voltage has started to rise, the quench antenna signals remain almost zero. In simulations this is best reproduced by

having a nearly symmetric heat pulse distribution. In this way, two opposing magnetic dipole moments travel towards each other, canceling each other out. This is an opposite, but similar effect to starting a quench from a single strand in the center of the cable. Then, as the cable's cross-section is fully normal, a small spike appears, and the magneto-resistance effect is yet again visible.

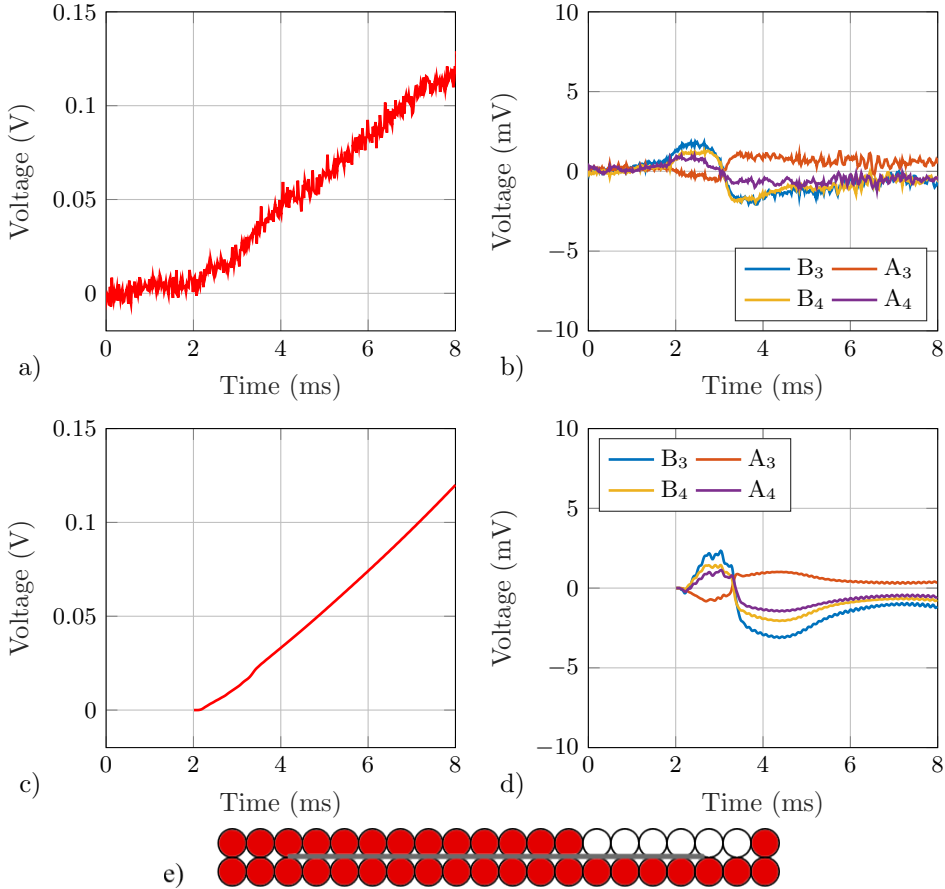


Figure 5.21: a) Measured resistive voltage build-up during the first training quench of the MQXFB2 quadrupole magnet at 15.9 kA and (b) the corresponding quench antenna signals. The best match of the simulated voltage build-up (c) and quench antenna signals (d). e) Cross-sectional view of the heat pulse used in the simulation, the heat pulse is applied to the red strands.

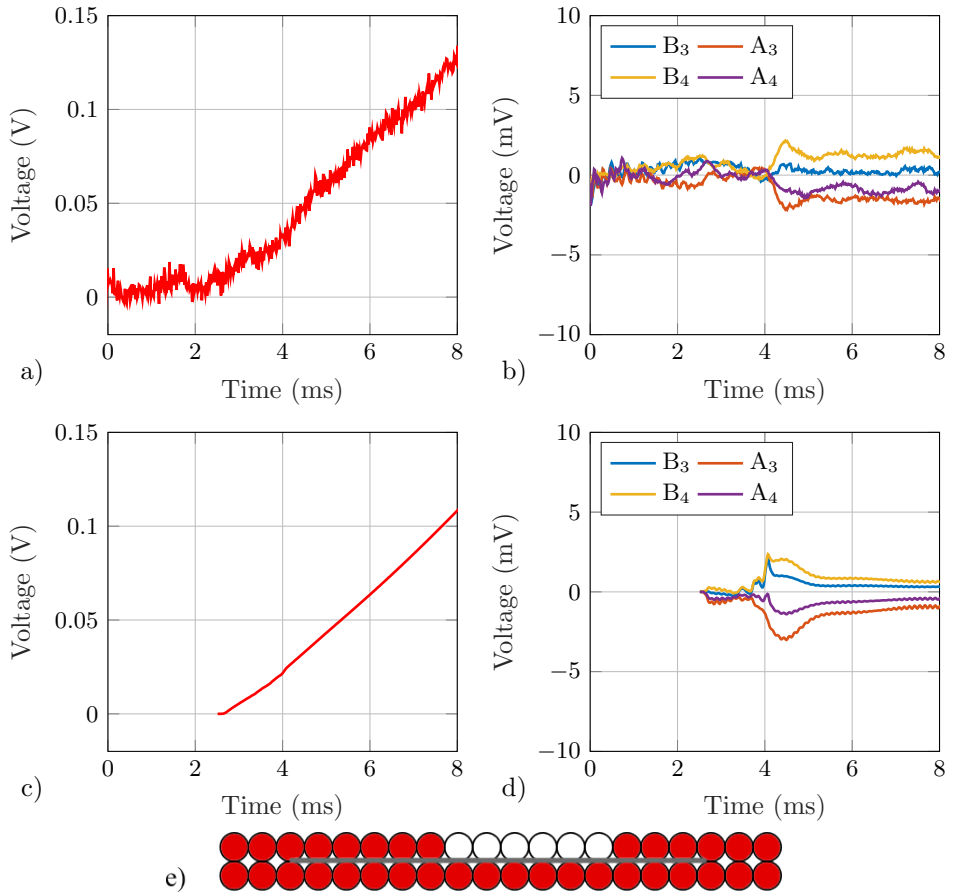


Figure 5.22: a) Resistive voltage versus time as measured during the second training quench of the MQXFB2 quadrupole magnet at 16.4 kA and (b) the corresponding induced voltage versus time of the four pick-up coils of the HQA. The best match of the simulated voltage build-up (c) and corresponding induced voltages of the HQA (d). e) Cross-sectional view of the heat pulse used in the simulation, the heat pulse is applied to the red strands.

5.3 Normal zone propagation in Rutherford cables

Once the initial quench development phase has completed, the normal zone propagates longitudinally along the cable in both directions with a certain propagation velocity. This normal zone propagation velocity (NZPV) depends on a range of local variables such as temperature, current distribution, magnetic field, and strain. Typically, the NZPV of cables is modeled either with a 1D approach or by homogenizing the cable, which allows for computational efficiency [160] - [169]. In most cases, such as studying quench protection scenarios or mechanical aspects, these methods are sufficiently accurate. In such models, on cable level, only the thermal domain is taken into account with the normal zone propagation velocity being a function of internal joule heating, thermal impedance and specific heat. The electromagnetic domain is then only modeled on magnet scale with a circuit representation to model the transport current decay in the magnet.

In the case of a non-uniform current distribution, for example caused by conductor damage, these simplifications are no longer valid. The electromagnetic domain also needs to be modeled on cable scale to account for current redistribution between strands [171, 172, 173]. In this section, the influence of such a non-uniform current distribution on the normal zone propagation is presented.

In magnets under test, the normal zone propagation is tracked using the voltage build-up and a local quench antenna system, as described in Section 3.2. Different measured quench propagation modes in 11 T dipole magnets are taken as case studies, with the likely causes of anomalous measurements reconstructed with the help of simulations performed with RuNe. The choice of dipoles for the case studies, rather than quadrupoles as in the previous section, is motivated by the relative abundance of experimental data.

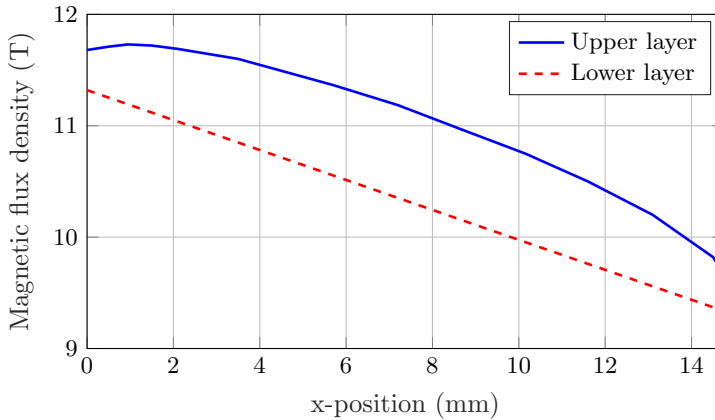
5.3.1 Normal zone propagation in cables with a uniform current distribution

First, we consider the model predictions for a quench in the pole turn of an 11 T dipole magnet under normal conditions. The magnet current is 11.6 kA and the operating temperature is 4.5 K. The cable parameters are listed in Table 5.2. The initial current distribution is assumed to be uniform. Compared to the earlier quadrupoles, where the magnetic field varies non-monotonically over the width of the cable within a range of 0.5 T, in 11 T dipole magnets, the magnetic field gradient in the pole turn varies close to linear with a gradient of about 2 T, as shown in Figure 5.23.

In Figures 5.24a and b the simulated strand current and temperature evolution during the quench are shown. Once a section of the cable has quenched and the transient effects associated with normal zone development have subsided, the current is redistributed according to the magneto-resistance of the copper matrix.

Table 5.2: Cable parameters used in the simulation of 11 T dipole cables.

Parameter	Value
Number of strands	40
Cable twist pitch	100 mm
Cable width	14.75 mm
Cable mid thickness	1.26 mm
Strand diameter	0.70 mm
Cu/non-Cu ratio	1.20
Copper RRR	100
R_a & R_c	10 & 2000 $\mu\Omega$
α_a & α_c	0.01 & 0.002 $\text{W m}^{-1} \text{K}^{-(1+b)}$
b	1.54
Temperature	4.5 K
Strand I_c at 12 T, 4.5 K	450 A

**Figure 5.23:** Calculated magnetic flux density as a function of position in the two layers of the Rutherford cable in the pole turn of an 11 T dipole magnet at a nominal cable current of 11.85 kA.

At higher magnetic field levels, the copper has a higher resistivity. The magnetic field gradient also causes the quench front to be at an angle. In the ‘top view’ of Figure 5.24a, this causes a visible ‘bow shock’ on the normal zone front that is traveling to the right. On the left side this bow shock is on the occluded other side of the cable. This bow shock is caused by the direction of the cable twist, which angles strands into the high magnetic field side of the cable, thus causing them to quench slightly earlier and push current to the lower magnetic field side in a cascade.

In Figure 5.25a, the strand current is shown along the length of the cable, 4 ms after quench initiation. The line color indicates the strand’s local temperature.

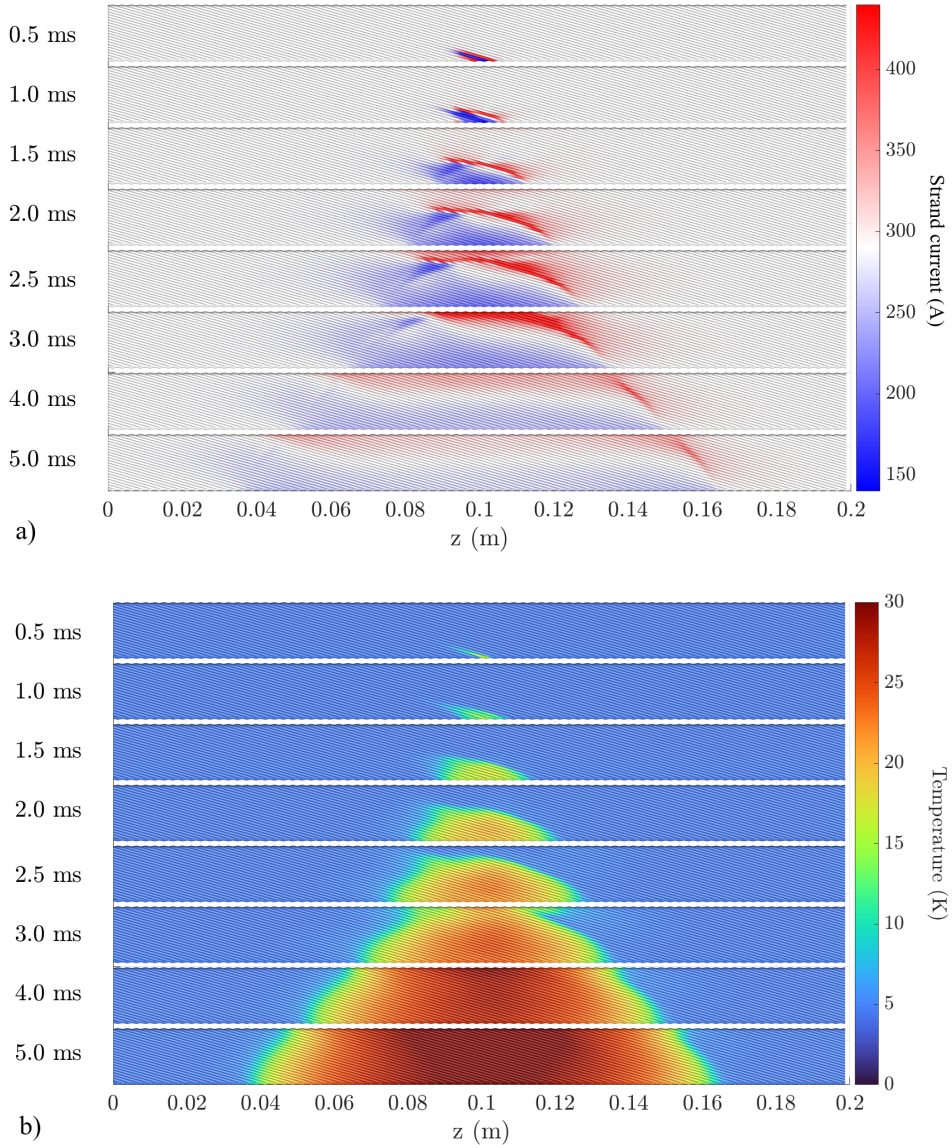


Figure 5.24: Top view on a cable section for 8 time steps showing the simulated evolution of strand currents (a) and temperatures (b) during a quench starting at $t = 0$ ms and $z = 0.1$ m. In this view, the higher magnetic field side of the cable is at the bottom.

The current imbalance is greatest at the thermal quench front. Behind the quench front, the ‘braided’ pattern in the current distribution occurs due to the copper magneto-resistance, with individual strands ‘zig-zagging’ between the high-field edge and the low-field edge of the cable. Ahead of the normal zone front, a diffusion effect is visible where the current already redistributes before the actual

thermal normal zone front arrives.

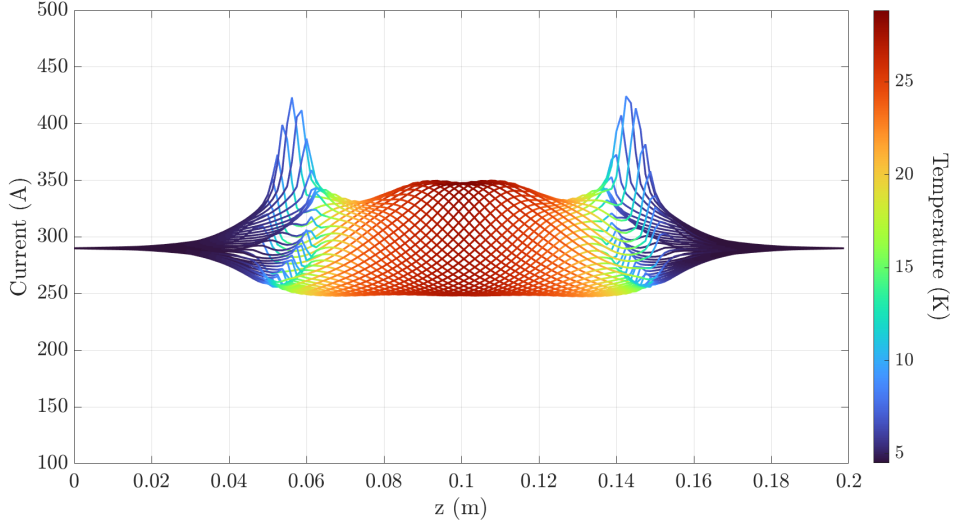


Figure 5.25: Simulated strand currents and temperatures versus position along the cable 4 ms after the quench starts at $z = 0.1$ m.

Quench antenna signals

As a normal zone front propagates through the cable, the current within the normal zone behind it will be shifted more towards the low-field side of the cable whereas ahead of the normal zone front it is evenly distributed. Therefore, the evolution of the current distribution in the cable can once more be modeled as a loop of current that generates a magnetic dipole moment, which travels along the cable with the normal zone front [106, 107]. This process is schematically depicted in Figure 5.26.

Such a traveling dipole moment will induce identical voltages in the successive pick-up coils of a quench antenna as it travels past, as also reported in [107]. A simulated example of a quench front traveling past the pick-up coils of the local quench antenna system, described in Section 3.2, is shown in Figure 5.27. The quench is initiated at some distance to the left of the array. As the normal zone front then travels past the array of pick-up coils, the voltage increases when it approaches a given pick-up coil, then decreases again as the front moves further to the right. Note that the integral of the resulting $V(t)$ waveform does not yield zero, since the current distribution before and behind the normal zone front, and therefore the total flux through a given coil, is different. The small peak that is visible in the first two coils around 2 ms results from the initial development phase of the normal zone. From the phase difference between the waveform of the different pick-up coils, the NZPV is determined to be about $15 \text{ m s}^{-1} \pm 1 \text{ m s}^{-1}$.

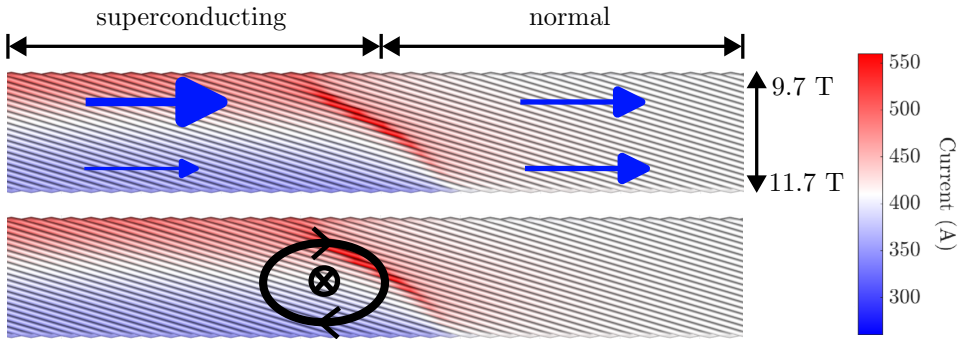


Figure 5.26: *Traveling normal zone front in a Rutherford cable. The magnetic field gradient causes a different current distribution in front and behind the normal zone front, due to the magneto-resistance effect in the copper matrix. This current redistribution manifests itself as a moving magnetic dipole moment traveling along with the normal zone front.*

Influence of lateral magnetic field gradient

In the pole-turn of the magnet, the magnetic field gradient from the inner to the outer edge of the cable is limited to about 2 T, but near the mid-plane of the magnet, the cable experiences a much larger magnetic field gradient. The magnetic field in the mid-plane turn of an 11 T dipole magnet ranges from 3.7 T to 11.3 T at a nominal operating current of 11.85 kA (see also Figure 1.8). This affects the angle of the thermal normal zone front, as the current-margin of the strands at the low field side of the cable is much higher, and it therefore takes longer for this side to turn normal, see Figure 5.28.

In Figure 5.29, the strand current is shown as a function of position along the cable at a time 5 ms after quench start. A color-coded temperature scale has been added to the line plot to visualize the thermal normal zone front. Due to the significantly lower magnetic field at the outer edge of the cable and the corresponding increase in current margin, the current imbalance in the ‘bow shock’ of the quench is now pushed to a level five times higher than in the case of the pole-turn quench shown in Figure 5.25.

5.3.2 Normal zone propagation in cables with a non-uniform current distribution

The model described above of a traveling dipole moment that can be used to analyze the quench antenna data only applies in the case of a nearly uniform current distribution in the cable. In Figure 5.30, a measurement with the local quench antenna system is shown, with quite a different behaviour from Figure 5.27. The

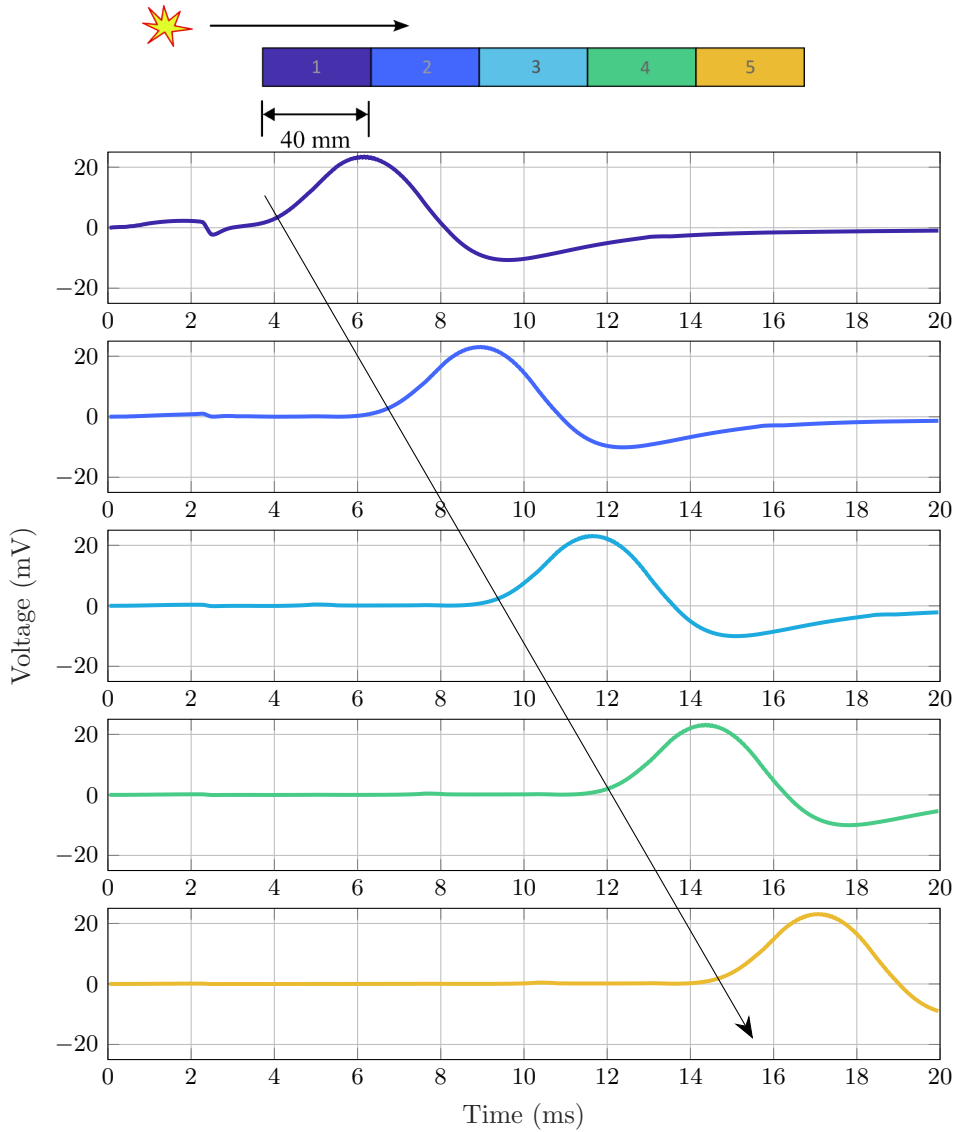


Figure 5.27: Simulated voltage of a pick-up coil versus time in the local quench antenna system as a normal zone front travels past, the cable has an initial uniform current distribution.

NZPV is more than three times higher, about 55 ms^{-1} . In addition, the polarity of the signal switches for consecutive pick-up coils as the normal zone front travels past them. Finally, the magnitude of the pick-up coil signals is at least five times higher and saturates the data acquisition system.

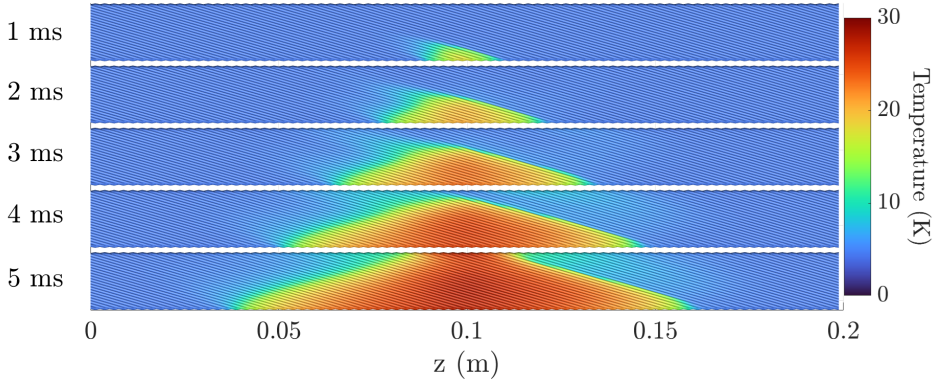


Figure 5.28: Top view on a Rutherford cable section for 5 time steps showing snapshots of the temperature distribution during a mid-plane quench. Compared to a pole-turn quench (Figure 5.24), the transverse magnetic field gradient is significantly larger ($\Delta B \sim 7.6$ T instead of ~ 2 T). The higher magnetic field side is at the bottom of these views.

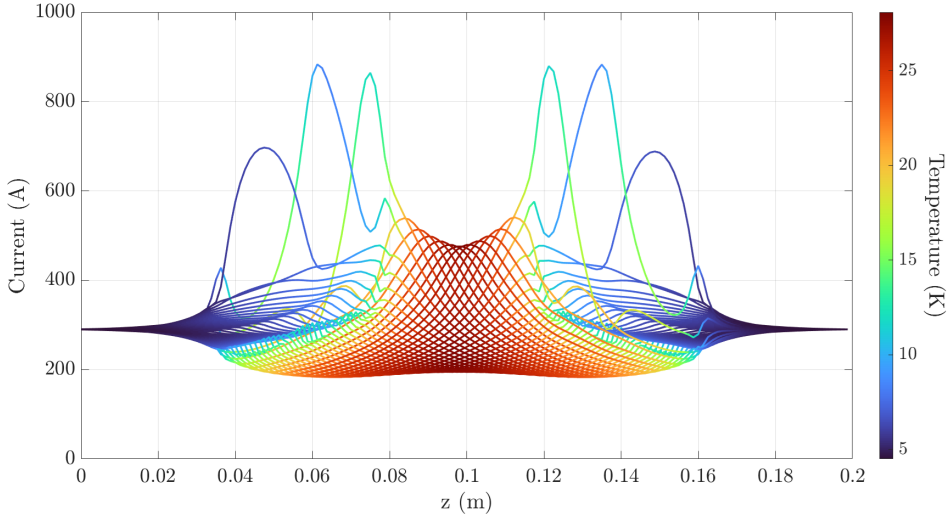


Figure 5.29: Simulated strand current as a function of position at time $t = 5$ ms for a quench in the mid-plane turn with a large magnetic field gradient in an 11 T dipole magnet. The line color corresponds to the local strand temperature.

In Figure 5.31, the resistive voltage build-up for the same quench is shown, where after a short linear start of 0.5 ms, the voltage rises with a varying slope. Such a modulation in the voltage build-up may indicate that the normal zone propagation velocity is not constant. After 3.5 ms the slope decreases, indicating a slow down of the NZPV, likely due to one of the quench fronts reaching the head of the magnet. Given the overall rate of voltage increase and the measured NZPV

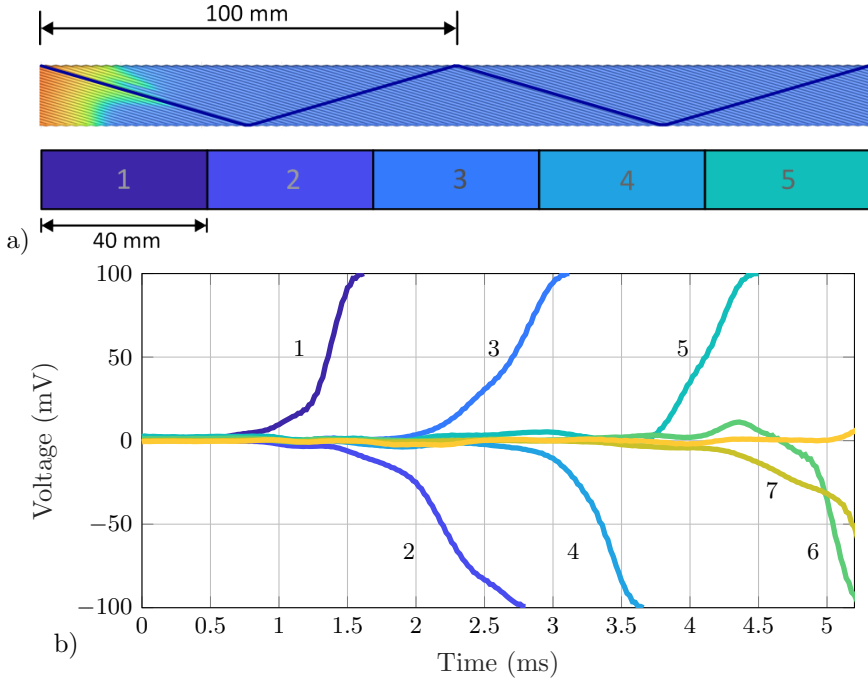


Figure 5.30: a) Relation between cable twist of 100 mm versus pick-up coil length of 40 mm. b) Measured voltage induced in the pick-up coils of the quench antenna versus time of an 11 T dipole magnet with performance issues. Compared to Figure 5.27 the voltage traces show anomalous behaviour, with a switching polarity of the signal on consecutive pick-up coils and an unexpectedly high magnitude that saturates the DAQ system at a 100 mV level.

(from Figure 5.30), two turns seem to be quenching simultaneously.

The pick-up coils have a length of 40 mm and the twist pitch of the cable is 100 mm, see Figure 5.30. A non-uniform current distribution causing the normal zone front to propagate first along the overloaded strands immediately next to a degraded one is assumed to be the cause of these anomalous observations. This hypothesis is supported by the independent V-I measurement of a non-zero voltage in this same magnet, which included a tell-tale decaying component indicating current redistribution already prior to the quench, as discussed in Chapter 4. The variation in the slope of the resistive voltage build-up is likely caused by the saturated strands traveling back and forth between the high - and low-field side of the cable.

The simulation as presented in Figures 5.24 through 5.27 for a uniform current distribution is repeated to investigate such a scenario with an initial non-uniform current distribution. There are two overloaded strands on either side of the broken strands, which become saturated at 507 A. In Figure 5.32, the simulated current

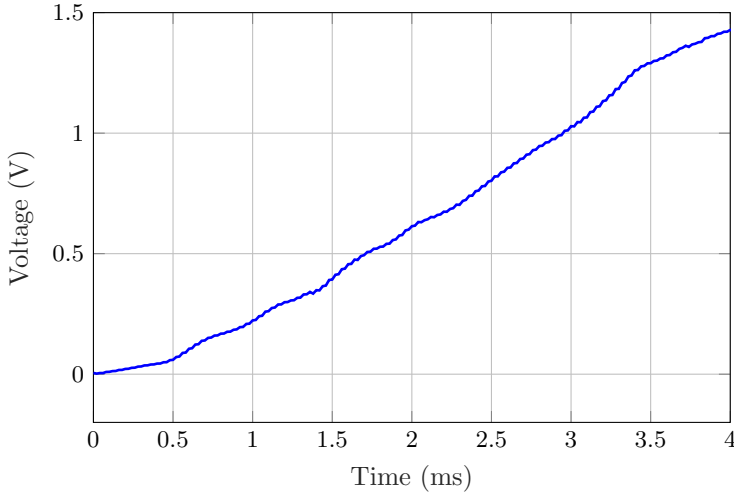


Figure 5.31: *Measured resistive voltage versus time corresponding to the quench with anomalous quench antenna signals in Figure 5.30.*

(a) - and temperature (b) evolution of a quench in a Rutherford cable with such a non-uniform current distribution is shown. The overloaded strands lead the normal zone front followed by a gradually expanding ‘wake’ of other quenching strands. Also in the simulation, the NZPV increases significantly with respect to the uniform case.

The individual strand currents as a function of position after 3 ms are shown in Figure 5.33. The magnitude of the current redistribution effect near the quench front is greater by a factor of two with respect to the uniform case of Figure 5.25. Since the overloaded strands always turn normal ahead of the others, the quench front never stabilizes into a configuration where all strands in the cross-section quench nearly at the same time. This means that more current can be accumulated as the normal zone grows transversely, as seen by the ‘wake’ following the overloaded strands. Combined with the larger NZPV, this explains the much stronger inductive signal that saturated the data acquisition system in the quench antenna measurement shown in Figure 5.30.

Parametric study of inter-strand contacts

In Figure 5.34, the simulated NZPV is shown as a function of the adjacent electrical contact resistance R_a , with the value for the thermal inter-strand contact conductance α_a fixed at $0.01 \text{ W m}^{-1} \text{ K}^{-(1+b)}$. The simulation is performed for the pole-turn cable of an 11 T dipole magnet at a current of 11.6 kA and a temperature of 4.5 K, with a strand saturation level of 500 A. In the case of a uniform current distribution, R_a has no influence on the NZPV. However, when saturated

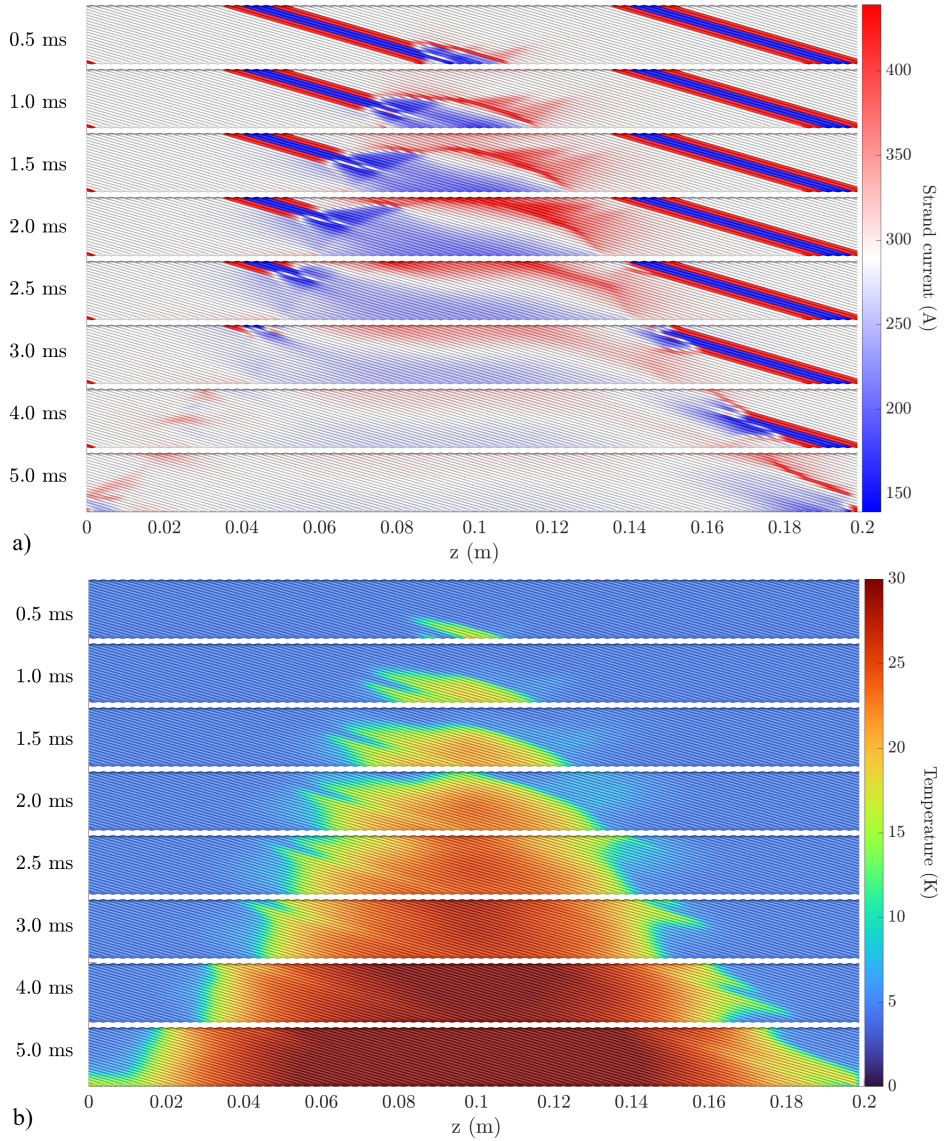


Figure 5.32: Top view on a Rutherford cable section for 8 time steps showing simulated time evolution of the current - (a) and temperature distribution (b) during a quench in a cable with damaged strands starting at $z = 0.1$ m and $t = 0$ ms. The higher magnetic field side is at the bottom edge of the pictures.

strands are present, we observe three different regimes. Starting from low values of R_a , the NZPV is near-constant ranging from $0.1 \mu\Omega$ to between $1 \mu\Omega$ and $10 \mu\Omega$ (regime I). As R_a increases further, the NZPV starts to decrease (regime II). The transition from regime I to regime II occurs for higher values of R_a when more

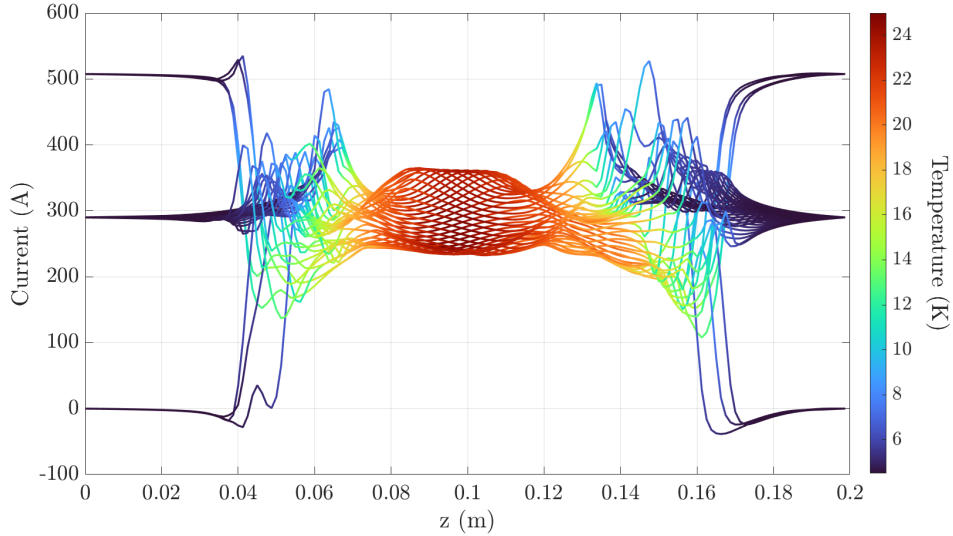


Figure 5.33: Strand current and temperature distribution at $t = 3$ ms as a function of position for the case of non-uniform current distribution, the quench starts at $z = 0.1$ m and $t = 0$ ms.

strands are saturated. Finally, as R_a is increased even further, the NZPV starts to increase sharply again (regime III).

The change-over from the relatively fast normal zone propagation in regime I to a slower NZPV in regime II, can be understood as a transition from a thermally-driven to an electromagnetically driven propagation mode. In terms of characteristic time scales, current redistribution between strands in the vicinity of the normal zone front occurs on a scale $\tau_{em} = \frac{L_{em}}{R_a}$ (with L_{em} the self-inductance of the current loops involved), while the natural time scale for the thermal propagation is $\tau_{th} = R_{th}C_{th}$ (with R_{th} and C_{th} the characteristic thermal impedance and heat capacity, respectively). As R_a increases, τ_{em} becomes smaller and current redistribution more rapid. The current redistribution front runs ahead of the thermal front and by the time the temperature of the saturated strand reaches the current-sharing temperature T_{cs} , where current starts to flow in the copper matrix and heat starts to be generated, a significant amount of current has already been driven to neighboring strands. This is illustrated in Figure 5.35, where the top graph shows the thermally driven regime ($R_a = 1 \mu\Omega$), while in the bottom graph the propagation is more electromagnetically driven ($R_a = 10 \mu\Omega$). In the former case, when the thermal quench front arrives ($T \approx T_{cs}$, indicated by the vertical dashed line), the saturated strand still carries nearly all its current ($I_{sat} = 507$ A), while in the latter case more than 25 % has already been driven into the neighboring strands since the current transfer length is much longer. As a consequence, the current density J_s of the strand is correspondingly lower, the heat generation $\rho_s J_s^2$ even more so (with ρ_s the strand resistivity) and the normal zone propagates

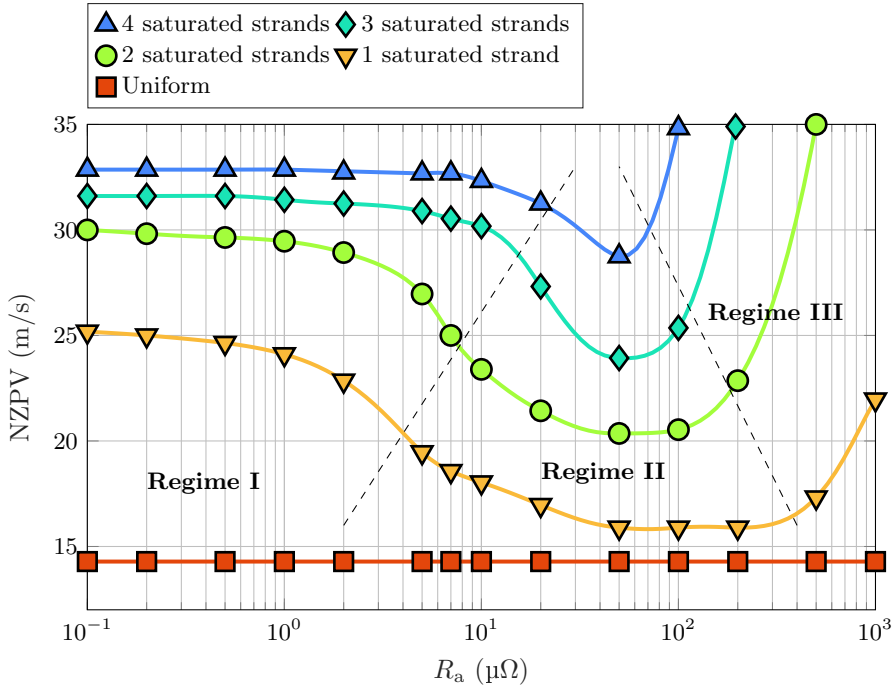


Figure 5.34: Simulated NZPV versus R_a (with $\alpha_a = 0.01 \text{ W m}^{-1} \text{ K}^{-(1+b)}$), in the case of non-uniform current distributions with 1 to 4 saturated strands on either side of a defect, compared to a uniform current distribution. Markers represent calculated points, the lines are a cubic spline fit. Three different regimes, separated by the dashed lines, are observed. Regime I is mostly thermally driven. In Regime II current diffusion starts to play a larger role and current is driven out of the saturated strands ahead of the normal zone front, which slows down the NZPV. Finally, in Regime III heating due to inter-strand current transfer starts to significantly pre-heat the cable ahead of the normal zone front, which again increases the NZPV.

slower.

For even larger values of R_a , an increase in the NZPV value is observed in Figure 5.34 (regime III), indicating the transition to another propagation mode. This third propagation mode likely emerges due to heating from the redistributing current passing through the inter-strand contacts as it is pushed from the normal zone into the adjacent strands. The heat load from this current diffusion process causes significant pre-heating of the cable ahead of the actual normal zone front, as illustrated in Figure 5.36. This hypothesis was confirmed by turning heating due to inter-strand current transfer off in the model, which causes regime III to disappear. As more strands are saturated, regime II becomes smaller and eventually regime I transitions directly to regime III. It has to be noted however, that such high values of R_a ($> 100 \mu\Omega$), when regarding the straight section of a magnet, lie well

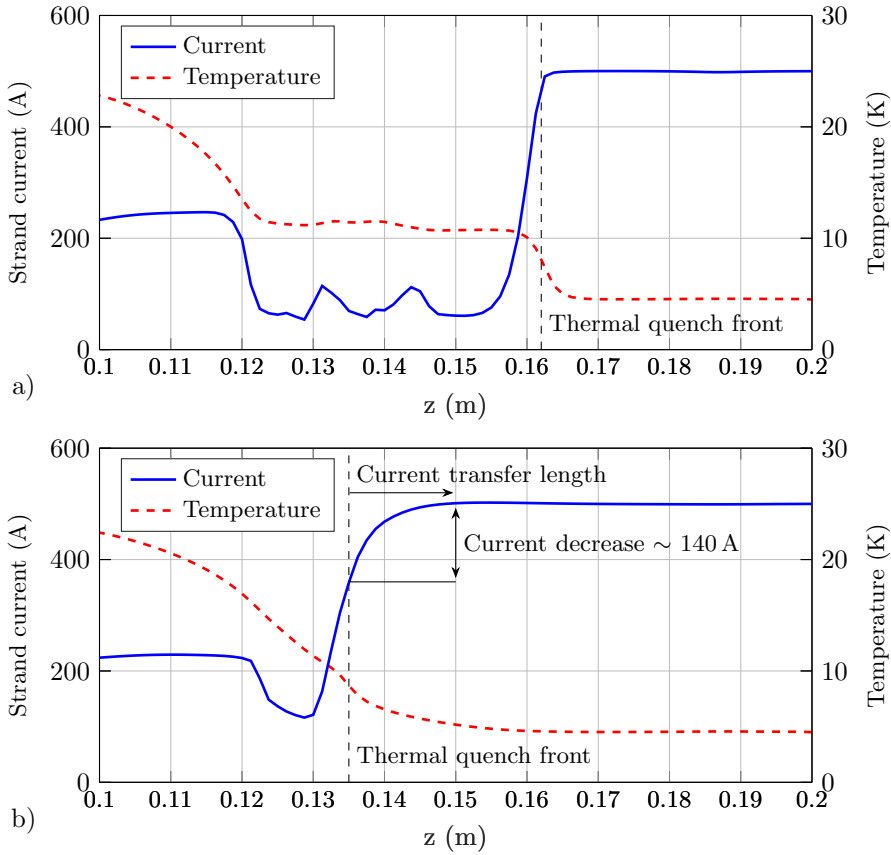


Figure 5.35: Calculated current and temperature of the overloaded strand as a function of position for (a) $R_a = 1 \mu\Omega$ and (b) $R_a = 10 \mu\Omega$ at $t = 3$ ms. In (b) the current in the overloaded strand is already much reduced by the time the thermal normal zone front (indicated by the vertical dashed line) passes by, resulting in a lower NZPV.

outside the expected range for the type of Rutherford cables treated in this thesis. However, locally in the head of the magnet, where the cable bends and gaps can open up between strands, the R_a values can be much higher. Note how Rutherford cables with such high values of R_a are unlikely to be able to sustain many adjacent saturated strands.

In Figure 5.37, the simulated NZPV is shown as a function of the adjacent thermal inter-strand contact parameter α_a , this time keeping the value for R_a fixed at $1 \mu\Omega$, which corresponds to the thermally driven regime I of Figure 5.34. For the case of one saturated strand, there is a similar transition from a relatively high NZPV to a lower NZPV around $\alpha_a = 0.06 \text{ W m}^{-1} \text{ K}^{-(1+b)}$. For a higher number of saturated strands the transition shifts to a higher α_a . The dependence of NZPV on α_a is more straightforwardly understood than on R_a . As α_a increases,

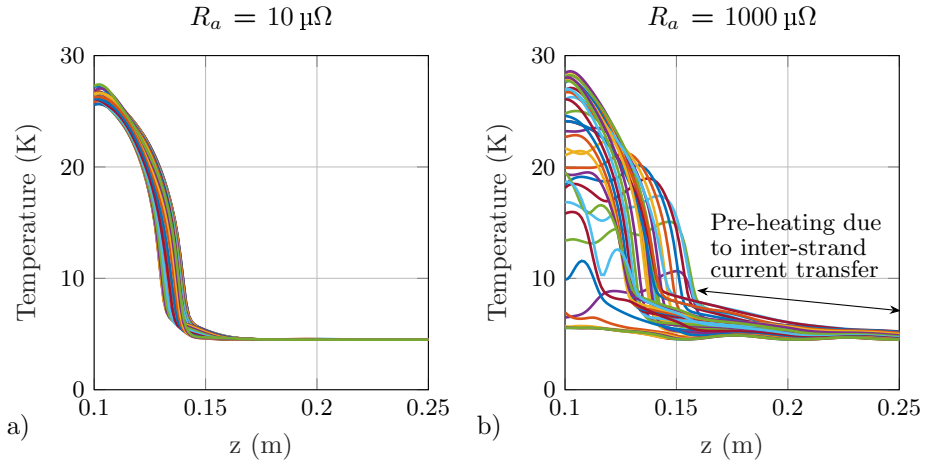


Figure 5.36: Simulated temperature distribution versus position in the cable for the case of 1 saturated strand at $t = 2.5$ ms, for a cable with $R_a = 10 \mu\Omega$ (a, regime II) and for a cable with $R_a = 1000 \mu\Omega$ (b, regime III). As visible in the $T(z)$ profiles, the NZPV is larger in (b) because strands ahead of the normal zone front are pre-heated due to current diffusion.

more of the generated heat in the saturated strand is evacuated to the neighbors and less is available to drive the thermal normal zone front in the saturated strand itself.

Influence of number of saturated strands

In the simulated case of Figures 5.32 and 5.33 with two saturated strands on either side of the damaged strands, the NZPV value is increased to about 25 m s^{-1} with respect to the predicted value of about 15 m s^{-1} for uniform current distribution, whereas in the measured example of Figure 5.30 the value of NZPV is more than double this simulated value, about 50 m s^{-1} . Even for four saturated strands and low values of R_a , the NZPV value does not rise much beyond 30 m s^{-1} . This may indicate that even a more severe current imbalance is present in this 11 T dipole magnet than the situations simulated up to this point. During the experiment with the SMC, which has a similar Rutherford cable but without a stainless steel core, it was observed that with five broken strands the magnet could still operate close to 90 % of the short-sample limit, with around four adjacent saturated strands. It is therefore not unreasonable to hypothesize that the 11 T dipole magnet has four or more saturated strands around 75 % of the short sample limit. The presence of the core in the dipole cable also means that current redistribution through adjacent strands is more dominant, which means that a similar defect in the 11 T dipole magnets as the one in our SMC magnet, will lead to earlier strand saturation. As the quench front in the saturated strands passes through the low-field side of

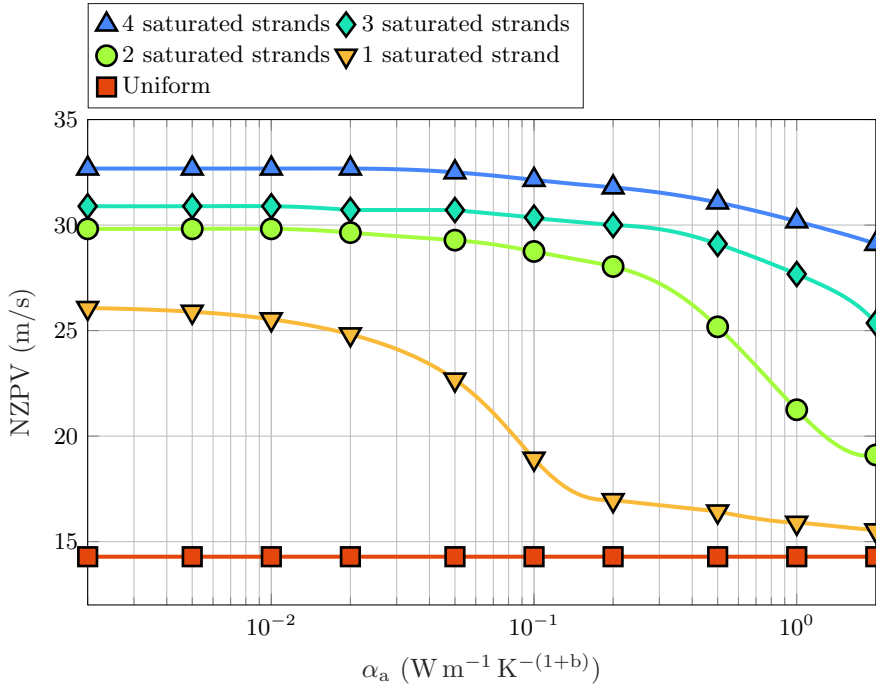


Figure 5.37: Simulated NZPV versus adjacent inter-strand thermal conductivity α_a , in the case of non-uniform current distributions with 1 to 4 saturated strands, compared to a uniform distribution. R_a is kept constant at $1 \mu\Omega$, which corresponds to the thermal driven regime I of Figure 5.34.

the cable, the NZPV is significantly reduced. In the case of a large defect, with more saturated strands that are also spaced wider apart on either side of a larger defect, there is a saturated strand on the high-field side of the cable for a longer proportion of the cable length, which results in an increased NZPV. In Figure 5.38, the time evolution of a quenching cable with a larger initial current imbalance is shown. The NZPV now indeed approaches 50 m s^{-1} , similar to the measured case in Figure 5.30.

In Figure 5.39, the NZPV is shown as a function of the number of saturated strands. In this case $R_a = 5 \mu\Omega$ and $\alpha_a = 0.01 \text{ W m}^{-1} \text{K}^{-(1+b)}$. After an initial roughly linear increase of the NZPV with increasing damage, the dependence at higher numbers of saturated strands eventually levels off.

Finally, also the induced voltage response of the pick-up coils of the local quench antenna system is simulated for the case of a strongly non-uniform current distribution, with eight saturated strands on either side of a defect, as shown in Figure 5.40. The voltage over successive pick-up coils now shows qualitatively identical behaviour as the measurement of Figure 5.30, with switching polarity

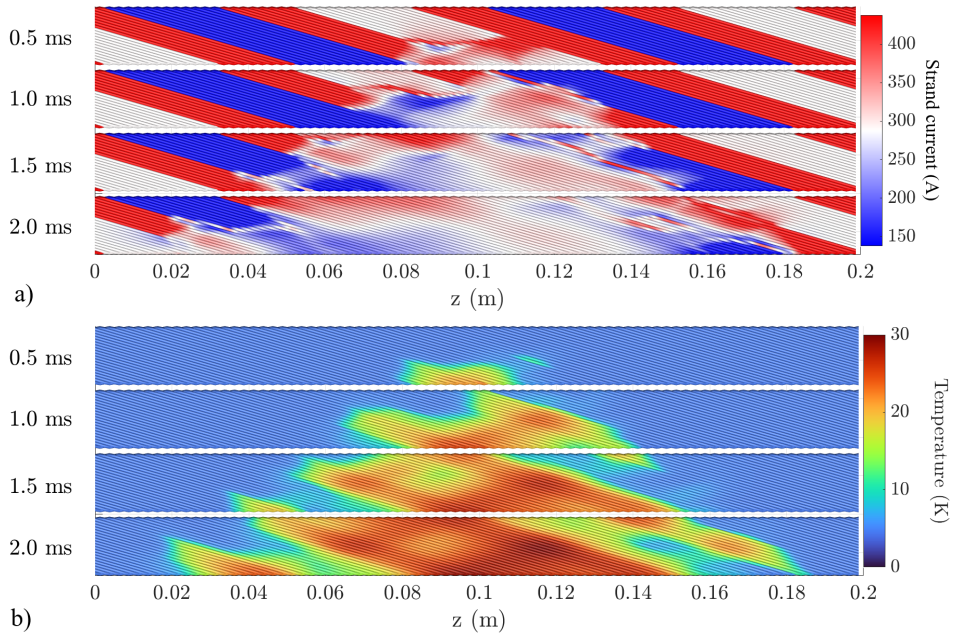


Figure 5.38: Top view on a Rutherford cable section with 4 time steps showing the time evolution of strand currents (a) and temperatures (b) during a quench, with a large non-uniform initial current distribution, with 8 saturated strands on either side of the defect.

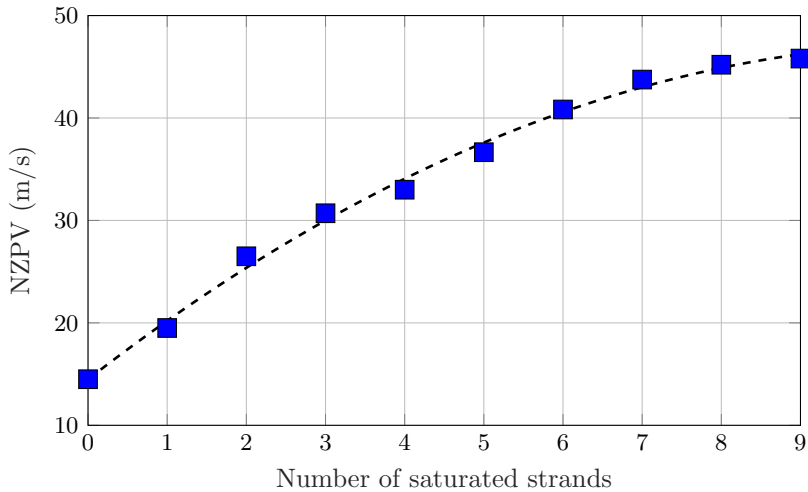


Figure 5.39: Calculated NZPV as a function of the number of saturated strands. The dashed line is fitted as guide for the eye and is only valid for integer number of strands.

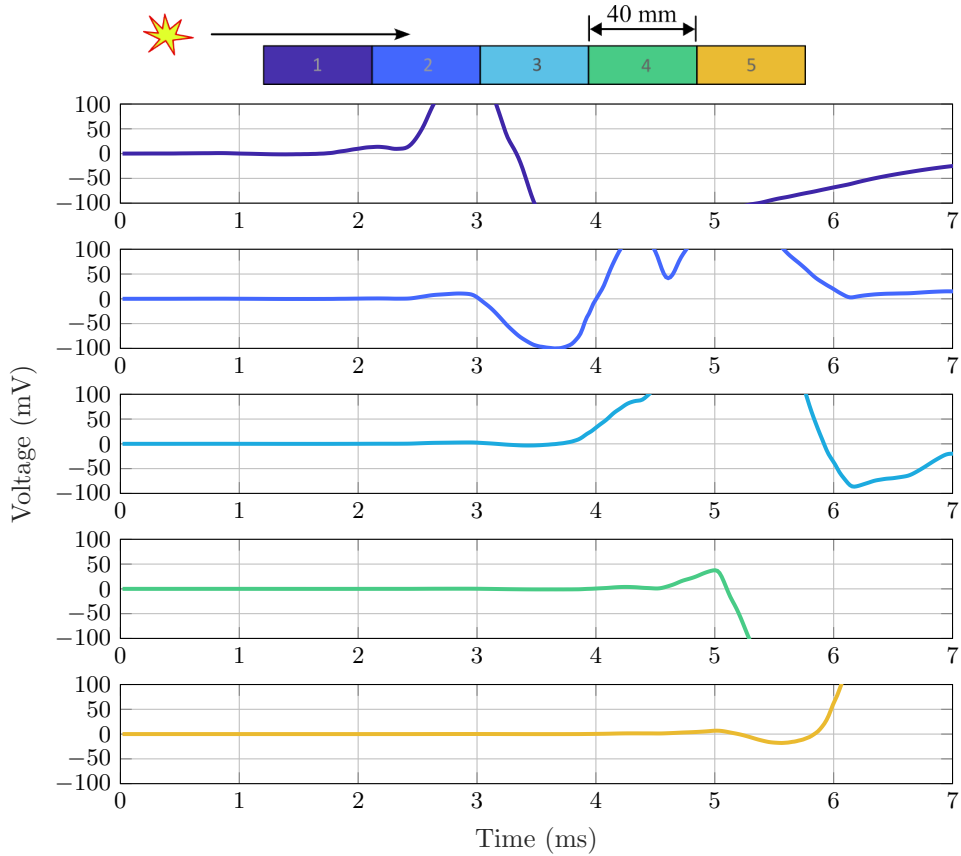


Figure 5.40: Simulated pick-up coil signals of a normal zone front passing by the local quench antenna system. The Rutherford cable carries a non-uniform initial current distribution with eight saturated strands on either side of the defect.

on successive pick-up coils; induced voltage levels that would indeed saturate the DAQ at 100 mV; and a NZPV around 50 ms^{-1} . The polarity flips between the consecutive pick-up coils, because the current redistribution is dominated by the non-uniform initial current distribution, which reverses direction from one side of the cable to the other every half twist pitch following the strand trajectory (see also Figure 5.30a).

5.4 Quench analysis of an SMC coil with introduced defect

In this final section, the results of the quench studies performed on the SMC magnet described in Sections 3.3 and 3.4 are presented. The effect of the introduced damage on the quench behaviour of the magnet as well as the quench propagation velocity are discussed. For ease of reference, Figures 3.14 and 3.15, showing the overall coil lay-out and the voltage tap positions, respectively, are repeated here.

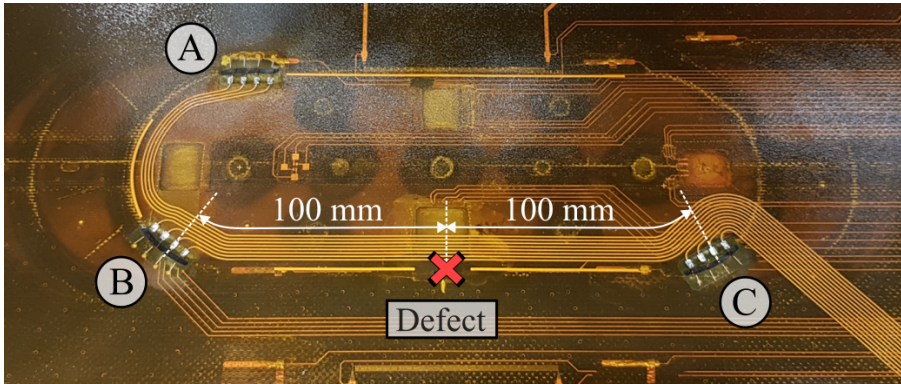


Figure 3.14: Top view on the SMC coil, showing locations of the defect and the voltage tap groups A, B, and C. Also visible are the copper traces that connect the new voltage taps to the instrumentation feed-through.

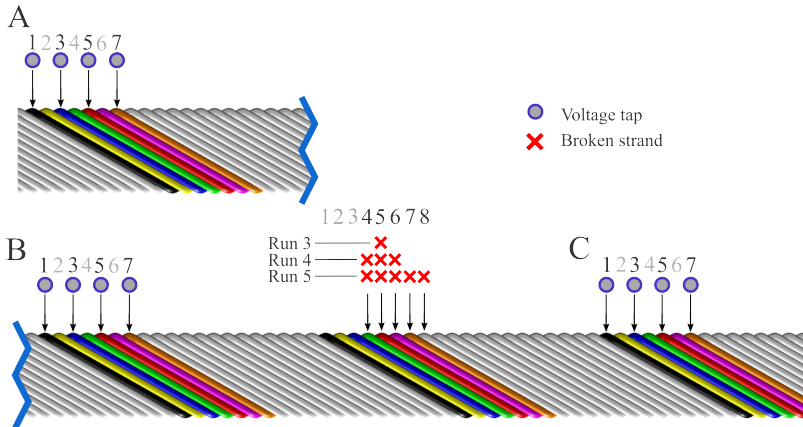


Figure 3.15: Schematic of the relative position of voltage taps and interrupted strands in the cable. For clarity, the number of strands in the figure is reduced to 20, as opposed to the real cable that features 40 strands.

5.4.1 Voltage build-up

In Figure 5.41, the resistive voltage as measured over the different voltage tap pairs is shown. At each of the locations A, B and C, four voltage taps are located (Figure 3.15). In this case, the quench starts at the location of the intentionally introduced defect, which may be deduced from the fact the voltage rise starts at locations B and C simultaneously. The voltage build-up over section B-C starts to slow down, presumably as the quench front gets closer to the ends of the straight segment of the coil where the magnetic field is slightly lower and hence the current margin is higher. Then, at about 2.5 ms, as the normal zone front passes the voltage taps at locations B and C, the voltage build-up slows down significantly: the full section B-C is in the normal state and the much slower subsequent voltage rise is caused simply by the heating of the cable and its ensuing resistivity increase.

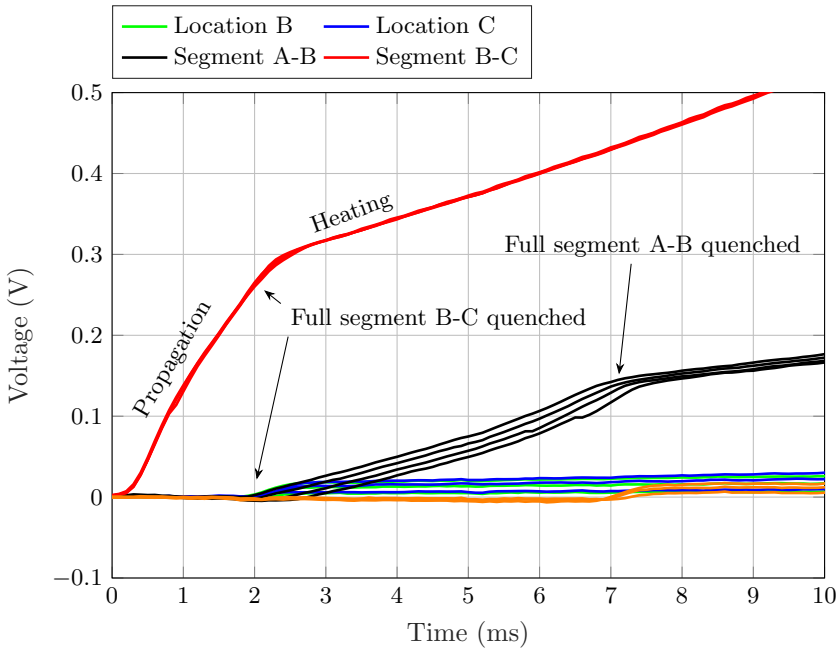


Figure 5.41: Resistive voltage versus time as measured over different segments of the SMC coil, with one intentionally broken strand in the cable. The quench occurred at 14.68 kA at a temperature of 4.5 K.

Once the quench front passes location B, the voltage over strand sections A-B start to rise as the normal zone propagates along the coil's head. Note that the slope of the voltage rise here is about half the one in section B-C, which confirms our hypothesis of a quench starting at the defect location: section B-C essentially shows the length of a normal zone expanding on two symmetric fronts, whereas section A-B only 'sees' one side of the expanding normal zone.

5.4.2 Normal zone propagation velocity

In Figure 5.42, the voltage drop along strand 5 as recorded by voltage tap pair B5-C5 is shown for quenches of the SMC coil with different levels of intentional damage. The quenches all initiated at the damage location. For each case, the quench occurred spontaneously as the current was ramped up with a rate of 20 A s^{-1} . As discussed in Section 4.5 (Figure 4.31) when the number of intentionally broken strands is increased, the quench current is reduced. Under normal circumstances in a magnet, a lower transport current and hence a lower magnetic field, leads to a lower normal zone propagation velocity. However, this is not observed in the SMC experiment, as shown in Figure 5.42. Instead, the NZPV tends to increase somewhat as more strands are broken.

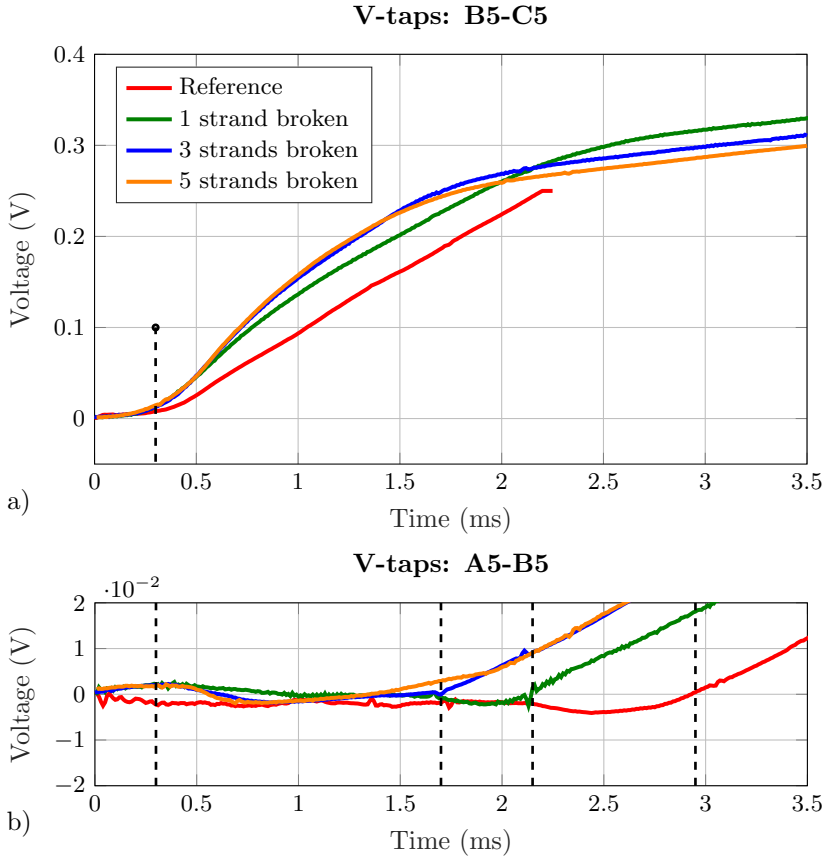


Figure 5.42: Measured resistive voltage drops along strand 5 plotted against time during a quench occurring after a ramp with a rate of 20 A s^{-1} for different levels of damage. The NZPV is determined by finding the time delay between the start of the voltage rise in segment B5-C5 and A5-B5, indicated by the vertical dashed lines.

As discussed in Section 5.3.2, the NZPV strongly depends on the local conditions of the most overloaded strands in the cable. If the magnetic field is lower, the overloaded strands have a higher critical current and thus can carry more current before being saturated. The ‘knee’ in the curves of Figure 5.42a, signifying that the entire section B-C is in the normal state, also occurs at a somewhat different voltage for all the cases. This is a result of the lower transport current, but also due to the difference in magnetic field. At higher magnetic field, the magneto-resistance effect is stronger and the copper matrix has a higher resistivity. In addition, when the NZPV is lower the normal zone is heated a bit longer before the entire is turned normal and the voltage increases accordingly.

In Figure 5.42b, the voltage over section A5-B5 is shown. By taking the time delay between the start of the voltage rise in section B5-C5 and in section A5-B5, an estimate of the NZPV can be made. It is assumed that the quench starts exactly in the center of section B5-C5. This assumption is verified by comparing the voltage rise of voltage tap pairs B1-B7 and C1-C7, which occur at the same time. The thus measured NZPVs are listed in Table 5.3. A strong increase in NZPV is observed at higher damage levels, with the case of 3 and 5 broken strands having almost double the NZPV value of the reference test.

Such a higher NZPV at higher damage levels, in spite of the lower transport current, is in agreement with the model predictions discussed in Section 5.3.2, with the quench propagating along overloaded strands.

Table 5.3: *Measured normal zone propagation velocity for different levels of damage at 4.5 K.*

Damage level	NZPV (m/s)
4.5 K	
reference	38 $\pm$ 2
1 strand broken	54 $\pm$ 3
3 strands broken	71 $\pm$ 4
5 strands broken	71 $\pm$ 4

5.4.3 Quench location

In Table 5.4, an overview is given of quench initiation sites (at the damage location or not) for different current ramp rates and damage levels. The corresponding tests were performed both at 1.9 K and 4.5 K. Focusing first on the ramp rate studies at 1.9 K, for a single broken strand no clear correlation between the quench location and the ramp rate was observed. However, for 3 and 5 broken strands, the quench initiates not at the position of the drilled strand sites for lower ramp rates, while at the higher ramp rates it does.

At 4.5 K, for a single broken strand the magnet never quenched at the defect

location. However, for the case of 3 broken strands it always quenched there. The fact that the coil does not always quench at the defect location, suggests the presence of another weak spot in this coil. As discussed in Chapter 4, current redistribution around the intentional defect causes a current imbalance that easily extends several meters along the cable and could interact with a weak spot somewhere else, or simply with the high magnetic field region at the other side of the pole turn. As shown in Figure 3.13, the drilled defects in the strands are located at the short edge of the cable where the magnetic field is at its lowest in the cable cross-section. Since the current imbalance extends to the other side of the pole turn, the overloaded strands pass through a higher magnetic field than the one that they experienced at the defect itself. The dependence of the ramp rate on the quench location indicates the importance of local damage and the resulting current redistribution on magnet stability.

Table 5.4: *Quench location at intentional damage location ($\checkmark$ = yes, $\times$ = no, - = no data) for several ramp rates at 1.9 K and 4.5 K.*

Damage level	10 A s ⁻¹	20 A s ⁻¹	50 A s ⁻¹	100 A s ⁻¹
1.9 K				
1 strand broken	$\checkmark$	—	$\times$	$\checkmark$
3 strands broken	$\times$	$\times$	$\checkmark$	$\checkmark$
5 strands broken	$\times$	$\checkmark$	$\checkmark$	$\checkmark$
4.5 K				
1 strand broken	—	$\times$	$\times$	$\times$
3 strands broken	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
5 strands broken	$\times$	$\checkmark$	$\checkmark$	$\checkmark$

5.4.4 Comparison with RuNe

Finally, a comparison is made between the observed quench behavior of the intentionally damaged cable in the SMC coil and the corresponding RuNe predictions to see if the non-uniform current distribution can explain the counter-intuitive increase in NZPV at lower quench current. Specifically, the pole-turn section of the Rutherford cable in the SMC coil with the broken strands is simulated, see Figure 5.43. The nodal voltages in RuNe at the location of the voltage taps in the experiment are then used for comparison with measured values.

The NZPV strongly depends on the initial conditions. The precise initial current distribution is not known, but we do know the following:

- the current in a saturated strand must be roughly equal to the strand's critical current where it experiences the peak magnetic field;
- there are about four adjacent saturated strands on either side of the intentionally broken strands, as derived in Section 4.7.5.

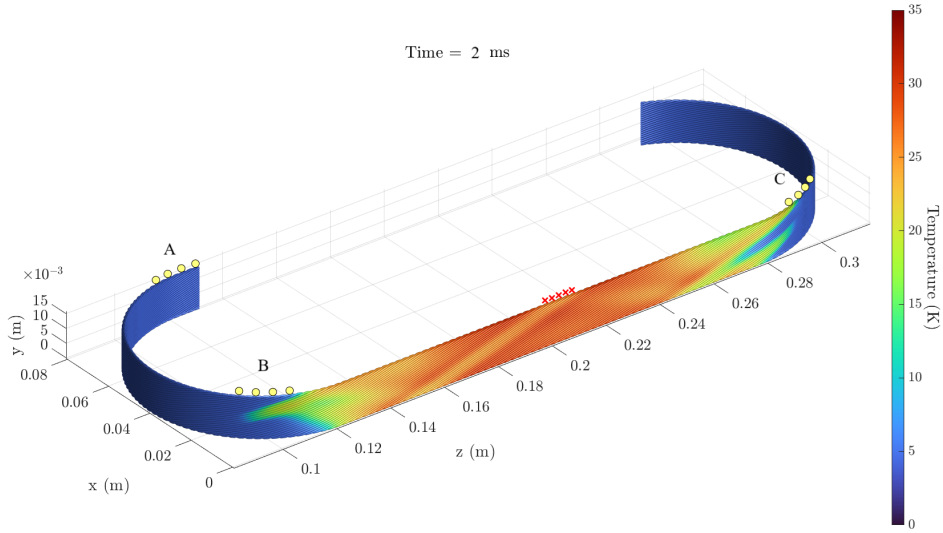


Figure 5.43: Simulated temperature distribution about 2 ms after the quench start in the pole turn of the SMC coil. An initial non-uniform current distribution was present. The voltage taps locations (yellow dots) and the broken strands (red crosses) are indicated. Since the broken strands initially carry practically no current, they do not heat up as fast as the adjacent overloaded strands.

With this in mind, we can make an estimate of the current in the saturated strands by plotting the strand critical current as a function of cable current (i.e. as a function of the peak magnetic field). The strand critical current values for the cases corresponding to Figure 5.42 are indicated in Figure 5.44. Since the SMC is relatively short in relation to the twist pitch of the cable, the peak magnetic field of the coil is not experienced by all strands. The broken strands are located on the low-field side of the cable, so once the saturated strands pass through the peak magnetic field in the cable cross-section, the field has already decayed slightly closer to the coil head (see Section 3.3.1), implying that there is some uncertainty in the current that the saturated strands carry.

In Figure 5.45, a comparison is made between measured and modeled voltage build-up during a quench of the SMC at 4.5 K, with a single broken strand. This comparison shows that the main physics of the problem is captured well by the model. However, there are too many adjustable parameters for a unique fit. The shaded area around the voltage rise over segment B5-C5 shows the variation in the simulation result that occurs with just a 10 A variation of the assumed current in the overloaded strands. The presence of collateral damage in the strands adjacent to the interrupted ones, as discussed in Chapter 4, makes it difficult to determine the initial conditions more accurately.

In Figure 5.46, the experimental data of Figure 5.42 are simulated. The as-

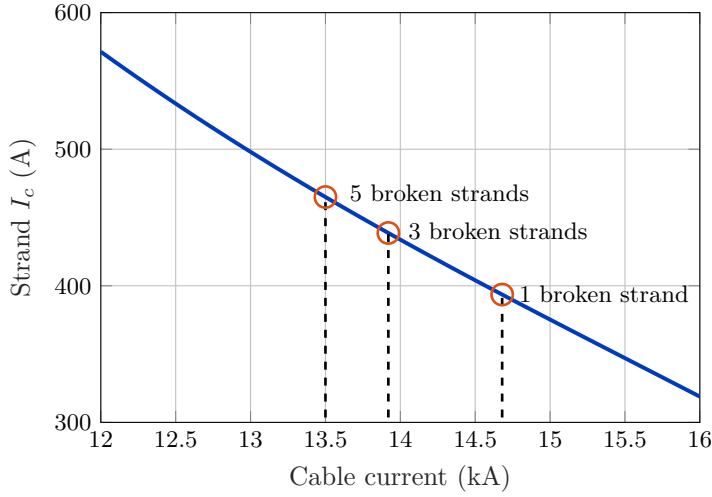


Figure 5.44: *Estimated strand critical current as a function of the overall cable current at 4.5 K. The measured quench currents for the three levels of damage with a ramp rate of 20 A s^{-1} are indicated with the vertical dashed lines. At higher damage levels the coil quenches at a lower cable current, which implies that the strands have a higher critical current at the moment of quench.*

summed value of the current in the overloaded strand is estimated from Figure 5.44. The simulation results show a similar behaviour as the measurement results, with indeed a higher NZPV at lower damage levels. The kink at the moment that the normal zone front leaves section B5-C5 is more pronounced in the simulation and even seems to be preceded by a slight acceleration. The overloaded strands pass through the high magnetic field region just before they leave the B-C section (see also Figure 3.14), causing the acceleration. This feature may well be absent in the measurement, due to an additional longitudinal variation in initial current distribution. As discussed in Section 4.2.3, the current imbalance caused by the defect, decays with distance (see for example Figure 4.7).

Also, the simulated NZPV value for the uniform case is about 40% lower than the actual measured one. This may be yet another indication of the presence of pre-existing damage in the cable during the reference test.

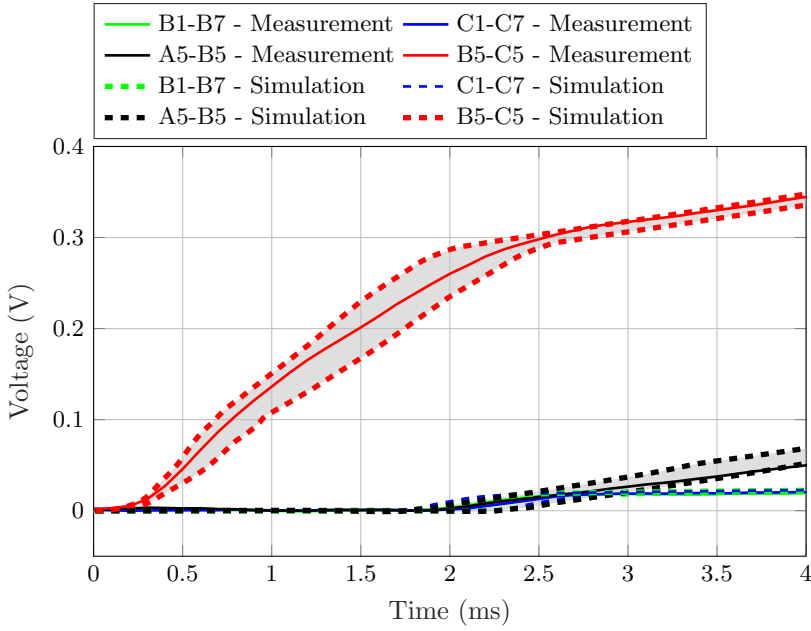


Figure 5.45: Comparison of measured (solid lines) and modeled (dashed envelopes) voltage tap signals during a quench in the pole-turn of the SMC coil. The envelopes show the predicted voltage range corresponding to a 10 A uncertainty in the estimated current carried by the saturated strands.

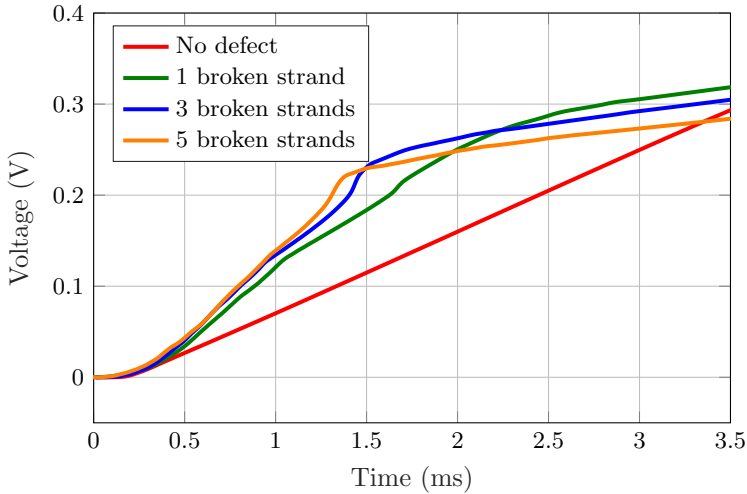


Figure 5.46: Influence of the damage level on the simulated resistive voltage versus time during a quench in the pole-turn of the SMC coil at 4.5 K.

5.5 Conclusion

In this chapter, the quench development and normal zone propagation of quenches in Nb₃Sn Rutherford cables in a magnet environment have been studied. A detailed study of the influence of several intrinsic and extrinsic variables on the time-evolution of a quench was performed. In particular, it was studied how these variables give rise to specific features in the data collected with the two main diagnostic tools for accelerator magnets: voltage taps and quench antennas.

During the development phase of a quench in one or a few strands, the normal zone is still expanding laterally in the cable cross-section. As the normal zone grows, it pushes current out of the quenching strands, which then accumulates near the zone edges. This process accelerates as more current accumulates and finally concludes once all strands in the cable cross-section are in the normal state. With voltage tap measurements, the development phase is observed characteristically as an accelerating voltage build-up followed by a ‘kink’ in the $V(t)$ curve that indicates the moment the full cable cross-section is in the normal state. From there on, the voltage tends to rise further linearly as the quench has entered its propagation phase. On the inductive quench antenna signals, this resistive kink corresponds to a prominent peak as it is the moment of maximum current redistribution.

The details of the initial quench development depend strongly on both the thermal and the electric inter-strand contact resistances. For both parameters, a higher value causes a slow down the lateral quench development. The electrical contact resistance has a stronger influence than the thermal one, and also affects the length-scale of the current redistribution, which reduces the magnetic dipole moment that is generated by the quench.

This magnetic dipole moment can be used to determine at which location in the cross-section of the cable the quench initiated, as long as one can assume that it originates from a single spot in the cable. The time integral of the dipole moment from its start up until the conclusion of the development phase is strongly linked to the quench start position. This way, the quench initiation position can be estimated without a labour-intensive comparison of the general shape of the quench antenna signals to all possible permutations calculated with the model.

In contrast with conductor-limited quenches, training quenches in the MQXF magnets studied typically involve initiation in a sizeable fraction of the strands at once. For localizing *training* quenches, an extra important parameter is thus the size of the initial perturbation. For several training quenches in MQXF magnets, the quench start position could be reconstructed by matching the features in the measured quench antenna signals with the simulated ones.

With a uniform initial current distribution, once the quench enters its propagation phase, the current distribution behind the normal front is dictated by the

magneto-resistance of the copper matrix. Once the steady-state propagating normal zone front is fully developed, it produces a ‘bow shock’ of accumulated current that preceeds the thermal normal front. When the magnetic field gradient in the cable is higher, as e.g. in the mid-plane turn of a dipole magnet, the magnitude of the accumulated current is larger. The current redistribution in the normal zone front, that moves from the high-field to the low-field side, can be modeled as a magnetic dipole moment traveling along with the normal front. As the normal zone front propagates past a series of consecutive pick-up coils, in each coil the same voltage is induced, but with a time delay between them.

In the case of a non-uniform initial current distribution, for example due to one or multiple degraded strands, the quench will propagate along the overloaded strands next to the degraded one(s). This significantly increases the NZPV and also affects the pick-up coil signals, as it generates a much larger magnetic dipole moment that moreover changes polarity as it travels along the cable. Due to the magnetic field gradient, the overloaded strands pass through the high-field and low-field side of the cable, which speeds up and slows down the normal zone propagation. This may be observed in the resistive voltage measured over the coil as a modulation on the $V(t)$ slope.

The quench analysis of the controlled SMC experiment with intentionally broken strands showed some counter-intuitive behaviour in which the observed NZPV increased with lower quench currents. From comparison with RuNe model predictions, this behaviour can be well explained by taking into account the fact that at lower overall quench current the strand critical current is higher, which means that the saturated strands adjacent to the defect carry actually a higher current thereby increasing the heat generated and therefore the NZPV.

Chapter 6

Conclusion

In this thesis, the effects are studied of local conductor damage in Nb₃Sn Rutherford cables that are used in accelerator type of magnets on the magnets' performance. In addition, measurement methods and tools are developed for advanced experimental diagnostics of such magnets. The underlying goal is to extract as much information as possible from such diagnostic equipment. To this end, comprehensive simulation efforts to characterize the electro-thermal behaviour of Rutherford cables under nominal - and defect conditions were performed. The simulation results were compared with measurement data from full-scale HL-LHC dipole - and quadrupole magnets and with dedicated experiments on a small race-track coil with controlled intentional damage.

First, in Sections 6.1 and 6.2, a brief summary of the numerical and experimental methods is given. Then, the main conclusions regarding current redistribution during normal operation (Section 6.3) and regarding quench initiation and normal zone propagation (Section 6.4) are presented, followed by concluding remarks in Section 6.5. For ease of reference, some of the key figures in the thesis are also repeated here.

6.1 Numerical techniques

Two simulation codes, **THEA** and **RuNe**, were used to study the thermal and electromagnetic behaviour of Nb₃Sn cables in accelerator magnets. The modelling efforts were focussed on two physical phenomena:

- Current redistribution in the cable around defective strands during a magnet current ramp.
- Quench development and normal zone propagation.

The existing code **THEA** was used to study current redistribution around a defect over long time - and length scales in Rutherford cables with locally damaged

strands.

In addition, a new 3D thermal-electric PEEC-FEM simulation code called **RuNe** was developed to study both current redistribution and quench behaviour. It has a similar modelling philosophy as the earlier model CUDI, but with a modified electrical network lay-out, now taking into account all current paths in the calculation of inductive voltages. To reduce calculation time, required to cover the large parameter space efficiently, RuNe employs hierarchical matrices that exploit the cable's periodicity. The resulting code gives a very detailed insight in normal zone propagation but also captures early quench development. The voltage signals measured with the pick-up coils in quench antennas and with voltage taps can be predicted quite accurately, so that RuNe proves to be a powerful tool to perform advanced data analysis and thus detailed diagnostics for studying magnet performance.

6.2 Experimental techniques

The two main diagnostic tools for assessing magnet performance are **voltage taps** and **quench antennas**.

Voltage build-up in series-produced accelerator magnets can often only be monitored over the coil ends. With differential measurements between coils parasitic inductive components can be eliminated, which makes it possible to detect the much smaller voltages specifically associated with conductor damage, even in 7 m long Nb₃Sn accelerator magnets. Using THEA, the hypothesis was confirmed that uniform damage causes a stable voltage at a constant-current plateau in the ramp sequence, while non-uniform damage causes a decaying voltage at constant current. In addition, it was shown that, depending on the size of the defect and on the inter-strand contact resistances, the absence of a detectable voltage does not guarantee the absence of a defect.

Two types of quench antenna systems for studying current redistribution in the Rutherford cable during quench transients are considered:

- A **local quench antenna** system with axially short pick-up coils proved to be efficient for longitudinal localization and tracking of a propagating normal zone front.
- A **harmonic quench antenna** designed for transverse localization of the quench origin allows for a detailed study of current redistribution during initial quench development and normal zone propagation.

A dedicated experiment was performed on a Short Model Coil (SMC) with intentionally interrupted strands. In the pole turn of the SMC, over a series of consecutive tests, 0, 1, 3 and 5 strands were interrupted. Extra voltage taps were positioned at strategic locations around the defect zone, which proved to be an

effective way to monitor the current redistribution process.

6.3 Current redistribution in damaged Rutherford cables

When a Rutherford cable has **local damage** in one or several of its strands, current will redistribute to bypass the damaged strands. Through simulation with both RuNe and THEA and with a dedicated experiment, we have shown that **this current redistribution process can be detected by the presence of a time-decaying component in the voltage** measured during the constant current plateaus of a step-wise current ramp sequence. Such decaying voltage signals were indeed measured on several HL-LHC magnets that also showed a sub-optimal quench performance. However, when the damage is affecting all strands equally, no decaying voltage component is present.

For a well-defined local defect and with well-known inter-strand resistances, the current redistribution and the corresponding V-I measurements can be quantified precisely using the model. Solving the inverse problem of localizing and quantifying the defect based on a limited amount of voltage measurements is difficult due to the many possible defect configurations.

The current redistribution process around a local defect is thus well-described and has the following features:

- Current redistribution around a local non-uniform defect can be considered to be a **diffusion process**. With time, the current imbalance in the cable extends further away from the defect position, which results in a decaying voltage as the current is redistributed over an increasing length.
- The **current mainly redistributes through the adjacent strands** rather than crossing strands, due to the typically much lower contact resistance and the higher mutual inductance between adjacent strand pairs.
- A higher adjacent inter-strand contact resistance R_a leads to a larger characteristic length of the current redistribution. Both, the magnitude of the measured **voltage** over the defect and the **characteristic length** of the current imbalance, are proportional to the **square root of R_a** .
- For commonly encountered values of R_a , in the range of $1\,\mu\Omega$ to $100\,\mu\Omega$, **significant current imbalance can extend over tens of meters** away from the defect location over the course of a nominal magnet ramp with a ramp rate of $10\,\text{A s}^{-1}$ to $20\,\text{A s}^{-1}$, see key Figure 6.1 (identical to Figure 4.7).
- The **number of damaged strands does not affect the characteristic length** of the current redistribution profile.

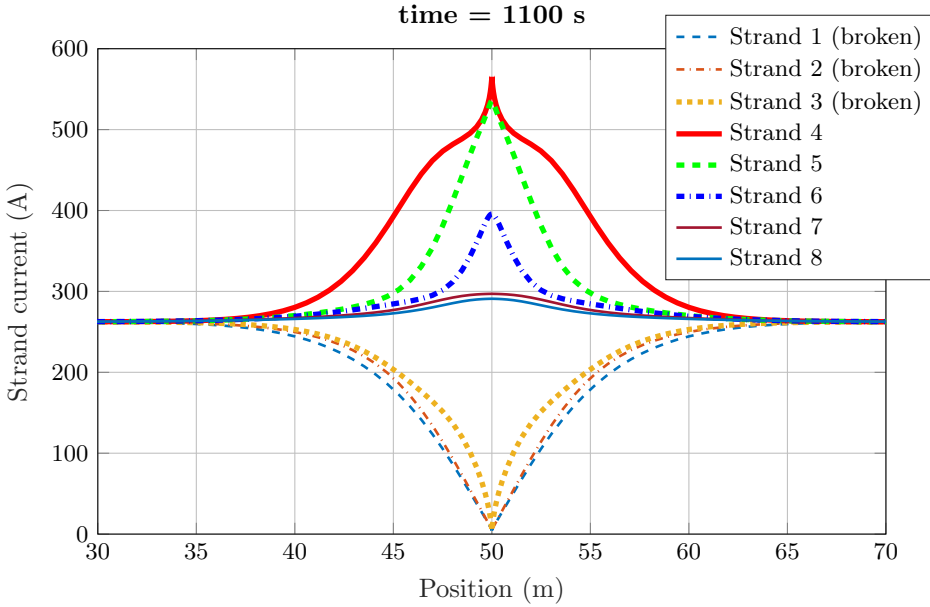


Figure 6.1: Calculated strand currents as a function of position along a 100 m long Rutherford cable with 16 strands (only 8 are shown because of symmetry), with 6 strands broken at 50 m, shown 50 s after the end of a ramp to 10.5 kA with a ramp rate of 10 A s^{-1} . The adjacent inter-strand contact resistance R_a is $5 \mu\Omega$. The local defect causes a current imbalance over a large section of the cable that expands in time.

- The magnitude of the voltage drop measured over the entire cable is proportional to the square of the number of adjacent broken strands.
- For *higher* ramp rates, the two strands directly adjacent to the damaged ones carry a *lower* excess current than for relatively low ramp rates, which leads to a delayed saturation of these strands.
- At a lower temperature, the current imbalance can rise to higher values leading to a thermally less stable situation.
- After a ramp down, the voltage drop over the damage location changes polarity close to the defect because the direction of the current is locally inverted.
- An analytic diffusion model shows that the voltage over a cable with a defect decays with time as $\frac{1}{\sqrt{t}}$ after a step-like ramp, and as $\sqrt{t} - \text{Re}(\sqrt{t - t_{\text{ramp}}})$ after a ramp of finite duration (with t_{ramp} the ramp duration).

The significant length over which the current redistribution process around a non-uniform defect occurs implies that **local damage can influence a magnet's stability over a large section**. For instance, a defect in a relatively low magnetic field area of the magnet can cause excess currents in strands far away in

a high magnetic field area, which are then more susceptible to (training) quenches. Similarly, a defect that causes a high current in strands that pass through a low magnetic field zone can lead to a self-field instability and thus also render the magnet susceptible to quench.

Relatively high ramp rates allow for less time for the current imbalance to diffuse away from the defect, making the **quench location during such high ramp rates the best indicator for the location of the damage.**

Current redistribution effects cause also lateral voltage differences between strands, which means that **the measured longitudinal voltage drop over a cable section depends on which strands are being probed** with the voltage taps. This is especially relevant for voltage taps placed in the winding pack close to a defect, where one can measure **negative voltage drops over conductor sections if a damaged strand is probed directly.**

If a cable has relatively low values for the adjacent inter-strand contact resistance R_a , in the order of $5\ \mu\Omega$ or less, it may be hard to detect a ‘small’ defect of just a few strands in a Rutherford cable with several tens of strands from V-I measurements. For example over a 40-strand cable ramped at a nominal rate of $10\ \text{A s}^{-1}$ to $20\ \text{A s}^{-1}$, the voltage difference will be about $0.5\ \mu\text{V}$ or less. Performing the V-I measurements at higher ramp rates can increase the redistribution-related voltage and makes it easier to detect.

The SMC coil with controlled damage was able to reliably reach its operational target **even with multiple saturated strands** during the measurement campaign. However, a strand in its saturated state is highly unstable due to its strongly reduced minimum quench energy. Judging the reliability and stability of a magnet by only considering the value of the quench current may thus not be the optimal way to diagnose cable damage. This has implications for magnet qualification criteria. It is recommended that besides testing whether the magnet is able to achieve a specified current, it must do so while staying below a certain threshold of the voltage difference on the steady plateau at that magnet current. In addition, the required quench current has to be sufficiently close to the short sample limit and not be limited to just the intended maximum operational current of the magnet.

6.4 Quench development and normal zone propagation

The quench process in a Nb_3Sn Rutherford cable can be divided in two distinct phases: the **initial development** and the subsequent **normal zone propagation**. Diagnostic signals during the development phase of a quench contain information on the event that initiated the quench. If a quench is **initiated in a single strand** or in a few strands, **an accelerating resistive voltage build-up**

is seen, as shown in Figure 6.2 (identical to Figure 5.2). This accelerating phase is then **followed by a kink in the $V(t)$ curve**, after which the voltage rises linearly. The kink marks the transition from the development - to the propagation phase.

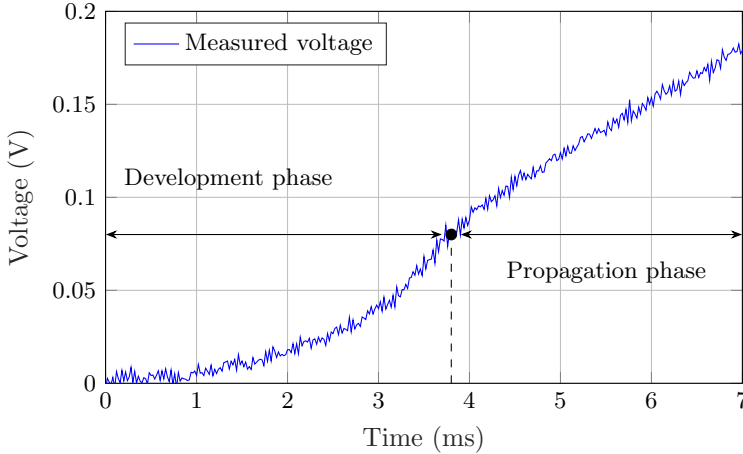


Figure 6.2: Voltage versus time as measured over a coil of a Nb_3Sn quadrupole magnet (MQXFBP2). The voltage build-up initially accelerates (i.e. $\frac{d^2V}{dt^2} > 0$) until the normal zone has expanded over the entire cross-section. Then the voltage continues to rise linearly as the fully-formed normal zone front propagates along the cable in both directions.

6.4.1 Quench development

The duration of the initial development phase and thus the voltage level at which the kink occurs, strongly depends on the electrical contact resistance between the strands and to a lesser extent on the thermal contact resistance. For weakly-coupled strands, the development of the normal zone into a propagating normal zone front takes longer and the acceleration in the voltage build-up is more pronounced. By comparing RuNe simulations with experimental data, an estimate of the local values of the inter-strand contact resistances can be made.

If the quench starts immediately with a linear voltage rise, this likely indicates that it initiated in many strands in parallel at once. For instance, this is observed during training quenches, where **the development phase is cut short because many strands in the same location turn normal simultaneously triggered by the training event**. This implies that, during magnet training, in many cases Rutherford cables cannot rely on the so-called ‘cable stability’ regime as defined for minimum quench energy measurements, were a normal zone in one strand or a few strands can still shrink due to current redistribution between strands.

The current redistribution associated with a quench generates a magnetic

dipole moment that can be picked up by quench antennas in the magnet bore. By matching the shapes of measured and simulated quench antenna signals, an estimate of the quench's transverse start location in the cable cross-section can be made. This method can also be applied to training quenches to find the group of strands that turn normal initially, but is computationally laborious. A more straightforward method of finding this location is by determining the time integral of the magnet dipole moment during the development phase, which strongly depends on the initial quench start position.

The effect on the quench antenna signals of the collapse of the superconductor's magnetization, occurring on an intra-strand filamentary level, during a quench was found to be small compared to the redistribution of transport current, which occurs on a much larger cable level.

The simulation code RuNe has demonstrated to be a very effective tool to reconstruct quench origins for both conductor-limited quenches as well as training quenches.

6.4.2 Normal zone propagation

During the normal zone propagation phase, the current redistribution and the shape of the thermal normal zone front depend strongly on the background magnetic field. The normal zone propagates faster on the high magnetic field side of the cable compared to the low magnetic field side due to the corresponding gradient in the strands' critical current, causing a propagation front that is at an angle with respect to the longitudinal cable direction. The current distribution in the normal zone itself, once the propagation front has passed, is governed by the magneto-resistance effect in the copper matrix of the strands.

A traveling normal zone front in a cable with a **uniform current distribution** will result in a succession of **identical signals** recorded by longitudinally adjacent pick-up coils of a quench antenna in the magnet bore.

When a **non-uniform current distribution** is present in the cable, as for example caused by a **local defect**, the normal zone propagation is strongly affected, see Figure 6.3. The normal zone front now **propagates first along the overloaded strands, which causes a significant increase in the normal zone propagation velocity**. This also increases the magnitude of induced signals in the quench antenna. Moreover, the signals on adjacent pick-up coils are no longer identical and may have an alternating polarity, depending on the ratio between the cable's twist pitch and the pick-up coil length. These types of anomalous readings on quench antenna signals are thus a tell-tale indicator of conductor damage.

For lower numbers of adjacent saturated strands, the normal zone propagation velocity initially decreases with increasing electric inter-strand contact resistance as the correspondingly longer current transfer length causes the overloaded strand

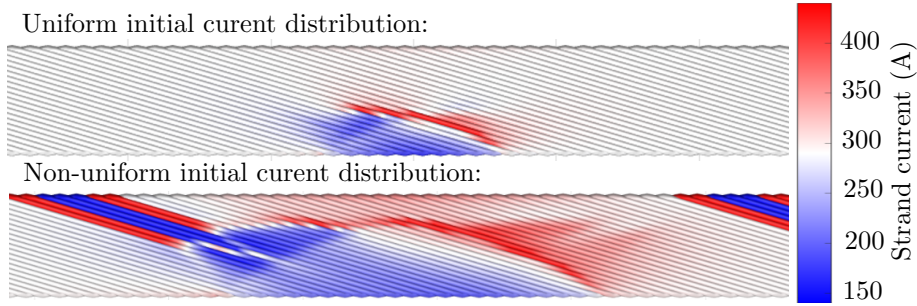


Figure 6.3: *Simulated strand currents at 1.5 ms after the start of the normal zone in a cable in the pole turn of an 11 T dipole magnet. In the case of a non-uniform initial current distribution, the normal zone propagates along the overloaded strands, resulting in an increased normal zone propagation velocity.*

to carry less current at the thermal normal front. However, when the contact resistance increases further to an uncommonly high level, the inter-strand current transfer pre-heats the cable ahead of the normal zone front, which causes the propagation velocity to increase again. The normal zone propagation velocity increases as more adjacent strands are saturated.

The simulation predictions were indeed observed in the SMC experiment. The normal zone propagation velocity became significantly higher after local damage was introduced in the magnet. It was also confirmed that the normal zone propagation velocity increased as more strands were broken, even though the magnet quenched at a lower current. The normal zone propagation velocity is thus shown to depend on the overall current in the saturated strands at either side of the defect, which have a higher critical current at lower magnet current and hence magnetic field.

6.5 Concluding remarks

In conclusion, an improved set of diagnostic tools for performance studies and characterization of Nb₃Sn Rutherford cables in accelerator magnets has been developed. Likely, the most serious performance degradation stems from local conductor damage that introduces a non-uniform current distribution over cable lengths that are comparable to the magnet size. This negatively affects the margin to quench and increases the normal zone propagation velocity. To investigate the root-cause of such non-uniform performance degradation, high-precision and low-noise voltage measurements and quench antenna measurements are required. Due to the large parameter space, multiple degradation modes are possible. With the newly developed RuNe simulation code, degradation assumptions can be studied and validated as well.

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Nomenclature

Symbols

A	Surface area	m^2
A_{cu}	Surface area copper	m^2
A_{n}	Magnetic field skew harmonic coefficients	T
A_{s}	Surface area of strand	m^2
b	Temperature scaling factor therm. cont. res.	-
B_{c}	Critical magnetic field (Type I sc)	T
B_{c1}	First critical magnetic field (Type II sc)	T
B_{c2}	Second critical magnetic field (Type II sc)	T
B_{d}	Dipole magnetic flux density	T
B_{n}	Magnetic field normal harmonic coefficients	T
B_{r}	Radial magnetic field component	T
$\dot{B}_{\perp}$	Perpendicular field change rate	T s^{-1}
ΔB	Magnetic field gradient	T
c_{p}	Specific heat	$\text{J kg}^{-1} \text{K}^{-1}$
$C_{1/2}$	Voltage coefficient of double exponential fit	s
C_{n}	Magnetic field harmonic	T
C_{th}	Heat capacity	J K^{-1}
d	Distance between strands	m
$d\tau$	Rate of change of τ_{var}	$\sqrt{\text{s}}$
D	Diffusivity	s m^{-2}
D_{b}	Distance between blocks of elements	m
e	Electron charge	C
E	Electric field	V m^{-1}
E_0	Electric field criterion	V m^{-1}
E_{b}	Particle beam energy	TeV
$\mathcal{E}$	Electromotive force	V
f_{d}	Discretization factor	-
f_{t}	Transport current correction factor	-
F	Inter-strand contact function	m^{-1}
F_{res}	Residual function	-
$\mathcal{F}$	Weighting function	-

g_a	Adjacent inter-strand contact conductance	S m^{-1}
g_c	Crossing inter-strand contact conductance	S m^{-1}
h	Planck's constant	$\text{m kg}^2 \text{s}^{-1}$
$H_{i,j}$	Thermal inter-strand impedance	K W^{-1}
I_{adj}	Adjacent strand current	A
I_{broken}	Broken strand current	A
I_c	Critical current	A
I_q	Quench current	A
I_{sc}	Current in superconductor	A
I_{ss}	Short sample limit current	A
I_{tot}	Total cable current	A
J_c	Critical current density	A m^{-2}
J_t	Transport current	A
k	Thermal contact conductance	$\text{W m}^{-1} \text{K}$
K	Surface current density	A m^{-1}
l_s	Length of strand element	m
L_b	Luminosity of particle beam	$\text{cm}^{-1} \text{s}^{-2}$
L_e	Length of element	m
L_{em}	Characteristic self-inductance	H
L_l	Leakage inductance per unit length	H m^{-1}
L_m	Mutual inductance per unit length	H m^{-1}
L_p	Twist pitch	m
L_s	Self-inductance per unit length	H m^{-1}
m	Magnetic moment	A m^2
M	Superconductor magnetization	A m^{-1}
M_{ij}	Mutual inductance between elements	H
M_{int}	Time integral of dipole moment	$\kappa \text{A m}^2 \text{s}$
M_{hollow}	Magnetization hollow filament	A m^{-1}
M_{round}	Magnetization value round filament	A m^{-1}
n	n-value of power law of sc. to normal transition	-
n_{ant}	Number of turns in quench antenna	-
n_s	Number of strands	-
N_e	Number of elements	-
p_{ext}	External heating	W
p_{int}	Internal heating	W
q_i	External strand heating	W
$q_{\text{Joule},i}$	Internal joule heating	W
r	Strand radius	m
r_0	Reference radius	m
r_a	Radius of particle accelerator	km
r_c	Radial position of dipole moment	m
r_{ij}	Distance between elements	m
r_{in}	Inner radius hollow filament	m
r_{out}	Outer radius hollow filament	m

R_a	Adjacent inter-strand contact resistance	$\mu\Omega$
R_c	Crossing inter-strand contact resistance	$\mu\Omega$
$R_{c,edge}$	Crossing inter-strand contact resistance of edge	$\mu\Omega$
R_d	Radius of block of elements	m
R_{th}	Characteristic thermal impedance	$K W^{-1}$
t_1	Thickness of cable thin edge 1	m
t_2	Thickness of cable thin edge 2	m
t_{ramp}	Duration of ramp	s
T_c	Critical temperature	K
V_{adj}	Voltage of adjacent strand	V
V_e	Voltage over strand element	V
V_{taps}	Voltage over voltage taps	V
w_c	Cable width	m
z_c	Transverse position of dipole moment (complex notation)	m
z_{vt}	Distance from voltage taps to defect	m

Greek symbols

α	Scaling factor thermal contact resistance	$W m^{-1} K^{-(1+b)}$
γ	Inverse of time-step	s^{-1}
Γ	FEM domain boundary	-
η	Admissibility criterion	-
θ_c	Angle of dipole moment	rad
θ_k	Keystone angle	rad
θ_l	Cable lay angle	rad
κ	Thermal conductivity	$W m^{-1} K$
λ	Quench antenna sensitivity	-
μ_0	Permeability of free space	$H m^{-1}$
ν	Ratio inner/outer radius hollow filament	-
ρ	Mass density	$kg m^{-3}$
ρ_{cu}	Electrical resistivity copper	Ωm
σ_n	Singular value	-
$\tau_{1/2}$	Time constant of double exponential fit	s
τ_{em}	Electro-magnetic time constant	s
τ_{th}	Thermal time constant	s
τ_{var}	Variable time ‘constant’	s
ϕ_c	Angular position of dipole moment	rad
ϕ_i	Nodal shape function	-
Φ	Magnetic flux	Wb
Φ_0	Magnetic flux quantum	Wb
Ω	FEM domain	-

Vectors

$\mathbf{A}$	Magnetic vector potential	Tm
$\mathbf{B}$	Magnetic field	T
$\mathbf{f}$	Load vector	-
$\boldsymbol{\theta}$	Vector of nodal temperatures	K
$\mathbf{i}$	Vector of element currents	A
i_{ext}	Externally applied current	A
$\mathbf{J}$	Current density	A m ⁻²
$\mathbf{m}$	Magnetic dipole moment (polar coord.)	A m ²
$\mathbf{m}$	Magnetic dipole moment (3D cart. coord.)	A m ²
$\hat{\mathbf{n}}$	Perpendicular unit vector	-
$\mathbf{p}_e$	Heat load on elements	W
$\mathbf{p}_{\text{ext}}$	Externally applied power	K
$\mathbf{p}_n$	Heat load on nodes	W
$\mathbf{r}$	Location vector	m
$\mathbf{v}$	Vector of nodal voltages	V
$\mathbf{v}_{\text{ext}}$	Externally applied voltage	V
$\Delta \mathbf{v}_{\text{ext}}$	Externally applied voltage	V m ⁻¹
$\mathbf{v}_e$	Resistive voltage over strand elements	V
$\mathbf{v}_r$	Resistive voltage over all elements	V

Matrices

$\mathbf{A}_{mn}$	Rectangular matrix	-
$\tilde{\mathbf{A}}_{mn}$	Compressed rectangular matrix	-
$\mathbf{C}$	Mass matrix	-
$\mathbf{G}$	Conductance matrix (per unit length)	S m^{-1}
$\mathbf{J}$	Jacobian matrix	-
$\mathbf{K}$	Stiffness matrix	-
$\mathbf{L}$	Inductance matrix (per unit length)	H m^{-1}
$\mathbf{L}_m$	Inductance matrix	H
$\mathbf{L}_{nf}$	Inductance matrix (near-field contribution only)	H
$\mathbf{L}_{\mathcal{H}}$	Hierarchical inductance matrix	H
$\mathbf{M}_{e \rightarrow n}$	Element-to-node connectivity matrix	-
$\mathbf{M}_{kcl}$	Kirchhoff's current law incidence matrix	-
$\mathbf{M}_{kvl}$	Kirchhoff's voltage law incidence matrix	-
$\mathbf{P}_l$	Preconditioner matrix	-
$\mathbf{R}$	Resistance matrix (per unit length)	$\Omega \text{ m}^{-1}$
$\mathbf{R}_m$	Resistance matrix	Ω
$\mathbf{R}_{nl}$	Non-linear element resistance matrix	Ω
Σ_{mn}	Diagonal matrix with singular values	-
$\tilde{\Sigma}_{mn}$	Reduced diagonal matrix with singular values	-
$\mathbf{U}_{mn}$	Matrix with left singular vectors	-
$\tilde{\mathbf{U}}_{mn}$	Reduced matrix with left singular vectors	-
$\mathbf{V}_{mn}$	Matrix with right singular vectors	-
$\tilde{\mathbf{V}}_{mn}$	Reduced matrix with right singular vectors	-